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Nonlinear Processes in Geophysics
Discussions
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Interactive comment on "Fluctuations of finite-time Lyapunov exponents in an intermediate-complexity atmospheric model: a multivariate and large-deviation perspective" *by* Frank Kwasniok

Anonymous Referee #1

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This manuscript investigates the fluctuations of the finite-time Lyapunov exponents in a three level quasi-geostrophic model. Fluctuations as well as the correlations of these fluctuations are analysed by means of empirical orthogonal functions and the large-deviations formalism. This work constitutes a relevant contribution to the general question of how "chaoticity" fluctuates, which can be potentially interesting for forecasting purposes. The scientific content is novel.

My main criticism to this work is its confusing presentation, which seriously hinders the readability and the appeal of this work. My judgement is that a substantial revision of

the manuscript is needed prior to publication.

I have several technical questions that I organize by sections:

* Section 2

In the model, if I correctly understand, the "h" term is only a parameter (not a function). If so, I think the model is invariant under a zonal shift. This neutral transformation would imply a second vanishing Lyapunov exponent in the spectrum. Please, clarify this point.

* Section 6.1

Concerning the last sentence in Sec. 6.1. I note that the equivalence between Lyapunov exponent fluctuations measured from Gram-Schmidt vectors and from covariant vectors, was detected already in Figs. 5 and 7 of Ref. [1].

In fact, the large fluctuations observed at the edges of the spectrum are not really surprising, at the light of the previous results on the diffusion coefficients in (Kuptsov and Polity, 2011) and [1].

* Section 6.2

It is absolutely necessary to include one formula defining the fraction of explained variance, in order to ensure the self-consistency of the text.

* Section 6.3

It is not said which is the total length of the time series used.

The value of τ_r is "hidden" in Sec. 3.

The three methods used to measure $D_{j,j}$ are not fully clear to me. I think the author should make a list with the three methods specifying which formulas are used in each one. And which parameters are used. Now the explanation is hidden in the caption of Fig. 5, and is hardly understandable.

After reading it several times I'm not sure if I correctly understand: Method one (red line) is computed directly from Eq. (14). The other two methods use the curvature of the rate function at its maximum. In one case (green line) the rate function is interpolated following Eq. (26) for several τ values. In any case, it is clear to me that this cannot be so good as method 1, because of the interpolations needed (details of this are unfortunately skipped). The last method (black line) uses the rate function in Eq. (29), which depends on previously estimated correlation lengths. All this information should be presented in a much more clear fashion. Now it is a mess.

Apart from the previous questions I have other minor recommendations/typos, (but I encourage the author to implement any other improvements in the presentation he may think of):

1. Two lines after Eq. (13), I would write scalar product instead of norm, because this is what matters for Gram-Schmidt orthogonalisation. I suppose the energy norm is trivially related to the scalar product used.

2. Vectors should be always typed in bold face, also for Greek letters.

3. Orthogonality of the eigenvectors, Eq. (18), is better written after Eq. (16).

4. The equation in the text preceding Eq. (18) is apparently lacking of $-\lambda$.

5. When introducing Eq. (27), it would be important to cite at least (Touchette, 2009) again and to mention this is the Gärtner–Ellis Theorem (if I'm not wrong).

5. The "log" symbol is missing in Eq. (35).

6. Figures 4-7 should be introduced in the text, one by one.

7. Last line of page 10. τ_c has not been defined.

8. Page 12, line 5. I don't appreciate smaller deviations of the rate function in this case

than for the zero exponent.

9. In figures 5-8, I would use different colours for the lines in panel (b), at least for the coloured ones since they are not related to the same colours in panel (a).

10. In the conclusions, it is mentioned that the most unstable exponents exhibit slower convergence to the large-deviation limit. Let me to point out that this is fully consistent with [1].

11. Labels (a), (b), etc. need to be included all the figures. This is critical in Fig. 9.

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[1] Pazo et al, Phys. Rev. E 87, 062909 (2013).

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