Dear Editor,

Thank you for handling our manuscript. We are pleased to see that our reply has been found satisfactory, that our paper notably improved and the referee now only raised minor comments to our revised manuscript.

We have further modified our paper to take into account these last remarks to the extent we consider them really beneficial. In fact, since some of these comments concern more the style of the presentation than the substance of the work, in a few occasions we have preferred to keep our presentation (see the detailed reply, here below).

Our changes are highlighted in red in the revised manuscript. Finally, we have also changed the vertical labels in Fig. 8a as we spotted a misprint.

Sincerely,

The authors

Reply to Referee 1

1. Presentation of the L96 model: A standard (with the exception of small-scale foreign) version L96 model using X and Y variables is presented in equation 1 and a rescaled version using X and Z variables in equation (4). The later is the version the authors appear to make use of, but most of the other appearances of the L96 model (equations 8, 18, and 24) are in terms of the X and Y variables and the original scaling. It would be clearer to pick a version and stick with it. Using the X and Z variables throughout seems like it would make the most sense.

We agree that the switch from Y to Z to refer to the fast variables can partially puzzle a reader. However, a thorough use of the rescaled variable Z would have consequence on the definition (scale) of the fraction Φ; additionally, the variable Y is much more appropriate for an analysis of the underlying "energetics". Finally, Z has been introduced only to show how the proper structure would look like in a context where the number of variables is changed.
Therefore, we have preferred to leave the notations as they are and added a remark on page 6, to clarify our choice of notations.

2. Table 2: It is not clear what the numbers in parentheses are. They are not described in the caption and I was unable to find an explanation in the text.

They measure the uncertainty in the estimated value of of Λ_S and of our estimate of i_S . This is now detailed in the table caption.

Reply to Referee 2

We acknowledge that the referee has appreciated the efforts made to improve the quality of the presentation. We have made further adjustments to take into account the last remarks to the extent we consider possible.

In the introduction of the report, the referee criticizes our reference to the limit case γ=0 and suggests to remove the corresponding paragraph. While we agree that for γ=0 the corresponding norm is not properly defined (not being positive definite), it is also true that any γ value strictly larger than 0 is a meaningful choice. Therefore, we have decided to carefully reword the text to avoid reference to $\gamma = 0$, mentioning instead small γ values (as we indeed already did in the previous version in most of the cases).

As for the following remark about the orientation of the CLVs, now, instead of referring to the "orientation" we now refer to “mutual angles”, which do depend on the norm adopted, therefore avoiding any possible misunderstanding.

For what concern the list of Minor points

1. Page 3, lines 12 - 13: this should not start a new paragraph, include this in the previous.

Done

2. Page 7, line 21: citation of Benettin et al. has an extra period.
3. Page 14, Figure 4 caption: quotes are reversed in “central band".

Both corrected.

4. Figure 8: remove $\gamma = 0$ case.

As discussed above, the discussion has been rephrased rather than being removed.

5. Page 22, line 14: what is fully linear? I understood the perturbation as being evolved in the nonlinear model. The difference between the control trajectory and the perturbation might evolve weakly nonlinearly, but this is again not fully linear.

The referee is right: "fully linear" is too strong and invalid statement. We replaced it with "approximately linear"

6. Page 23, lines 1 - 4 and lines 14 - 15: in lines 1 - 4 it is stated that in all cases, the two plateaus are clearly visible. However, lines 14 - 15 state that when h = 1 it is practically impossible to find a threshold. This is contradictory and I agree with the second statement in lines 14 - 15, I didn't find a second plateau for h = 1 easy to detect whatsoever. The authors should explain clearly and in detail how the plateaus are chosen.

The referee is right: for h=1, the presence of a second plateau is not entirely obvious. We have weakened the statement in the beginning of page 23, restricting it to h<1. Additionally, we have added a sentence to clarify the protocol adopted to define the height of the plateau.

7. Page 23, lines 20 - 21: it is stated that the γ parameter proves to be useful in improving the accuracy of the two plateaus exhibited by the FSLEs." I don't understand why. The case where the choice of γ is used to identify the onset of the plateau is in Figure 8.a in the inset where h = 1/4 , which was already discussed as having clear plateau onsets with respect to the standard Euclidean norm. The case where a choice of may be helpful in identifying a plateau would be for h = 1 as discussed above, but this was not performed. Currently, I don't think this statement is justified. The choice of has only helped with the qualitative discussion of the saturation of errors on page 22, lines 1 - 12.

Here we disagree with the referee: the parameter γ is very useful, as already noted in the main text and in the caption: the curves plotted in Fig. 8 have been obtained by using a small γ value (10^{-3}), since this allows for a maximal extension of the plateau. If this is still not enough to have a clear evidence for h=1, this does not mean that γ is not useful to play with.

8. Page 24, lines 4 - 5: this should not start a new paragraph.

Done

9. Page 24, lines 15 - 17, page 25 Fig. 9: a new figure and results should not be introduced in the conclusion. This figure and the discussion should be moved to section 4.

Shifting Fig. 9 in section 4 would make the reading of the manuscript yet heavier. We have preferred to leave the discussion into the final section renaming it as "Discussion and conclusions" so as to make it clear its content.