

## ***Interactive comment on “Nonlinear effects in 4D-Var” by Massimo Bonavita et al.***

**Massimo Bonavita et al.**

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Response to Reviewer of “Nonlinear effects in 4D-Var” by Massimo Bonavita, Peter Lean and Elias Holm.

Anonymous Referee #2 Received and published: 12 July 2018 We thank the Reviewer for the careful reading of our manuscript and the constructive suggestions. We provide below detailed answers to her/his comments.

1) P4, line 24: “Holm et al., 2003 should be Holm et al. 2002. “.

Thanks for spotting this typo, it will be corrected in the final version.

2) P6, section 4: “The experimental design is not clearly defined in this section. The details about the multi-incremental strategy, like the number of outer loops, the horizontal model resolution and the type of simplified physics (dry vs full) in each loop

should be described. Part of this information is found in the caption of Fig.6 as well as in section 5 for the operational ECMWF 4D-Var. It would be clearer if all these pieces of information were gathered at the beginning of this section.

The details of the multi-incremental 4D-Var setups are described in Sec 5 and in Table 1. They will be referred to in Sect. 4 as well, as requested by the Reviewer.

3) P7, line 2-3: It is stated that ‘... the magnitude of the analysis increments appears to be asymptote to a relatively small value ...’ In principle, the analysis increments should converge to zero after a given number of outer loops. Is there an explanation why the analysis increments seem to converge to an asymptote for the temperature and in the stratosphere?

The Reviewer has raised an interesting point. The lack of convergence at outer loop level apparent in the stratospheric temperature analysis increments is mirrored in observation space by the increased analysis and background departures visible in the stratospheric-peaking channels of satellite sounding instruments (Fig. 11, bottom right panel). After some further sensitivity studies, the cause of this behaviour has been traced to the different timesteps used in the inner and outer loops. The difference in timestep leads to different speed of propagation of stratospheric gravity waves, which then leads to oscillating behaviour in the minimization. This will be discussed in the revision, but we defer a more complete treatment of this interesting effect to a future paper.

4) P8, line 29-31: It is stated that the number of inner loop iterations is approximatively 30 for each outer loop and this number is not sensitive to the resolution and outer loop number when using a convergence criterion based on the information content. It is assumed here that a satisfactory convergence is reached for each outer loop. For the experiment with only one outer loop at TL255, is the minimization also stopped after 30 iterations? In this case it is possible that the convergence for all spatial scales may not be reached, which may explain why the results shown in Fig. 13 are much

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worse for one outer loop than for three and four outer loops. In effect, a degradation of  $\Delta Lij20\%$  in Z500 for 12h forecast lead time in the southern hemisphere extra-tropics is surprisingly large. Furthermore, this result seems not sensitive to the degree of non-linearity of the observation operations according to Fig. 13 (linear operators only) and Fig. 12 (all operators). I recall here that the main advantage of the multi-incremental strategy proposed by Veerse and Thepaut (1998) is to find the minimum of the 4D-Var cost function at a lower and affordable computational cost by exploiting the fast convergence of large-scale increments at the beginning of the minimization. The full convergence of the minimization problem for the first outer loops is not necessary since the re-linearization and the change in resolution modify the shape of the quadratic cost function. Most importantly is the convergence of the last outer loop. The convergence of smaller-scale increments is slower and hence embedded in the inner/outer loops process. For a fair comparison, the total number of inner loop iterations should ideally be the same for the experiments with one and three outer loops ( $\Delta Lij90$  iterations). This is what it is done in Veerse and Thepaut (1998) where 80 iterations are used for both the one outer loop (80) and four outer loops ( $4 \times 20$ ) experiments. If only about 30 iterations are used in the one outer loop experiment, then it is important to verify that the gradient norm decreases by at least two or three order of magnitude between the first and last iteration. I suggest that a brief discussion about this issue be added to the text.

As stated on Page 8, Sec. 5, of the manuscript “The number of inner loop iterations in the minimisations is not prescribed, because minimisations stop when a convergence criterion based on the information content of the minimisation is reached. Convergence is usually reached in approx. 30 iterations, and this number has been found not to be sensitive to resolution and number of outer loop relinearizations”. The convergence criterium of the ECMWF 4D-Var minimization is based on an information content type of measure as described in Fisher, 2003 (“Estimation of entropy reduction and degrees of freedom for signal for large variational analysis systems”, ECMWF Tech. memorandum n. 397) and it is typically reached in about 30 iterations, with little sensitivity to

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resolution and number of outer loop relinearisations. It is important to note that this convergence criterium is always met before the hard stop criterium of 50 iterations is reached in the experiments described in the paper. Regarding the convergence properties of the coniugate gradient/Lanczos algorithm for different spatial scales, it is worth noting that the one outer loop 4D-Var runs with a T255 minimization resolution, which is the highest resolution used in the multi-incremental experiments (Table 1), and it runs to convergence (within the criterium described above). We deduce that the reduced skill of the single outer loop experiment is not due to convergence issues or insufficient resolution of the analysis increments, but to its inability to account for nonlinear effects in both the model and the observations. In terms of how degraded the single outer loop analyses and forecasts are, it is well-known that day 1,2 skill scores are sensitive to the verifying truth and its error correlations with the forecast fields. After day 3, when we can reasonably assume that the verifying analysis errors are small in a relative sense with respect to the forecast errors, then the degradation is seen to be in the 3 to 5% range. This is in line with the results of O-B statistics for different instruments (Fig. 11). As a final point, our main objective in the investigation of the impact of additional outer loops is not to compare performance in an “equal cost” scenario, but to approximately quantify how much skill the additional cost of running more outer loops might give. We will add this discussion in the revised version of our manuscript.

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