

## Response to Reviewer #2

First of all, I would like to thank this reviewer for his or her thoughtful comments and the potential simplifications he or she has pointed out.

A crucial statement can be found towards the end of the first page of the report: “all the developments around the so-called ‘exponential sampling’ seem rather useless”, and a few lines later this reviewer proposes dyadic expansions *ad hoc*.

Let me answer this misunderstanding with the following figure, which should explain why exponential sampling is the foundation of the whole article:

Basic Building Block	Bernoulli $B(p)$	
↓	Ordinary Sampling	<b>Exponential Sampling</b>
Derived Distribution	Binomial $B(n, p)$	Weaver $W(n, p)$
↓	(Standardized) Limit	
Limit Distribution	Normal $N(0, 1)$	Weaver’s hem $W(p)$

From a combinatorial point of view, there’s Pascal’s triangle on the left-hand side, and the multiplicative ‘triangle’ introduced in the manuscript on the right-hand side. From a dynamic system perspective, the paths split and merge on the left, but they only split on the right, which is a crucial feature of chaos and turbulence (with vortices, eddies or boxes multiplying, but not fusing).

Moreover, starting with two constant random variables  $X_0 \equiv 0$  and  $X_1 \equiv 1$ , exponential sampling readily *implies* dyadic expansions. The latter expansions are also crucial for (non-constant) random variables  $X_0, X_1$ , such that  $EX_0 \neq EX_1$ , since the expected values can be treated (w.l.o.g) in the same way. Building on this structure, the logical - and by no means ‘abrupt’ - next step is the general treatment in Section 6, which explicitly includes the random variables’ ‘inner variability’.

I assume that this reviewer is extremely familiar with cascades, the corresponding models ( $\alpha, \beta, p$  and their variants), and many other details of nonlinear processes. Perhaps that is why he seems to have difficulties in grasping the description of the process in the introduction. In algorithmic terms the model studied is:

- 0) There are two populations, and  $n = 0$ .
- 1) Select one of the populations at random
- 2) Draw an iid sample of size  $2^n$  from that population
- 3)  $n = n + 1$
- 4) Proceed to step 1)

Since the first reviewer and all other readers of this manuscript (at least 20, according to Researchgate) have not had any difficulties at this point, I presume that there is no ‘missing information’ here.

At least in my understanding, the antonym of complicated is not ‘trivial’, but simple. Starting with an elementary building block, transparent sampling procedures produce basic distributions with straightforward properties ( $B(n, p), W(n, p), \dots$ ). At least with the wisdom of hindsight, these properties may appear ‘trivial’; however, they nevertheless require proof. Moreover, it turns out that, despite - or rather due to - their simple structure,

the Bernoulli and its descendants are pervasive and can be generalized considerably (see the last section for many potentially interesting ‘extensions’ of exponential sampling).

Finally, I think most scientists would agree that it is worth the effort to reduce complicated matters to reasonable first principles (see, for instance, Feynman’s ‘prepare a freshman lecture’ test’, or Lovejoy and Scherzer 2013, “The weather and climate”, Chap. 2). In this vein, the referee rightly criticizes that ‘tedious algebra’ contributes to the length of the manuscript, which the author would be glad to shorten, rectify and *simplify* (see my response to Reviewer #1).