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Role of nonlinear interaction between water and plant in stability analysis of nonspatial plants

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Abstract: In this study, a theoretical ecosystem model is applied to discuss a stability of plant using linear and nonlinear methods. Two common linear methods are employed to analyze a linear stability of plant through judging the positive or negative of eigenvalues (Lyapunov method), and solving a linear singular vector (LSV). To explore the nonlinear stability of plant, a conditional nonlinear optimal perturbation (CNOP) approach is used. The CNOP, which is a type of initial perturbation, could cause the nonlinearly most unstable for an equilibrium state. The CNOP is a nonlinear development of the LSV which is the rapidest initial perturbation with a linear framework. The numerical results show that two linear stable equilibrium states (plant and desert) with the linear methods are nonlinear unstable with the CNOP method. When there is large enough magnitude of initial perturbation, the linear stable plant (desert) equilibrium state will be evolved into desert (plant) equilibrium state using the CNOP-type initial perturbation. This character disappears using the LSV-type initial perturbation. The above results are effective for two types of plant, namely grasslands and trees. Through analyzing the nonlinear dynamics of the theoretical model, it is found that the nonlinear interaction between plant and water play more important role to a transitions between two equilibriums states than the evaporation and the plant losses expressed by linear terms in the theoretical model. The findings could be exhibited by using the nonlinear method (the CNOP method), but fail by using the linear methods.

1. Introduction

Arid and semiarid regions cover more than 40% of the globe, and are extremely sensitive to environmental condition and human activities (Charney, 1975; Fu and An, 2002; Sankey et al., 2012; Huang et al., 2017). There are already plenty of evidences that plant degeneration often occurs in arid and semi-arid regions (Ni, 2004; Okin et al., 2009; Sun et al., 2017). The instability of plant impacts not only animal biodiversity, soil productivity, and so on, but largescale climate change, the transfer of radiation, water and energy, etc (Xue and Shukla, 1993; Xue, 1996; Eklundh and Olsson, 2003; Lu and Ji, 2006; Notaro et al., 2006; Piao et al., 2007; Xu et al., 2012). Hence, understanding the drivers and dynamics of plant stability in arid and semi-arid region is motivated by analyzing the effect of disturbances regimes on plant degradation.

Abrupt changes of plant in arid and semi-arid regions may be considered as the transitions between two stable equilibrium states (Mauchamp et al., 1994). Precipitation was thought to the key factors to sharp transitions between different vegetation states (Hardenberg et al., 2001; Motchell and Csillag, 2001; Sun and Mu, 2013, 2014). It had been reproduced to the transitions from bare soil at limited rainfall to homogeneous vegetation at high rainfall observed in arid and semi-arid regions using a theoretical model. For same rainfall, the transitions between different stable equilibrium states are also discovered. Zeng et al. (2004, 2005, 2006) build a theoretical model, which could reconstruct different plant pattern along a moisture index in North China, to reveal the coexistence of the grassland and the desert equilibrium states at the same climate condition. The stabilities of the grassland and the desert equilibrium states were demonstrated by using the Lyapunov method. The shading mechanism of the wilted biomass was announced as the key mechanism of the maintenance of the grassland equilibrium state. The transition between two plant equilibrium states is also investigated. Okin (2009) proposed a theoretical model to examine the grassland-shrubland dynamics. Their finding suggested that a feedback between grass biomass and soil erosion may cause an abrupt transition from grassland to a shrubland state observed throughout the southwestern U.S. in the past 150 years.

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The bistability character was explained by the theoretical model and the Lyapunov method. However, the above models are the nonlinear model. It is inappropriate that the linear method (Lyapunov method) is applied to explore the stability of equilibrium states. A nonlinear stability analysis method (the condition nonlinear optimal perturbation method, the CNOP method, Mu et al., 2003) was employed to illuminate a nonlinear stability of the grassland and the desert equilibrium states (Mu and Wang, 2007; Sun and Mu, 2009, 2011; Sun and Xie, 2017). The CNOP is a type initial perturbation, which could cause the most unstable state compared to the linear stable grassland and the desert equilibrium states. If the CNOP could bring to the transition from one linear stable equilibrium state to another, the linear stable equilibrium state was considered as the nonlinear stable. Mu and Wang (2007), and Sun and Mu (2009, 2011) found the CNOP-type initial perturbation, which brought to the transition between two equilibrium states. The nonlinear mechanism played the key role in the transition between two equilibrium states opened out by their studies.

The purpose of this report is to investigate the nonlinear stability of equilibrium state, and reveal which dynamics mechanism is important to transition between two equilibrium states by using the CNOP method and a theoretical model. We argue further the roles of the linear terms and the nonlinear terms in the transition processes.

2. Model and method

2.1. The model

A simple theoretical model, which could descript the plant and water dynamics, and simulate the different types of plant patterns, is employed to explore the stability of plant (Klausmeier, 1999).

The model is presented as follows:

$$\begin{cases} \dfrac{\partial W}{\partial T} = A - LW - RWN^2 + V\dfrac{\partial W}{\partial X} \\ \dfrac{\partial N}{\partial T} = RJWN^2 - MN + D(\dfrac{\partial^2}{\partial X} + \dfrac{\partial^2}{\partial Y})N \end{cases} \quad (1)$$

W and *N* represent water and plant biomass in the two-dimensional domain indexed

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by *X* and *Y*. *A* is the rate of water input, *L* is the rate of evaporation. The expression of RWN^2 means the rate of plants taking up water, and is a nonlinear term. *V* is speed of water flowing downhill. *J* is the yield of plant biomass per unit water consumed. Plant biomass misses according to the density-independent mortality and maintenance at rate *MN*. Plant dispersal is simulated by a diffusion term with diffusion coefficient *D*. The above theoretical model could be nondimensionalized (Klausmeier, 1999) to

$$\begin{cases} \dfrac{\partial w}{\partial t} = a - W - wn^2 + v\dfrac{\partial w}{\partial x} \\ \dfrac{\partial n}{\partial t} = wn^2 - mn + (\dfrac{\partial^2}{\partial x} + \dfrac{\partial^2}{\partial y})n \end{cases} \quad (2)$$

In this study, to explore the linear and nonlinear stability of nonspatial plant, the space derivatives are set to zero. So, the below theoretical model is analyzed

$$\begin{cases} \dfrac{\partial w}{\partial t} = a - w - wn^2 \\ \dfrac{\partial n}{\partial t} = wn^2 - mn \end{cases} \quad (3)$$

2.2 Conditional nonlinear optimal perturbation method (CNOP)

To determine whether the nonlinearly stability or instability of the plant or not, the conditional nonlinear optimal perturbation approaches related to initial errors (CNOP) are applied (Mu et al., 2003, 2004). The CNOP method is introduced below for the convenience of the reader.

The target problem can be represented in following ordinary or partial differential equation:

$$\begin{cases} \dfrac{\partial U}{\partial t} = F(U, P) \quad U \in R^n, t \in [0, T] \\ U|_{t=0} = U_0 \end{cases} \quad (4)$$

where F is a nonlinear operator; P represents the model parameters; U_0

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contains the initial values of the state variables; M_τ represents the propagator of the ordinary or partial differential equations from the initial time 0 to τ; and U_τ is a solution of the ordinary or partial differential equations at time τ that satisfies $U(\tau) = M_\tau(U_0)$.

Let $U(T;U_0)$ and $U(T;U_0)+u(T;u_0)$ be the solutions of the ordinary or partial differential equations (2) with the initial and model vectors U_0 and U_0+u_0. u_0 indicates the errors and perturbations related to the initial values and model parameter values. $u(T;u_0)$ describes the variations in the reference state $U(T;U_0)$ caused by the initial errors and the model parameter errors u_0. $u(T;U_0)$ satisfies the following conditions:

$$\begin{cases} U(T;U_0) = M_T(U_0) \\ U(T;U_0)+u(T;u_0) = M_T(U_0+u_0) \end{cases}. \quad (5)$$

For the chosen norm $\| \ \|$, a perturbation u_σ is the CNOP if and only if

$$J(u_\sigma) = \max_{u_0 \in \sigma} J(u_0), \quad (6)$$

where

$$J(u_0) = \| M_T(U_0+u_0) - M_T(U_0) \| \quad (7)$$

Here, U_0 is the reference state; u_0 is the error of the initial conditions; $u_0 \in \sigma$ is the constraint condition. So, the CNOP represents a type of initial errors, which could cause the most unstable state. To obtain the maximum value of (2), the sequential quadratic programming (SQP) algorithm (Barclay et al., 1997) is employed. The gradients of the cost function are computed by the definition of the gradient.

2.2. Experimental design

A control factor in the theoretical model is water input (*a*). There are different equilibriums for different water input. According the climate character of arid and semi-arid (rainfall is about from 250 Kg H_2O m^{-2} $year^{-1}$ ~ 750 Kg H_2O m^{-2} $year^{-1}$). If the water input *a* is from 0.077 to 0.23, the plant is tree, and *m* is 0.045 (see appendix for details). For the grassland state, *a* is from 0.94 to 2.81, and *m* is 0.45. In our reports, two water inputs (*a*=1.2, 0.2) and two optimization times (T=20 and 30, 20 and 30 years) are considered in order to determine whether or not the numerical results are dependent upon the choices of the reference state and the optimization time. The model is discretized based on the fourth-order Runge-Kutta method with a time step of dt=1/24 (representing half of a month). L2 norm is chosen, and the constrained condition about the initial perturbation is $\|u_0\| \le \delta$.

To analyze the linear stability of plant, a traditional method is used to judge the positive and negative of eigenvalues. If the eigenvalues are positive (negative), the plant or desert is stable (unstable). To analyze the nonlinear stability of plant or desert, the CNOP is calculated to determine the nonlinear evolution of the initial perturbation. If the nonlinear evolution of the initial perturbation will be zero, the plant or desert is nonlinear stable. On the contrary, an abrupt change occurs, and the plant or desert is nonlinear unstable. In the same way, a linear singular vector (LSV) method is used to compare to the CNOP method.

3. Numerical results

3.1. Linear stability analysis

For the theoretical model, there are three equilibrium states. One is a desert state, and two is plant state. If the water input *a* is from 0.077 to 0.23, the plant is tree, and *m* is 0.045. For the grassland state, *a* is from 0.94 to 2.81, and *m* is 0.45. Table 1 shows examples about the stability analysis for different equilibrium states of grassland (*a*=1.2) and tree (*a*=0.2). It is found that there are three equilibrium states for grassland or tree. One is the linearly stable grassland or tree equilibrium state, and another is the linearly stable desert equilibrium state due to negative

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eigenvalues. The final one is the linearly unstable grassland or tree equilibrium state. Figure 1 shows the equilibrium states and stability analysis for different water inputs within grassland or tree.

3.2. Nonlinear stability analysis

To analyze the nonlinear stabilities of equilibrium states, the CNOP method and the LSV method are employed. If the grassland (desert, *a*=1.2) equilibrium state is transformed into the desert (grassland) equilibrium state due to the CNOP-type initial perturbation. The linear stable grassland (desert) equilibrium state is considered as nonlinearly unstable. When the constrained value (δ) is 1.0, it is found that the grassland equilibrium state is transformed into the desert equilibrium state due to the CNOP-type initial perturbation (*w'*=-0.489, *n'*=-0.872) (Figure 2a and 2b). However, the grassland equilibrium state fails to be transformed into the desert equilibrium state due to the LSV-type initial perturbation (*w'*=-0.741, *n'*=-0.671) for the same constrained value δ=1.0. There are similar results for the desert equilibrium state as the reference state. The above analysis results imply that the linearly stable grassland (desert) equilibrium state is nonlinearly unstable for grassland (*a*=1.2).

In addition, the tree and desert equilibrium states (*a*=0.2) are also analyzed. It is shown that the desert equilibrium state will be transformed into the tree desert equilibrium state due to the CNOP-type initial perturbation (*w'*=0.0125, *n'*=0.2397). However, the desert equilibrium state is kept due to the LSV-type initial perturbation (*w'*=0.0721, *n'*=0.2290). Our findings also illuminated that the turnover time for the tree (about 200 years) as the reference state is longer than that for the grassland (about 20 years) as the reference state. The variational ratios of three terms about the plant and the water in the theoretical model for the grassland are larger than those for the trees. The ratio of plant biomass losses (*m*) for the grassland is larger than that for the tree. The above results also hint that the tree is more stable than the grassland.

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4. Discussions

To analyze the dynamics of plant due to two types of initial perturbations, the nonlinear model (Eq. 8) and linear model (Eq. 9) of initial perturbation are showed for the Eq. 3.

$$\begin{cases} \dfrac{\partial w'}{\partial t} = -w' - (\bar{w}+w')(\bar{n}+n')^2 + \bar{w}\bar{n}^2 \\ \dfrac{\partial n'}{\partial t} = (\bar{w}+w')(\bar{n}+n')^2 - \bar{w}\bar{n}^2 - mn' \end{cases} \qquad (8)$$

$$\begin{cases} \dfrac{\partial w'}{\partial t} = -w' - w'\bar{n}^2 - 2nwn' \\ \dfrac{\partial n'}{\partial t} = w'\bar{n}^2 + 2\bar{n}\bar{w}n' - mn' \end{cases} \qquad (9)$$

$\bar{w}$ and $\bar{n}$ are the basic state of the water and the plant. w' and n' are the perturbation of the water and the plant. $(\bar{w}+w')(\bar{n}+n')^2 - \bar{w}\bar{n}^2$ represents the nolinear term about plants taking up water of initial perturbation. $w'\bar{n}^2 + 2\bar{n}\bar{w}n'$ is the linearization of the nonlinear ter. It is shown that the nonlinear evolutions of the CNOP and the LSV with Eq. 8 are similar to those of the CNOP and the LSV being superimposed on the basic state (Figure 2). In addition, the linear evolution of the LSV is also discussed. We find that the character is similar between the nonlinear evolution and linear evolution about the LSV. This suggests that the CNOP could cause the nonlinear stability for the nonlinear model, however the LSV fails under the same extent of constrained condition.

To analyze the difference of the dynamical mechanisms about the CNOP and the LSV, the right terms of Eq. 8 and 9 are computed. Figure 4 shows the evolutions of every term. For the grassland as the reference state, it is found that the variation extent (0.98) of the nonlinear term $(\bar{w}+w')(\bar{n}+n')^2 - \bar{w}\bar{n}^2$, which represents nonlinear interaction between water and plant in arid and semi-arid region, caused by the CNOP is greater than those of the linear terms w' (0.87) and mn'(0.10), which represent evaporation, and plant biomass loss through density-independent mortality and

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maintenance (Figure 4a). However, although the variation extent (2.06) of the linear term $w'\bar{n}^2 + 2\bar{n}\bar{w}n'$, which represents linear interaction between water and plant in arid and semi-arid region, caused by the LSV is greater than those of the linear terms w' (0.15) and mn'(0.87) at first period, the linear interaction between water and plant $w'\bar{n}^2 + 2\bar{n}\bar{w}n'$ rapidly decays to 0.15 with the developing of time, which is lower than effect of evaporation (0.21) and plant biomass loss (0.90) (Figure 4b). So, the nonlinear character may be not reflected by the linear model and the LSV. The high effect of the nonlinear terms brings to the loss of the water due to the CNOP-type initial perturbation, and the remaining water does not supply the plant. The desert equilibrium state is generated. However, the low effect between water and plant of the linear term also leads to the loss of the water due to the LSV-type initial perturbation, but the rest of water could support the plant. The grassland equilibrium state is kept.

For the desert state as the reference state (*a*=0.12), we find that the variation extent (0.26) of the nonlinear term $(\bar{w}+w')(\bar{n}+n')^2 - \bar{w}\bar{n}^2$ caused by the CNOP is smaller than that of the linear term w' (0.33), and is greater mn'(0.19) at first year (Figure 4c). However, after one year, the nonlinear effect is shown, and the variation extent (0.26) is greater than those of the linear terms evaporation (0.04) and plant biomass loss (0.21) (Figure 4c). The variation extent of the linear term of interaction between water and plant is lower than those of the linear terms of evaporation and plant biomass loss all the time caused by the LSV (Figure 4d). For the desert as the reference state (a=0.2), the effects of the nonlinear term (0.012) and two linear terms (0.012 and 0.011) due to the CNOP are identical. As the change of time, the variation extent of the nonlinear term (0.011) is greater those of two linear terms (0.003 and 0.010) (Figure 4e). The variation extent of the linear term of interaction between water and plant is always smaller than those of the linear terms of evaporation and plant biomass loss caused by the LSV (Figure 4f). For the two desert equilibrium states, the effect of the nonlinear term results in enough water to plant growing, and the desert equilibrium states are finally transformed into the grassland and tree equilibrium states. However, the effect of the linear term results in deficient water to plant growing, and the desert equilibrium states are kept.

For the tree as the reference state (a=0.2), it is found that the patterns of the CNOP (w'=-0.001380, n'=-0.9399) and the LSV (w'=-0.0002412, n'=-0.9999) are similar. So, the variations of the tree due to the CNOP and the LSV are equivalent. The linear stable tree equilibrium is also nonlinear stable. In addition, the plant in this study was chosen as nonspatial plant in Eq. 3. In fact, this model could be employed to explore the stability of spatial plant to consider the advective term ($v\frac{\partial w}{\partial x}$) and diffusion term ($(\frac{\partial^2}{\partial x}+\frac{\partial^2}{\partial y})n$) (Klausmeier, 1999; Sherratt and Lord, 2007; Sherratt, 2016) (Figure 5). It is interesting that the linear stability and nonlinear stability of spatial plant are discussed. In future, the issues will be answered.

Beyond all doubt, evapotranspiration (ET) is an important indicator factor for the transition between two equilibrium states in arid and semi-arid regions (Kurc and Small, 2004). The relationships of the ET-soil moisture impact on the plant stability. Consistent with the previous work, the evaporation was also an important factor in our studies (Huang et al., 2017). The evaporation caused the decreasing water in the soil layer, and the resulting would bring to a lack of supply for the plant. Hence, the transition of from the grassland to the desert easily occurred. Compared to the effect of the evaporation, the nonlinear interaction between the plant and the water was more important from our findings using the CNOP approach. And, this effect will directly result in the transition.

5. Conclusions

In this study, three common methods (the Lyapunov method, the LSV method, and the CNOP method) are employed to explore the stabilities of plant (including grassland and tree) and desert. The first two methods are used to analyze the linear stabilities of plant and desert. The last method is applied to discuss the nonlinear stabilities of plant and desert. It is found that the linear stable grassland and desert equilibrium states are nonlinear stable when there is enough larger variation of initial

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perturbation. Through computing the variations of nonlinear terms $(\overline{w}+w')(\overline{n}+n')^2-\overline{w}\overline{n}^2$, it is demonstrated that the nonlinear interaction between water and plant plays an important role in the stabilities of grassland and desert compared to the linear terms of evaporation and plant biomass losses. The CNOP approach could reflect this nonlinear character, but the Lyapunov method and the LSV method fail.

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Appendix

In the section, the dimensionless of the Eq. 1 is introduced (Klausmeier, 1999). In the studies of Klausmeier (1999), the dimensionless processes have been stated. Here, this treatment is introduced for readers' convenience. Let $w = R^{1/2}L^{-1/2}JW$, $n = R^{1/2}L^{-1/2}N$, $x = L^{1/2}D^{-1/2}X$, $y = L^{1/2}D^{-1/2}Y$, $t = LT$, $a = AR^{1/2}L^{-3/2}J$, $m = ML^{-1}$, and $v = VL^{-1/2}D^{-1/2}$. Klausmeier (1999) indicated that the rainfall (A) was about from 250 to 750 Kg H_2O in arid and semi-arid region. The evaporation rate was L=4 year^{-1}. According to the researches of Mauchamp et al. (1994), Klausmeier (1999) confined the four parameters values: J_{tree}=0.002 kg dry mass (kg H_2O)$^{-1}$, J_{grass}=0.003 kg dry mass (kg H_2O)$^{-1}$, M_{tree}=0.18 year^{-1}, and M_{grass}=1.8 year^{-1}. And, the R_{tree}=1.5 kg H_2O m^{-2} year^{-1} (kg dry mass)$^{-2}$ and R_{grass}=100 kg H_2O m^{-2} year^{-1} (kg dry mass)$^{-2}$ were also determined. According to the above the parameters values, the dimensionless a and m could be obtained for grass and tree as follows: a_{tree}=0.077 to 0.23, m_{tree}=0.045, a_{grass}=0.94 to 2.81, m_{grass}=0.45.

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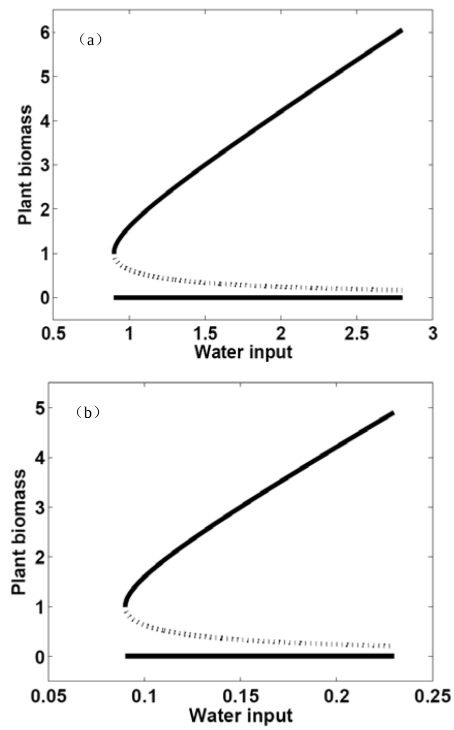

Figure 1. The equilibrium state of the vegetation for the theoretical model. Linearly stable equilibrium state is denoted by solid line corresponding to bare soil and vegetation. Linearly unstable equilibrium state is denoted by dash line corresponding to vegetation. (a): grass; (b) tree.

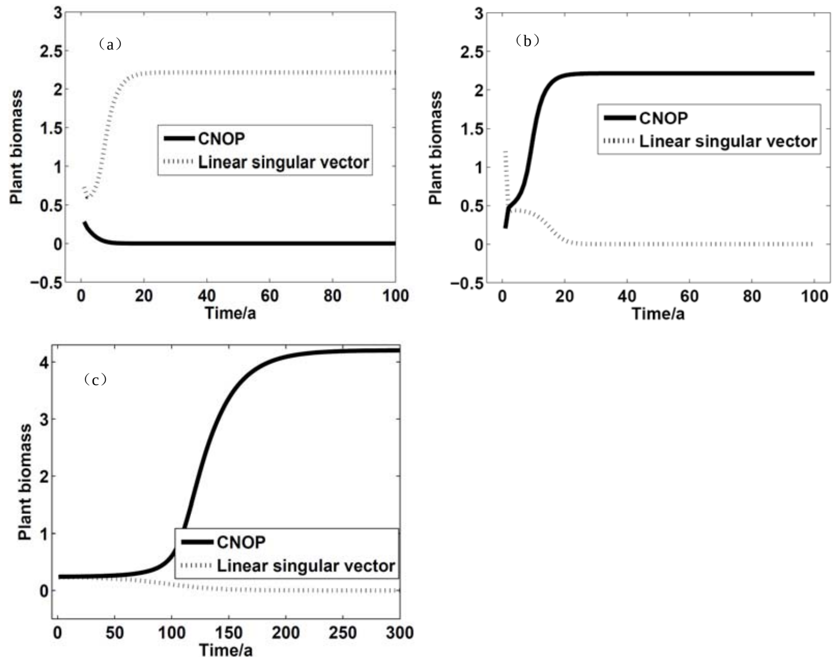

Figure 2. The nonlinear evolutions of CNOP and LSV. (a): grassland (a=1.2); (b): desert (a=1.2); (c): desert (a=0.2).

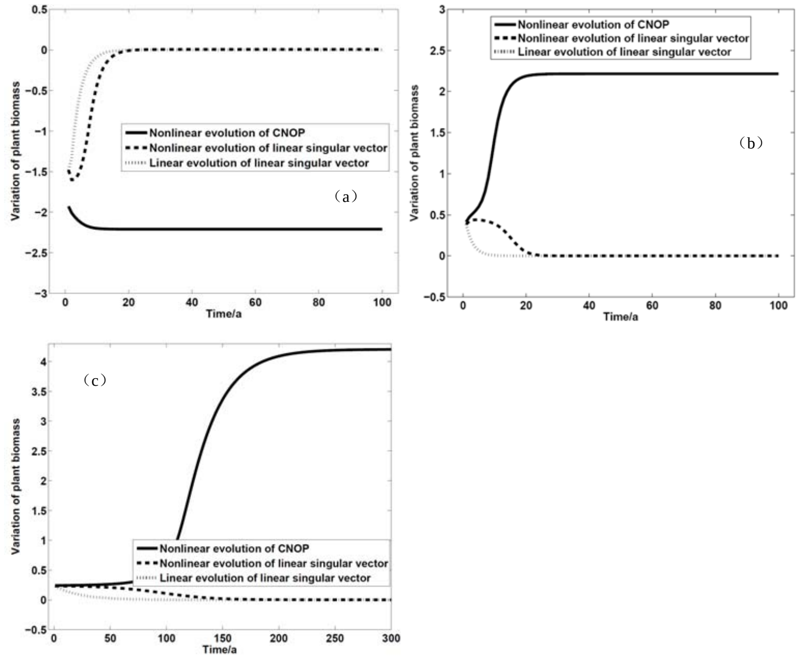

Figure 3. The nonlinear and linear evolution of CNOP and LSV with Eq. 8 and 9. (a): grassland (a=1.2); (b): desert (a=1.2); (c): desert (a=0.2).

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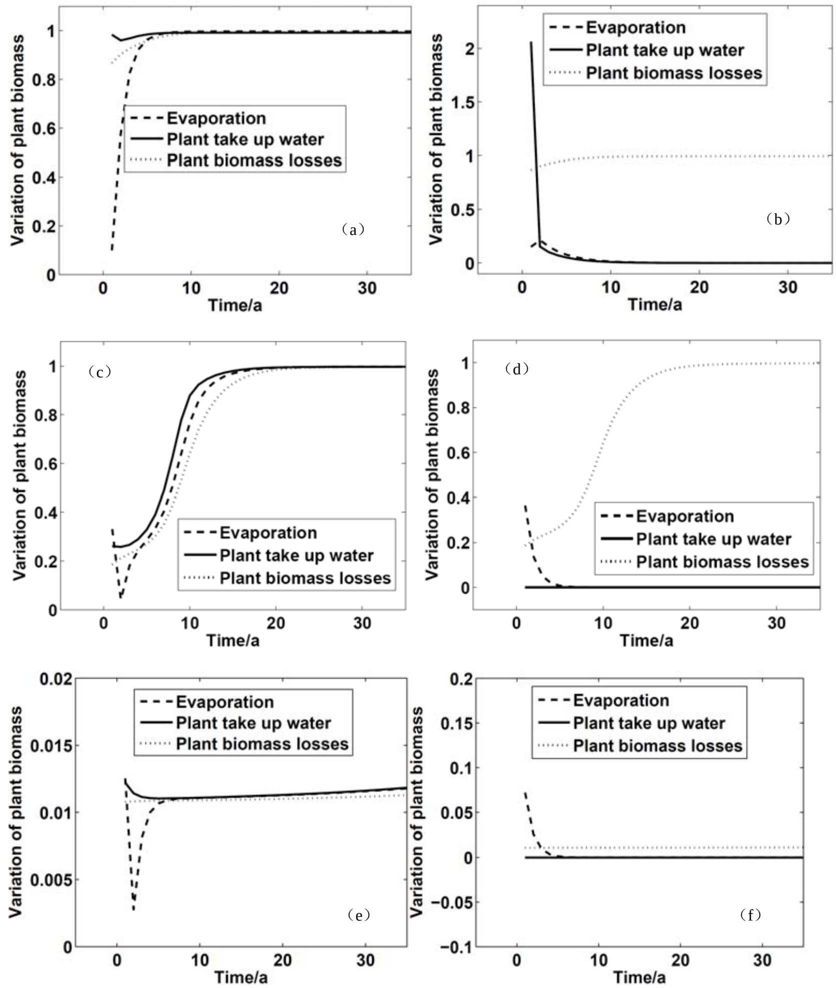

Figure 4. Absolute variations of right terms with Eq. 8 and 9 due to the CNOP and LSV. (a), (c), and (e): CNOP; (b), (d), and (f): LSV. (a) and (b): grassland (a=1.2); (c) and (d): desert (a=1.2); (e) and (f): desert (a=0.2).

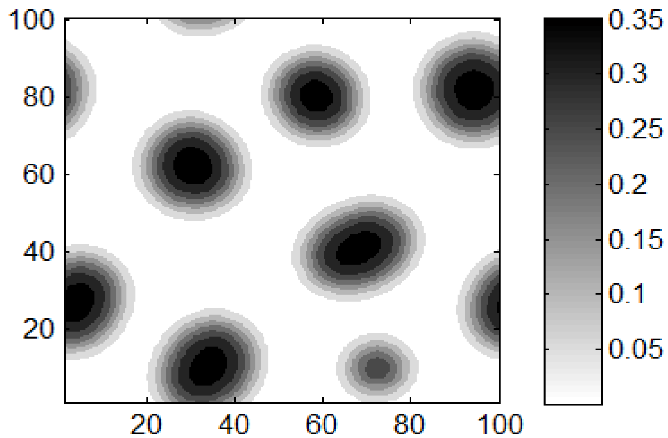

Figure 5. The distribution of plant for full model (Eq. 2)

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Table 1 The linear stability analysis for different equilibrium states

Types	Equilibrium *w*: water *n*: plant biomass	Eigenvalues	Stability
Grassland	Grassland (*w*=0.203, *n*=2.21)	-0.344, -5.093	Linear stable
(*a*=1.2)	Grassland (*w*=0.997, *n*=0.451)	0.330, -1.084	Linear unstable
	Desert (*w*=1.2, *n*=0)	-1.000, -0.450	Linear stable
Tree (*a*=0.2)	Tree (*w*=0.011, *n*=4.207)	-0.040, -18.613	Linear stable
	Tree (*w*=0.189, *n*=0.238)	0.040, -1.052	Linear unstable
	Desert (*w*=0.2, *n*=0)	-1.000, -0.045	Linear stable

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Table 2 The CNOP and LSV initial perturbations for different grassland or tree equilibrium state

Types	Reference state	δ	CNOP	LSV
Grassland (a=1.2)	Grassland (w=0.203, n=2.21)	1.0	(w'=-0.489, n'=-0.872)	(w'=-0.741, n'=-0.671)
	Desert (w=1.2, n=0)	0.53	(w'=0.332, n'=0.413)	(w'=0.365, n'=0.384)
Tree (a=0.2)	Tree (w=0.01069, n=4.207)	1.0	(w'=-0.001380, n'=-0.9399)	(w'=-0.0002412, n'=-0.9999)
	Desert (w=0.2, n=0)	0.24	(w'=0.0125, n'=0.2397)	(w'=0.0721, n'=0.2290)