

1 General response and main points

I first want to thank the authors for their revision, which I believe has greatly improved and clarified their work. However, while I am in favor of the work being accepted for publication, there are still some aspects that I believe should be improved, especially regarding section 5 which I was not formerly able to interpret.

I agree with the interpretation of the figures given in line 5 - line 12 (first sentence), page 22. However, there are a few issues following this. Firstly, the proposed γ -norm is not actually a norm when $\gamma = 0$; indeed, this is in fact not positive definite. This leads me to a great deal of confusion regarding lines 12 - 14 page 22, with the discussion on $\gamma = 0$. Under the proposed metric the components of the perturbation in the fast variables can be of arbitrarily large Euclidean norm and the measured size of the perturbation will still be the same. This makes the choice of perturbations completely unconstrained and I don't really understand the point this is trying to raise. I recommend removing the discussion on $\gamma = 0$ entirely.

In a separate but related point, I recommend removing the text in lines 14 - 15 page 11 and lines 1 - 9 page 12. Firstly, this discussion is distracting in section 3 as its relevance is only explored later in section 5. Moreover, I find this discussion fairly misleading in the way it describes the "orientation" of the CLV depending on the relative scales used to represent the variables in the γ -norm. The choice of the γ -norm or any other will not change the orientation of the Oseledec spaces (or their spanning vectors). In the simple case of distinct Lyapunov exponents, one can choose an arbitrary spanning set of covariant Lyapunov vectors; the choice of normalization can only define a new spanning set via a choice of non-zero scalar multiples of the original set. If it is the intention of the authors to simply discuss the relative timescale on which errors in the fast variables saturate with respect to the saturation of errors in the slow variables, this should be self contained within section 5, and discussions should focus on this point alone.

The manuscript is otherwise strong, and I think removing the two discussions above will greatly improve the work. In the following, I will outline several minor points that should also be addressed.

2 Minor Points

1. Page 3, lines 12 - 13: this should not start a new paragraph, include this in the previous.
2. Page 7, line 21: citation of Benettin et al. has an extra period.
3. Page 14, figure 4 caption: quotes are reversed in "central band".
4. Figure 8: remove $\gamma = 0$ case.
5. Page 22, line 14: what is fully linear? I understood the perturbation as being evolved in the nonlinear model. The difference between the control trajectory and the perturbation might evolve weakly nonlinearly, but this is again not fully linear.
6. Page 23, lines 1 - 4 and lines 14 - 15: in lines 1 - 4 it is stated that in all cases, the two plateaus are clearly visible. However, lines 14 - 15 state that when $h = 1$ it is practically impossible to find a threshold. This is contradictory and I agree with the second statement in lines 14 - 15, I didn't find a second plateau for $h = 1$ easy to detect whatsoever. The authors should explain clearly and in detail how the plateaus are chosen.
7. Page 23, lines 20 - 21: it is stated that "The parameter γ proves to be useful in improving the accuracy of the two plateaus exhibited by the FSLEs." I don't understand why. The case where the choice of γ is used to identify the onset of the plateau is in Figure 8.a in the inset — here $h = \frac{1}{4}$, which was already discussed as having clear plateau onsets with respect to the standard Euclidean norm. The case where a choice of γ may be helpful in identifying a plateau would be for $h = 1$ as discussed above, but this was not performed. Currently, I don't think this statement is justified. The choice of γ has only helped with the qualitative discussion of the saturation of errors on page 22, lines 1 - 12.
8. Page 24, lines 4 - 5: this should not start a new paragraph.

9. Page 24, lines 15 - 17, page 25 Fig. 9: a new figure and results should not be introduced in the conclusion. This figure and the discussion should be moved to section 4.