Nonlin. Processes Geophys. Discuss.,
https://doi.org/10.5194/npg-2018-45-AC1, 2019

Nonlinear Processes in Geophysics
Discussions
Open Access

Interactive comment on "A Bayesian Approach to Multivariate Adaptive Localization in Ensemble-Based Data Assimilation with Time-Dependent Extensions" *by* Andrey A. Popov and Adrian Sandu

Andrey A. Popov and Adrian Sandu

apopov@vt.edu

Received and published: 28 January 2019

1 Additions

We have added a citation to our own problems package that was used in the creation of this paper. We have also added rough results about the amount of Lyapunov exponents and fractal dimension of the QGSO model.

Some additional citations other than the ones asked for by the referees have been

added in order to paint a more complete picture of the current state of knowledge.

A sentence has been added thanking the referees for their input into the quality of the paper.

Oracles have been clarified a bit and the introduction has been expanded.

Some duplicate information has also been removed.

2 Response to Referee 1

We thank the referee for the many technical suggestions.

2.1 Specific Comments

> *1. The manuscript requires some rearrangement. It is unclear as to why the introduc- tion began by describing a model. This part is more suited for the "Background" section.*

This makes sense. Done.

> *2. The motivation, objectives, and methods of the study should be clearly stated in the introduction. Some of these are actually discussed in the "Conclusions." Please make appropriate revisions.*

The introduction has been fleshed out to include more of our motivations and aims.

> *3. Expand your literature review as to include other studies that have done applications of adaptive localization in ensemble methods*

This has been done, in conjunction to the recommendations of referee 2.

> *4. P5: the uncertainty in space is represented by $pi(x|y,v)$, but according to equation (12), it should read $pi(x|y,v)$*

We are not sure what this comment is referring to.

> *5. P6: for equation (15), you may want to add that the chain rule was also used, in case you want to reach out to students, as derivations might not be too trivial*

This is reasonable, and has been done. An additional comment about symmetric semi-positive definite matrices was also added.

> *6. P7: equation (18) takes the form of an EnVar-like cost function. Comment on the applicability of your univariate or multivariate adaptive localization technique for hy- brid EnVar data assimilation systems and also discuss limitations*

An additional comment has been made discussing this.

> *7. P9, L3-10: Clarify if univariate localization functions were used and whether the extension of the localiza- tion matrix was done in the present study. Would this help in making your methodology suitable for testing with more complex geophysical models?*

We have clarified that we did not use multiple localization functions in this paper, removing doubt that multivariate localization was used. We are not sure how this point

related to more complex geophysical models, however we have clarified that point separately.

> *8. P10, L11-17: How do oracles compare to optimal parameter estimation as employed in control theory, with many applications in variational data assimilation?*

A comment was added clarifying that oracles are a type of minimal variance estimator of a full space in a restricted space.

> *9. P11, L22: Comment on the choice of 10 members for the ensemble and clarify if varying this number can impact your results*

A comment was added clarifying the choice of 10 ensemble members in Lorenz '96. The question of varying the amount of ensemble members has been explored in other literature, and is outside the scope of this paper.

> *10. There are some mathematical notation typos, please revise all equations thoroughly, and make sure to conform to the mathematical notation standards of NPG for all scalars, vectors, matrices, etc.*

All mathematical notation was changed to conform with the NPG guidelines, which mainly meant changing vectors to be boldface italics. The only Exception being the localization matrix ρ, as we could not find a way to have non-italic greek letters in conventional latex. Some other notation has been clarified (like indexing into columns using MATLAB syntax) in order to improve readability.

> *11. Given the results with a simple geophysical model, provide a brief overview on how this methodology could be tested with a complex numerical weather prediction model (e.g. regional/convection allowing mod- els)?*

A sentence was added in the conclusion about the possibility of applying this approach to the WRF model.

> *12. Provide more quantitative estimates in the "Results" and in the "Conclusions" sections*

Quantitative estimates were previously provided in the figures. The figure captions have not been changed, but discussion about the results has also been provided where the figures are cited, with some additional quantitative estimates.

2.2 Technical Corrections

All relevant typos have been addressed. Some comments in the typo section would have erroneously changed the semantics of several key statements and have thus been ignored.

More detailed information was added to the places where the figures are referenced, though we feel full duplication of the information would not be a good presentation of the information in the final publication.

3 Response to Referee 2

We thank the referee for many detailed suggestions and interesting questions.

3.1 Specific comments

> *Several important references are missing from the Introduction, e.g. Menetrier et al, 2015, Flowerdew, 2015 considered optimal localization,*

> *Buehner et al., 2015 suggested scale-dependent localization.*

These have been added.

> *I suggest rearranging the text so that everything related to multivariate localization is in section 3.2 (Extension to multivariate localization functions). I think describ- ing univariate case first, and then introducing groups for different localization radii (currently P5, L7-13) when extending to multivariate localization functions might improve the manuscript readability.*

This is a good point. In order to make the transition smoother and emphasize the fact that the multivariate approach is important, it has been moved into its own section in front of the Bayesian Approach section.

> *There seems to be a contradiction between 4.2 and 4.3. P11, L27-28 state that the problem is better suited for multivariate localization, while P12, L11-12 state that the canonical L96 model is ill suited for multivariate adaptive localization.*

There is no contradiction between 4.2 and 4.3. Although it is a bit confusing, Lorenz '96 was only tested with univariate adaptive localization, and the prediction was that the problem is not suited for this. This has been clarified in (the previous) 4.2.3 by adding the word 'univariate'.

> *I would like to see more details on the L96 multivariate localization experiment setup (Figure 5). Were the groups fixed throughout the experiment? How were they chosen? Did the groups use the same mean and variance parameters at each assimilation cycle? If the groups were fixed, it would*

be interesting to see how the estimated localization radii vary for different groups throughout the experiment.

This segues into the next point about the multivariate Lorenz '96. Whereby in the normal Lorenz '96 there is no sensible way to create groupings (other than each variable being in its own group), We essentially create artificial groups through the varying forcing. All this has been clarified in the text.

I see that in L96 experiments half of the domain is more sparsely observed than the other. Introduction (P2, L34-35) states that optimal localization may depend on observation properties. In L96 experiments, did you see evidence of the optimal localization radii being dependent on observation density?

An graph of the multi-group localization radii has been generated and discussed in the text, showcasing the differences between the radii of sparsely and fully observed groups.

Section 4.4.1, Figure 8. If instead of fixing the mean parameter to be the same as the constant suboptimal radius at each assimilation cycle, the adaptive localization radius estimated at the previous DA cycle was used to estimate the mean parameter, would the adaptive localization radius converge to the optimal one after some DA cycling?

The idea of doing an online radius mean estimate has been explored, but unfortunately as the optimal radii between steps are weakly correlated, and are not normally distributed, this often lead to suboptimal convergence of the radius estimate, and often lead to filter divergence. This idea is outside the scope of this paper in our opinion.

3.2 Questions on Extensions

How does this method extend to the ensemble DA algorithms other than DEnKF?

A paragraph has been added discussing this in the 'Solving the Optimization Problem' section, as the answer is closely tied with that problem.

For large DA applications like NWP, ensemble filters similar to DEnKF typically assimilate observations sequentially, and use $P^f H^T$ localization instead of Schur-product P^f localization which becomes too expensive. Would the method still be applicable in this case, and if so, how would it change?

This is already done in the paper! This has been clarified in the section discussing this approach.

Do you have a recommendation on how the groups for multivariate adaptive localization should be chosen?

A paragraph discussing this has been added to the multivariate Lorenz '96 localization results.

3.3 Technical comments

What is subscript i in Equation 10?

The subscript i has been removed from equation (10). It was a typo holdover from a previous version of the manuscript.

Interactive comment on Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg-2018-45, 2018.