Nonlinear Processes
in Geophysics

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Discussions

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Interactive comment on "Explosive instability due to flow over a rippled bottom" *by* Raunak Raj and Anirban Guha

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We thank you for your useful comments, suggestions and constructive criticisms, which have significantly helped us in improving the paper. We have split your comments into several parts (marked in red), so that all the queries could be answered in a point-wise manner. Responses to your comments are marked in red in the paper.

Opening comments: The manuscript at hand presents a theory about resonant triadic interactions for water waves propagating on top of a sinusoidal bottom, in the presence of a linearly sheared current. In the absence of current, this interaction is well known, and the only possible interaction involves a propagating wave, a reflected one, and

the bottom undulations. In this work, the authors investigate the effect of a linearly sheared current and investigate the possible existence of an explosive instability (i.e. unboundedly growing with time). Indeed, since the dispersion equation admits new possible solutions, various triadic interactions are now made possible. At the end of the manuscript, a short discussion is introduced about this explosive instability in the presence of a two-layered flow, explaining this instability is more likely to occur in such configurations. My overall impression of the manuscript is not very good. It is unclear, and for this reason, the point made by the authors is rather unconvincing.

(1) The first reason concerns the way the dispersion equation is addressed. The authors present the roots of the dispersion equation as the intersection of the two branches named SG+ and SG−, but I could not find any definition of these two curves. As far as I understand, for each value of k, the authors find the two roots of equation 2.2, and then add kU to each of these roots, obtaining the curves SG+ and SG−. Unfortunately, in that process, two other branches are neglected (the branches $\omega_i n - kU$). Indeed, it is well known that even on a linearly sheared current, the dispersion equation admits four distinct solutions. This procedure has a strong impact on the following discussion, since the reader never realizes which wave is considered (as far as I understand, the counter propagating wave is always excluded, since the intrinsic frequency $\omega_i n = \omega + kU$ is not even considered here). And this is not a minor remark. It is important, I believe, to understand which waves are considered and discussed here, by comparison with the classical four solutions of the dispersion equation (see e.g. the review paper by Peregrine). Anyway, a classical way to proceed with this difficulty is to consider both $k < 0$ and $k > 0$, and seek for positive values of ω. I assume this modification would possibly impact the results presented here (possibly, equations 2.6 and 2.7 might be affected, and the overall discussion impacted.

Our response: Kindly note that we have actually mentioned in the paper what $\mathcal{SG}^+$

and $\mathcal{SG}^-$. It appears below Eq. (2.2): "*The positive intrinsic frequency branch has been labeled as $\mathcal{SG}^+$, while the negative one as $\mathcal{SG}^-$.*" We have assumed $k > 0$ without loss of generality, as apparent from Fig 2. The convention $\omega > 0$ could also be taken, but have chosen the former.

The dispersion relation obtained for waves on a sheared flow is given as

$$\omega_{in}^2 + \Omega \tanh(kH)\omega_{in} - gk \tanh(kH) = 0,$$

where, $\omega_{in} \equiv \omega - Uk$. In other form, this can be written as,

$(\omega - Uk)^2 + (\omega - Uk)\Omega - gk \tanh kH = 0$. This is an exact dispersion relation with no approximation or assumptions. It can be observed that if (ω_0, k_0) is a solution of above equation, then $(-\omega_0, -k_0)$ will also be a solution. In other words, the dispersion relation is symmetric about the origin. If we plot this dispersion relation, allowing k and ω to take any real value, then the dispersion relation will have 4 branches: $(|\omega|, |k|)$, $(|\omega|, -|k|), (-|\omega|, |k|)$ and $(-|\omega|, -|k|)$. As mentioned above, the two branches in the $k < 0$ plane are simply the reflection of branches in the $k > 0$ plane about the origin. We therefore can ignore one half of the dispersion curve which lies on $k < 0$ plane. Thus both $(|\omega|, |k|)$ and $(-|\omega|, -|k|)$ denote waves propagating along the positive x direction (+ve phase speed) and both $(|\omega|, -|k|)$ and $(-|\omega|, |k|)$ denote waves propagating along the negative x direction (-ve phase speed).

You have correctly mentioned that "Anyway, a classical way to proceed with this difficulty is to consider both $k < 0$ and $k > 0$, and seek for positive values of ω". We have done a very similar thing but in a different way. Rather than considering both $k < 0$ and $k > 0$ and fixing $\omega > 0$, we have fixed $k > 0$ and have considered both $\omega > 0$ and $\omega < 0$. In the convention suggested by you, $k < 0$ will denote counter-propating wave while in the convention used by us, $\omega < 0$ denotes the counter-propagating wave. We hope that the confusion regarding the convention is now addressed.

We also add that it is very common practice to consider only $k > 0$ plane while studying

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the dispersion curves. See for example, all the dispersion curves in Cairns (1979), Carpenter et al. (2011). However, just to clarify this point, in the revised version, we have added the following sentence after equation (2.2):

For this paper, without a loss of generality, we have considered a positive k and have allowed the frequency ω to be either positive or negative. As a consequence, $\omega > 0$ means a positively travelling wave while $\omega < 0$ means a negatively travelling wave in the stationary reference frame.

(2) The second point which remained dark concerns the obtention of equations 2.9. These equations are just dropped into the paper, without any reference, neither theoretical background.

Our response: The general triad formulation is well known and can be found, for example, in Craik (1988). In this paper, we are investigating a special kind of triad - the Bragg triad, where one of the constituent 'waves' is the bottom ripple. The details of the derivation has been provided in our recent paper, Raj and Guha (2019). We have made some small rewordings around equation (2.9) so that the reader knows where to find the Bragg resonance equations. We hope this would avoid both confusion as well as repitition.

We only know that a time dependence is assumed for both a_1 and a_2, but this sounds very surprising. What exactly is the physical problem at hand?

The physical problem at hand is that two waves are passing over a bottom topography and both of them are exchanging energy with each other via the mediation of the bottom ripples (as if the bottom ripple is a constant amplitude, stationary wave which forms a triad with the two real waves). In doing so, the amplitude of one of the constituent wave usually increases and that of the other wave decreases. However, for

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the particular case of 'explosive growth', the amplitudes of both the waves increase. Apparently it may appear as a violation of the conservation of energy, but it isn't; see Craik and Adam (1979). For clarity, we have included a sentence in the last paragraph of the first section:

Here, the system consists of a surface wave propagating over a rippled bottom topography and the fluid flowing with a mean velocity profile as shown in figure 1(a). For explosive instability, we expect this wave to grow along with its explosive counterpart while simultaneously conserving the energy of the system.

How is it possible that both these amplitudes suffer a time evolution simultaneously? Aren't we considering forcing by an incident wave (which would certainly have a constant amplitude)?

No, there is no other incident wave of constant amplitude. Firstly, it is well known that **three** different waves can exchange energy with each other with amplitude of all the waves changing (known as wave triad interaction; see Craik (1988)). Secondly, it is also known that there can be a case that **three waves interact but amplitude of one stays constant** and other two register an exponential growth (known as parametric subharmonic instability (PSI)). This is nothing but a subcase of the first case i.e. wave triad interaction. Thirdly, it is also known that only **two** different waves can exchange energy with each other provided that there is a bottom ripple present (Bragg resonance). Our paper is a sub-case of the third case in which the two waves interact with the bottom and both the waves register a growth.

This remark is also important. It is almost impossible for the reader to reach a clear idea of the problem considered, including its boundary conditions, or its spatial extension. These two points seem of major importance to me, and the impression of unclarity remains valid all along the manuscript. Therefore, I cannot recommend this manuscript for publication in its present form.

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We hope that now we have been able to clarify your doubts.

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