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Nonlinear Processes in Geophysics Discussions
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Interactive comment on "Nonlinear effects in 4D-Var" *by* Massimo Bonavita et al.

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We thank the Reviewer for the careful reading of our manuscript and the constructive suggestions. We provide below detailed answers to her/his comments. An edited version of the manuscript is provided in the attachment.

1) P4, line 25: ". . .a nonlinear dependence of the J_B part of the cost function on the state." The "J_B part of the cost function" is not defined, but I presume, based on common terminology in the literature that the "J_B term" is the first term to the right of the equal sign in (1). That is, of course, quadratic rather than linear. I'm guessing they mean that the nonlinear change of variables leads to a nonlinear dependence of the gradient of the J_B part of the cost function on the state.

The Reviewer is correct. Eq. (3) has been expanded to explicitly indicate the J_B and J_O terms of the cost function. Also, the sentence on P4, line 25 has been reworded

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to take the Reviewer's comment into account.

2) P6, line 10 and following, discussion of figure 3: The solid line in figure 3 appears to connect the centers of the circles with the centers of the triangles. This should be noted in the caption. I would be interested to see the prior estimate of the standard deviation, and/or the RMS instrument error, including representation error, marked on that graph. I understand that it might not be so easy to arrive at such numbers.

The caption of Figure 3 has been modified to highlight what the Reviewer noted. Also, the background error standard deviation in observation space for the observation type in the plot has been documented in the caption.

3) P6, lines 18-24, discussion of figures 4 and 5: "Figure 4 plots the correlation and standard deviation of these differences . . ." What are the units on the radial axes? Are standard deviations for different observed quantities normalized to give a single number for the total standard deviation? If so, how?

The units on the radial axes are Kelvin. We have plotted the standard deviation of the differences of the departures between the linear and non-linear stages of 4D-Var. For satellite observations, these are departures in the brightness temperature [K] and for radiosonde observations these are departures in the temperature [K]. We have improved the figure caption to provide a better description of what has been plotted. We have not normalised the standard deviations in this figure.

4) P7, line 1: "The magnitude of the analysis increments is seen to gradually decrease . . ." Can the authors extract a rate of convergence of the iteration process? Given some reasonable set of assumptions, e.g., local convexity of the nonlinear cost function, do we know what rate of convergence to expect?

To the Authors' knowledge there are no practically applicable convergence results for incremental 4D-Var. We could certainly estimate a convergence rate from our experimental results, but we do not think this would be of general interest, as it would be

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sensitively dependent on the specifics of the experimental setup, and in particular on the resolution and time step used in the outer and inner loops. Also, as it is apparent from the results shown in the paper, convergence may be achieved for a given number of outer loop iterations in some parts of the control space (e.g., stratosphere) and not in others (e.g., tropical troposphere).

5) It is well known that for the cost function based on linearized assumptions, the value of the cost function at its minimum should be a random variable with chi-square distribution on a number of degrees of freedom equal to the number of independent observations. How close do the cost functions discussed here come to satisfying the chi-square test at, say, 95% confidence?

The Reviewer raises an interesting point. Increasing the number of iterations reduces somewhat the value of the Jo term of the cost function, but not enough to significantly alter the chi_squared statistics. These statistics show that for conventional observations the current ECMWF assimilation system is close to statistical consistency (i.e., the normalised cost function is in the 0.4-0.5 range), while for satellite observations values of the cost function are typically in the 0.1-0.2 range. This is due to the significant spatial and spectral error correlations of these observing systems and thus the reduced number of effective independent pieces of information that these observations provide.

6) p8, line 11: “No further improvements were seen with further increases in the outer loop count, which points to residual model deficiencies . . .” Another possibility is the observation error, again including representation error. Can it be that the model is already fitting the data within the assumed level of data error?

The prescribed wind error standard deviations vary as a function of height and specific observing system, but in the atmospheric layer shown in Figure 10 (100-400 hPa) they are in the range of 2 to 2.5 m/s for both zonal and meridional components. This implies wind vector observation errors of $\sim$3 to 3.5 m/s. As it is evident from Figure 10, even

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after 5 outer loops the analysed nonlinear trajectory still presents localised wind vector differences significantly greater than 4m/s. We concur with the Reviewer in that, given the rather extreme weather conditions, this residual discrepancy could be accounted for by representativeness errors of the observations themselves. We have modified the text accordingly to allow for this possibility. One could also argue that if we think of representativeness errors as model errors projected into observation space, then this is just a semantic distinction.

7) Figures 6 and 7 need better labels on the ordinate axes. In figure 6, the ordinate axis is labeled “model level.” In figure 7 the ordinate axis is not labeled at all. Please change the labels in both to pressure.

This has been done.

Please also note the supplement to this comment:
https://www.nonlin-processes-geophys-discuss.net/npg-2018-20/npg-2018-20-AC1-supplement.pdf

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