

## ***Interactive comment on “Fluctuations of finite-time Lyapunov exponents in an intermediate-complexity atmospheric model: a multivariate and large-deviation perspective” by Frank Kwasniok***

**Anonymous Referee #2**

Received and published: 31 May 2018

In this manuscript the fluctuations of finite time Lyapunov exponents are studied in a meteorologically relevant system. The author uses an interesting approach for studying collective excitations observed within the Lyapunov spectrum. I think the work is relevant for understanding the properties of fluctuations in a high dimensional dynamical system. Nevertheless, I think there should still be some improvement of presentation and explanation of the results.

On section 1: - Line 8: based from what I understand: Could the covariance structure tell us something about how "close" or how "interactive" the various unstable and stable

C1

directions are? Could the covariance structure be related to the investigations of the inertial manifold using the angles between Lyapunov vectors. Maybe the work along the lines of Yang et al (2009) should be referenced here as a motivation.

On Section 2:

- I think a short concise table listing all parameters of the model with their dimensional and a dimensional values would be beneficial to introduce the model setup.

On Section 3:

- It should be noted that the mean of the finite time Lyapunov exponents are in fact average growth rates of linear perturbations of the system. But the finite time LEs are not directly the growth rate of those perturbation. In fact one can define backward, forward and covariant LEs. There is a good review paper on this by Kuptsov and Parlitz ([https://www.researchgate.net/publication/51961547\\_Theory\\_and\\_Computation\\_of\\_Covaria](https://www.researchgate.net/publication/51961547_Theory_and_Computation_of_Covaria)) which explains this distinction. I think using the FTLEs of the Gram Schmidt algorithm is alright, but it should be better clarified what type of FTLEs they actually are.

On Section 4:

I think this section should motivate better why one should use EOFs and what would be potentially alternatives to this approach.

On Section 6:

Section 6.1: - Since the model is zonally symmetric there should be two zero exponents. Can you verify this and could you include this in this discussion? - Figure 2: This result should be referenced with the findings about fluctuations of the LE for covariant, backward and forward exponents in Vannitsem, Lucarini (2016). I think when you study collective excitations this is an interesting different viewpoint.

Section 6.2: - I think it would be helpful to present the matrix D as well as a surface plot and also use the first EOF and second EOF to see what parts of the D matrix are

C2

actually reconstructed using the EOF method.

Section 7:

- That no clear time scale separation is found is probably because QG equations are scale filtered equations. Similar results were found before in QG models (Vannitsem 1997, Schubert 2015, Vannitsem 2016 ).
- Can the collective excitations be traced to any anomalous behavior in the non linear background state  $x$ ? I think that would be an interesting addition but of course not necessary in order to do this study.

References:

Vannitsem, S. and Lucarini, V. (2016) Statistical and dynamical properties of covariant Lyapunov vectors in a coupled atmosphere-ocean model—multiscale effects, geometric degeneracy, and error dynamics. *Journal of Physics A: Mathematical and Theoretical*, 49 (22). 224001. ISSN 1751-8113

"Hyperbolicity and the Effective Dimension of Spatially Extended Dissipative Systems"  
Hong-liu Yang, Kazumasa A. Takeuchi, Francesco Ginelli, Hugues Chaté, and Günter Radons *Phys. Rev. Lett.* 102, 074102 – Published 18 February 2009

VANNITSEM, S., & NICOLIS, C. (1997). Lyapunov vectors and error growth patterns in a T21L3 quasigeostrophic model. *Journal of the atmospheric sciences*, 54(2), 347-361.

Schubert, S. and Lucarini, V. (2015), Covariant Lyapunov vectors of a quasi-geostrophic baroclinic model: analysis of instabilities and feedbacks. *Q.J.R. Meteorol. Soc.*, 141: 3040-3055. doi:10.1002/qj.2588

---

Interactive comment on Nonlin. Processes Geophys. Discuss., <https://doi.org/10.5194/npg-2018-25>, 2018.