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Nonlinear Processes in Geophysics
Discussions
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Interactive comment on "The onset of chaos in nonautonomous dissipative dynamical systems: A low-order ocean–model case study" *by* Stefano Pierini et al.

Anonymous Referee #1

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The authors investigate the onset of chaos in general nonautonomous dissipative systems by means of an ensemble approach, on the example of a low-order ocean model. They use several indicators, among which the cross-correlation between two nearby trajectories is particularly promising since there is no consensus on how to identify chaos in systems of arbitrary time-dependence. The paper is interesting, and well-written. It certainly deserves publication after the clarification of a few points,

The classical literature on dynamical systems (see e.g. Eckmann-Ruelle or Ott) clearly considers chaotic attractors of periodically driven systems usual, i.e. non-pullback, attractors, since they can be observed on stroboscopic maps taken with the period of

driving on which the dynamics appears to be autonomous. With the exception of a short section (5.2), all the examples investigated in the paper are autonomous or periodically driven, their attractos are thus non-pullback attractors. The pullback concept is meaningful to reserve for systems of arbirtrary, non-periodic time-dependence. The authors represent, however, the attractors by ensembles instead of long time series, although the two methods are known to be equivalent in cases with constant or time-periodic driving. The novelty of their approach is that they do not take advantage of the simply form of the time-dependce, their conclusions might thus be valid for systems of arbitrary time dependence, too. I recommend to make clear in the text that the authors investigate non-pullback attractors as if they were pullback.

Further remarks p1, line 18: not all autonomous dissipative system exhibit period doubling cascade, etc. Think of nondriven mechanical systems in the presence of friction: all motion stops, there is no asympotic dynamics. The sentence should be reformulated.

Fig.1: what is the time t at which the attractors are plotted?

Eq.(5): This quantity is similar to the broadly used finite time Lyapunov exponent. Why do you think that sigma is better suited here?

Fig.2: what is the value of T*? How do the plots change if T* changes? Why exactly this T* is taken?

Fig.3, panel c: this is a chaotic case, and the blue curve cannot be distinguished from the red? This contradicts Fig.4a where the green points are spread even if started from the vicinity of P_1.

p7, l4: Fig.4a is taken at t=400, but the caption says t=300. What is the correct statement?

Fig.4a: I do not see the red dots mentioned in line 5 of page 7.

Fig.4b,d: I wonder if the use of this entropy is useful if it gives different values for the same chaotic attractor even after 400 time units. We know that S should asymptotically converge to a costant (exactly since the driving is constant). What if we take more initial points in the small square used in Fig.4a?

p.8, lines 2-5: you speak about a "sensitive dependence on initial data". I agree, there is some dependence but this is certainly different from the traditional sensitivity since the letter holds on the chaotic attractors. Yours is there for a periodic motion and disappears, as you say in line 5, on the attractor. It might be useful to use a different terminology here.

p9, line17: what is the meaning of "spans the attractor roughly six times"?

Fig.10: What is T and T* here? How do the results depend on their choice?

Section 5.2: The system treated here is governed by a real pullback attractor. The time-dependene shown in Fig. 17. is a general one but does not possess any drift. Since systems with drift are important in undertsanding e.g. climate change, the question arises: would <theta>_Gamma be a useful indicator also for this type of systems?

p21, line 7: "In this paper, we studied the transition from nonchaotic to chaotic PBAs in a nonautonomous system whose autonomous limit is is not chaotic. I am lost: is not the attractor of Fig. 1c chaotic?

p25,line 17: The sentence "the union of quasiperiodic orbits that . . . may exhibit a power spectrum that contains a multitude of local variations reminescent of those exhibited by power spectra of the chaotic orbit" is not convincing. Is the attractor quasiperiodic or chaotic? The blue dots in Fig. A2 do not clarify the situation either. Therfore, here at least I recommend to show this attractor also as the result of a long time series (after an appropriate removel of transients).

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