Nonlin. Processes Geophys. Discuss.,
https://doi.org/10.5194/npg-2019-16-AC1, 2019

Nonlinear Processes in Geophysics Discussions

Interactive comment on "Joint state-parameter estimation of a nonlinear stochastic energy balance model from sparse noisy data" *by* F. Lu et al.

F. Lu et al.

feilu@math.jhu.edu

Received and published: 28 June 2019

Dear Dr. Grudzien,

We thank you for carefully reading the manuscript and for the valuable feedback.

We have added the references in your comment 1) and corrected the typos mentioned in your comments 2) and 4).

About your comment 3): "Page 21, lines 19 - 25: I think this discussion is very interesting and useful to the reader. I would like this to be expanded to elaborate on the consequences for the analysis of paleo-climate in terms of the contributions of different

processes to past states, and how this may affect the inferences we may wish to make based on such models. Likewise, I would like this to elaborate further on the more realistic case of nonlinear observation operators relating proxy measurements to the actual climate system, and how the reconstruction of the climate and parameters will be affected by these additional complications. I think the work will benefit from a longer conclusion and discussion of the implications of the results for more realistic modeling settings."

We extended our discussion of the implications of our results for paleoclimate reconstructions by adding a subsection '5.4 Implications for paleoclimate reconstructions'. This section covers steps to be taken towards more realistic observation operators and discusses the implications for reconstructing physically meaningful state estimates and inference of parameters for energy sinks and sources. In turn, we shortened the respective paragraph in the conclusions.

5.4 Implications for paleoclimate reconstructions
Our analysis shows that assessing the well-posedness of the inverse problem of parameter estimation is a necessary first step for paleoclimate reconstructions making use of physically motivated parametric models. When the problem is ill-posed, a straightforward Bayesian inference will lead to biased and unphysical parameter estimates. We overcome this issue by using regularized posteriors, resulting in parameter estimates in the physically reasonable range with quantified uncertainty. However, it should be kept in mind that this approach relies strongly on high quality prior distributions.

The ill-posedness of the parameter estimation problem for the model we have considered is of particular interest because the form of the nonlinear function $g_\theta(u)$ is not arbitrary but is motivated by the physics of the energy budget of the atmosphere. The fact that wide ranges of the parameters θ_i are consistent with the "observations" even in this highly idealized setting indicates that surface temperature observations themselves may not be sufficient to constrain physically-important parameters such as albedo, graybody thermal emissivity, or air-sea exchange coefficients separately. While state-space modeling approaches allow reconstruction of past surface climate states, it may be the case that the associated climate forcing may not contain sufficient information to extract the relative contributions of the individual physical processes that produced it. Further research will be necessary to understand whether the contribution of e.g. a single process like graybody thermal emissivity can be reliably estimated from the observations if regularized posteriors are used to constrain the other parameters of $g_\theta(u)$.

If the purpose of using the SEBM is to induce physical structure into the state reconstructions without specific concern regarding the parametric form of g, reparametrization or nonparametric Bayesian inference can be used to estimate the form of the nonlinear function g but avoid the ill-posedness of the parameter estimation problem. This is an option if the interest is in the posterior of the climate state and not in the individual contributions of energy sink and source processes.

State-of-the-art observation operators in paleoclimatology are often non-linear and contain non-Gaussian elements (Haslett et al., 2006; Tolwinski-Ward et al., 2011). A locally linearized observation model with data coming from the interpolation of proxy data can be used in the modeling framework we have considered, along with the assumption of Gaussian observation noise. Alternatively, it is also possible to first compute off-line point-wise reconstructions by inverting the full observation operator, potentially interpolating the results in time, and using a Gaussian approximation of the point-wise posterior distributions as observations in the SEBM (e.g. Parnell et al., 2016). We anticipate that such simplified observation operators will limit the accuracy of the parameter estimation, but that the regularized posterior would still be able to distinguish the most likely states and quantify the uncertainty in the estimation. Directly using non-linear, non-Gaussian observation operators requires a more sophisticated particle filter as optimal filtering is no longer possible. Such approaches will increase the computational cost and face difficulties avoiding filter degeneracy.

The last paragraph in the conclusions is replaced by:
This work shows that it is necessary to assess the well-posedness of the inverse problem of parameter estimation when reconstructing paleoclimate fields with physically motivated parametric stochastic models. In our case, the natural physical formulation of the SEBM is ill-posed. While climate states can be reconstructed, values of individual parameters are not strongly constrained by the observations. Regularized posteriors are a way to overcome the ill-posedness but retain a specific parametric form of the non-linear function representing the climate forcings.

References

Haslett, J., Whiley, M., Bhattacharya, S., Salter-Townshend, M., Wilson, S. P., Allen, J., Huntley, B., and Mitchell, F.: Bayesian paleoclimate reconstruction, Journal of the Royal Statistical Society, Series A, 169, 395–438, 2006.

Parnell, A. C., Haslett, J., Sweeney, J., Doan, T. K., Allen, J. R., and Huntley, B.: Joint Palaeoclimate reconstruction from pollen data via forward models and climate histories, Quarternary Science Reviews, 151, 111–126, 2016.

Tolwinski-Ward, S. E., Evans, M. N., Hughes, M. K., and Anchukaitis, K. J.: An efficient forward model of the climate controls on interannual variation in tree-ring width, Climate Dynamics, 36, 2419–2439, 2011.

Interactive comment on Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg-2019-16, 2019.