Dear Editors and Reviewers:

Thank you for your letter and for the reviewers' comments concerning our manuscript entitled "Exploring the effects of missing data on the estimation of fractal and multifractal parameters" (ID: npg-2018-38). Those comments are all valuable and very helpful for revising and improving our paper, as well as the important guiding significance to our researches.

Anonymous Referee #1:

Firstly, we sincerely apologize for our poor English writing in our submitted manuscript.

As for the scientific issues discussed in this article, we mainly focused on the impacts of missing data on the estimation accuracy for fractal or multifractal series. To solve this problem, we think there should be many perspectives. It is an angle for stochastic process theory to solve this kind of problem. We think that non-parametric statistical inference based on simulated data is also a relatively simple method. The reason is that statistical inference theory can compare and pick out the advantages or disadvantages of multiple test schemes. As for why we choose various interpolation methods as candidates, such consideration arose from many previous articles using interpolation methods to preprocess them.

We recognized that this paper treated missing data roughly, and does not deeply considered the missing types such as stationary vs. nonstationary, irregular or blank, and so on. Many traces of artificial manipulations are found in the experimental scheme, indeed in this way the missing extent is also emphasized and is supposed to have most influence for the estimations. We think that this method can basically reflect the comparison of the estimation accuracy between the series with missing data and various interpolation methods. Of course, it can also determine whether the interpolation pretreatment is necessary or not.

We try to continue to reinforce some of the conclusions in this article. For instance, the simulation data of fractal Brownian motion with parameter H ranging between 0.1 and 0.9 and multifractal series with parameter C_1 ranging between 0.1 and 0.3 and α between 1.0 and 2.0 respectively are generated, and the amount of simulated data is many times larger than before. Monte Carlo and bootstrap statistical inference are used for multiple sample data and single sample data, respectively. Each validation will cover three parts: firstly, the median values are compared for all datasets reflect the degrees of deviation from the true values (**Fig.1 and Fig.2**); secondly, the percentage of estimates to the data with missing values and interpolated data falling into the estimated intervals of the original data are compared to reflect the discreteness of the estimates for the affected data (**Fig.3**); thirdly, the probabilities that the affected population are misjudged as another population, i.e., type I errors are calculated and used to compare the estimation accuracy (**Fig.4**).

Note:

1. 3 missing types, (The missing data are divided into three levels according to the missing degree, and it reflects the size of the missing quantity. The smaller the number, the smaller the degree of missing data.) ;

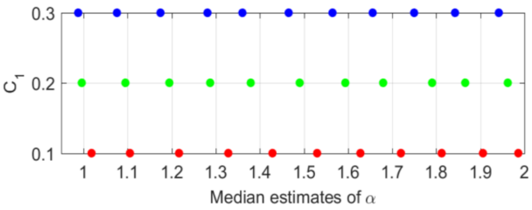
2. 5 interpolation methods include piecewise linear interpolation (PLI), Nearest-neighbor interpolation (NI), piecewise cubic spline interpolation (PCSI), piecewise cubic Hermite interpolation (PCHI) and piecewise Bessel interpolation (PBI)).

Anonymous Referee #2:

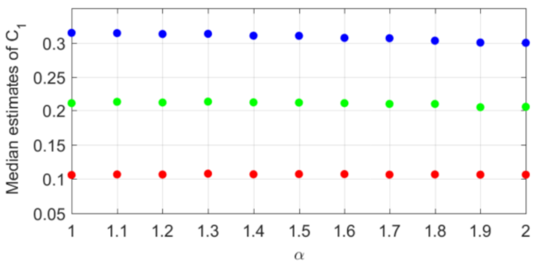
Firstly, we sincerely apologize to the reviewers for the poor quality of English writing. Thank you very much for giving this major review opportunity.

Since receiving the feedback, we have devoted ourselves to the revision of the manuscript. We deeply rethink the scientific issues discussed in this paper. Our thinking has changed from emphasizing the using of bootstrap method to the influence of missing data on estimation accuracy for fractal or multifractal series. Meanwhile，we also expand the breadth and depth of the experiment in the paper according to the comments. The amount of simulated data is many times larger than before, which makes it impossible to complete the revision within given time limit. Therefore, we hope to give us another three months to complete the revision.

In response to the reviewer's comments, the simulation data of fractal Brownian motion with parameter H ranging between 0.1 and 0.9 and multifractal series with parameter C_1 ranging between 0.1 and 0.3 and α between 1.0 and 2.0 respectively are generated. Monte Carlo and bootstrap statistical inference are used for multiple sample data and single sample data, respectively. The experimental data set include the original data (Monte Carlo simulations), the series with missing data (The missing data are divided into three levels according to the missing degree, and it reflects the size of the missing quantity. The smaller the number, the smaller the degree of missing data.) and interpolated data (For 3 missing types, the interpolated data are obtained using 5 interpolation methods including piecewise linear interpolation (PLI), Nearest-neighbor interpolation (NI), piecewise cubic spline interpolation (PCSI), piecewise cubic Hermite interpolation (PCHI) and piecewise Bessel interpolation (PBI)). Each validation will cover three parts: firstly, the median values are compared for all datasets reflect the degrees of deviation from the true values (**Fig.1 and Fig.2**); secondly, the percentage of estimates to the data with missing values and interpolated data falling into the estimated intervals of the original data are compared to reflect the discreteness of the estimates for the affected data (**Fig.3**); thirdly, the probabilities that the affected population are misjudged as another population, i.e., type I errors are calculated and used to compare the estimation accuracy (**Fig.4**).

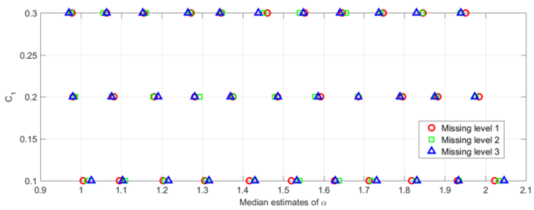


Median estimates of α when C_1=0.1, 0.2 and 0.3

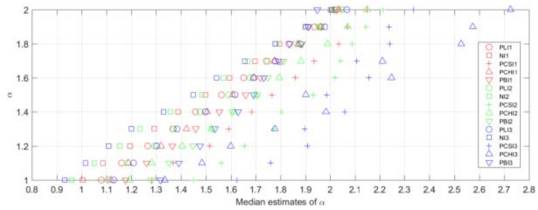


Median estimates of C_1 when α ranges between 1.0 and 2.0

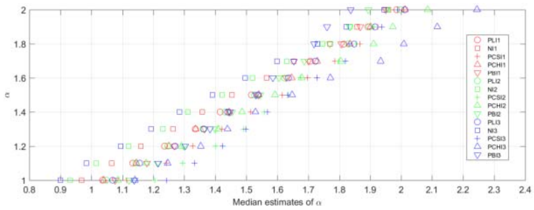
Fig.1



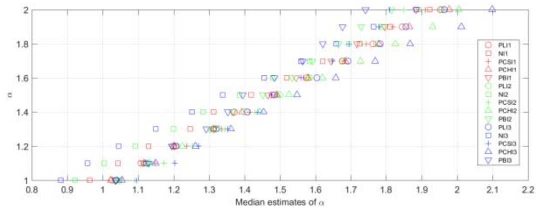
Median estimates of α for 3 missing types when C_1=0.1, 0.2 and 0.3



Median estimates of α for 5 interpolated series to 3 missing types when C_1=0.1

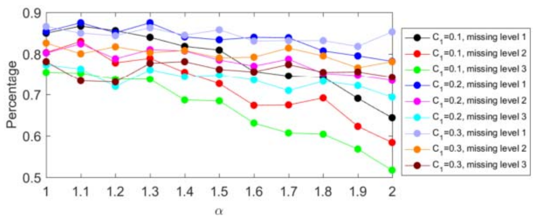


Median estimates of α for 5 interpolated series to 3 missing types when C_1=0.2

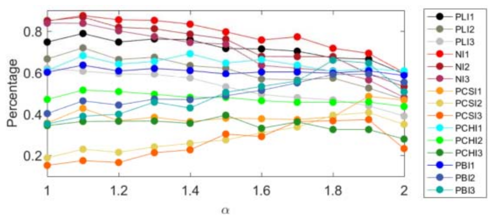


Median estimates of α for 5 interpolated series to 3 missing types when C_1=0.3

Fig.2

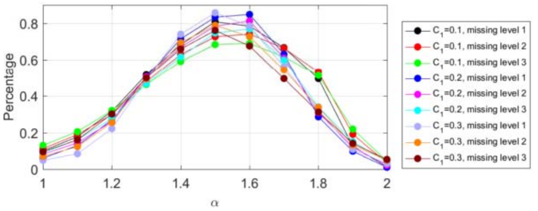


Percentage of population estimates for missing types falling into the Monte Carlo intervals

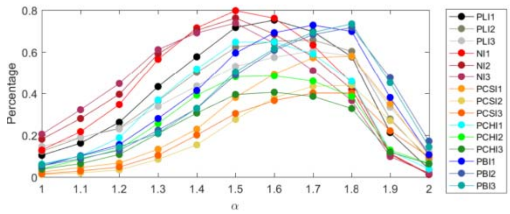


Percentage of population estimates for interpolated series falling into the Monte Carlo intervals

Fig.3



Type I errors of α=1.5 for 3 missing types



Type I errors of α=1.5 and C_1=0.1 for 5 interpolated series from 3 missing types

Fig.4