Responses to Reviewers' Comments

The author would like to thank both reviewers very much for their useful comments which helped further improve the manuscript.

Reviewer 1

- A. Large deviation theory
  Q:
  The presentation of the large deviation theory is a mess, specially considering the presentation of the block averaging method. Is not enough to say ...
  A:
  The presentation of the large-deviation theory has been simplified as suggested by the reviewer (see revised manuscript).

- B. Inertial manifold
  Q:
  At the end of Sec. 6.2 the new paragraph on the inertial manifold is not correct. It points to a dimensionality of the inertial manifold (≈ 125) much smaller than the dimensionality of the attarctor obtained by the Kaplan-Yorke formula. This is not possible: the dimension of the attractor is necessarily smaller than the dimension of the inertial manifold.
  A:
  Yes, the author is aware of this. It is not claimed that the two dimensions are the same; it is just stated that there may be a relationship between the standard deviations of the FTLEs and the existence of entangled and isolated Lyapunov modes which needs to be investigated further. This is made clearer in the revised manuscript.

- C. Correlation length

  1. Q:
     The formula for the correlation length in Eq.(57) is puzzling. Where it comes from? What hypotheses have been assumed?
     A:
     The expression for the correlation length occurs in the formulation of the central limit theorem for dependent random variables based on the assumption of a sufficiently mixing Markov process. This information is given in the revised manuscript.
  2. Q:
     The text suggests that the crossover τ for convergence to the large-deviation regime is related to the correlation time of the FTLE. I disagree with this interpretation for two reasons. First, the correlation lengths observed are not much different in comparison with the crossover τ's. Second, in spatially extended systems it is typical that correlation times are insensitive to the system size, while crossover τ scales with the system size as L^z, $z > 0$. This indicates a different operate in each case (Pazó et al., 2013).
     A:
     The author thinks the crossover τ for convergence to the large-deviation regime is determined by both the correlation length and the degree of non-Gaussianity of the distribution of the FTLEs. This is made more precise and linked explicitly

to the definition of the correlation length in the revised manuscript. The severely delayed convergence for the first and the last exponent is due to the strong non-Gaussianity of these FTLEs. The dependence of the crossover τ on the system size is not discussed in the manuscript.

- D. Legendre transform method
  Q:
  I encourage the author to emphasize in the conclusions that the Legendre transform approach appears to be superior to the probability density approach in order to find the rate function (assumed the conditions of the Gärtner-Ellis theorem hold). Moreover the Legendre transform method yields diffusion coefficients fully consistent their direct measurement from the data.
  A:
  Some remarks along these lines have been added to the conclusions.

- E. Figures
  1. Q:
     In Figs. 2(c) and 2(d) the factor $\tau^{1/2}$ should appear in the y-axis label.
     A:
     Done.
  2. Q:
     I have a suggestion, which can be followed or not: In Figs. 3(a) and 3(b), wouldn't it be clearer to set log scale for the y-axis?
     A:
     I tried this but it appears not to be really helpful.
  3. Q:
     Figure 4 is more clear writing $y_j^{(\tau)}$ in the y-axis label.
     A:
     $y_j^{(\tau)}$ is the amplitude of the principal component but Figure 4 shows the amplitude of the eigenvectors $\mathbf{e}_j^{(\tau)}$.
  4. Q:
     Contrary to what is claimed in the author's response, the labels '(a)', '(b)', etc. are not included in the revised ms.
     A:
     I think it will be more convenient to put the labels in during typesetting of the manuscript as some of these will need to be placed outside of the figures (e.g., for Figure 12).

- F. Typos
  1. Q:
     Page 17, line 8, $n \to L/n$.
     A:
     There are actually n subsets (or subsampled time series) and the length of each is L/n (or more precise the largest integer L' such that $m + (L' - 1)n \leq L - 1$. This is clarified in the revised manuscript.
  2. Q:
     Page 21, line 4, to $\to$ too.

A:
Corrected.

Reviewer 2

- Q:
  Line 10, page 2, the author indicates that the large deviation theory will be used to characterize the variability of the FTLEs. I suggest to introduce there a paragraph explaining what is the interest of the large deviation theory to characterize the FTLEs. Also some comments on the usefulness of the approach in line with the results obtained in the manuscript would be worth doing in the concluding section. This should help the reader who is not familiar with the approach to apprehend the importance of the technique.
  A:
  Some comments along these lines have been included in the revised manuscript.

- Q:
  Page 9, line 11, 2 references to Kwasniok (2019a, 2019b) are mentioned. When looking at the list of references, it appears that these works are in progress and are not effective references. This is not accepted by the journal. So these references have to be removed (if not accepted yet), and a possible solution is to provide a brief summary of the technicalities that were referred to in an Appendix. This can be short.
  A:
  The references have been removed.

- Q:
  Line 11 of page 13, please remove the reference to Vannitsem and Lucarini (2016). This is not referred at the appropriate place.
  A:
  The reference has been removed at this place.

- Q:
  Please check the labels in the panels of the different figures. Many are missing, if not all.
  A:
  I think it will be more convenient to put the labels in during typesetting of the manuscript as some of these will need to be placed outside of the figures (e.g., for Figure 12).

- Q:
  Figure 7. Please clarify what is LegTr.
  A:
  LegTr refers to the Legendre transform method and this is clarified now.

- Q:
  Line 4, page 21. “to” should be “too”.
  A:
  Corrected.

- Q:
  Page 23, line 15. Please remove the references to Kwasniok 2019a, 2019b.

A:

The references have been removed.