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Nonlinear Processes in Geophysics
Discussions
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Interactive comment on "The onset of chaos in nonautonomous dissipative dynamical systems: A low-order ocean–model case study" *by* Stefano Pierini et al.

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Anonymous Referee #2

We would like to thank the reviewer for his/her valuable comments that have helped us improve the manuscript. Please find below our responses to the reviewer's comments.

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REVIEWER: Section 3.2 is not needed in my opinion. I wander if it is correct to interpret a transient growth as a "sensitive dependence on initial conditions".

RESPONSE: We thank the reviewer for this comment. We have used instead the

expression "sensitive phase dependence on initial data" when referring to the non-chaotic case with sigma>1. Besides, we have also modified the first paragraph of section 3.2 in order to better characterize this specific form of sensitivity (p. 9, l. 12-20): "We conclude the analysis of the autonomous system by discussing an apparent paradox. We have just seen that, in regions of Gamma where sigma> 1, the trajectories for gamma = 1.1 exhibit sensitive phase dependence on initial data, as shown, for instance, by Fig. 3b, by the red dots in Fig. 4c and by the red curve in Fig. 4d. Sensitive dependence on initial data is usually associated with chaotic dynamics, but in this case the dynamics is periodic. This paradox is resolved by noting that such sensitivity concerns only the phase of the periodic trajectories, as already noticed in the previous subsection and, in addition, it occurs only if the initial data lie outside the attractor, e.g, elsewhere on Gamma; on the attractor, this phase sensitivity disappears, as we will show below. On the contrary, in the chaotic case gamma = 1.35, the sensitivity to initial data for trajectories with sigma > 1 always holds, off the attractor as well as on it, in excellent agreement with the chaotic character of the dynamics in the latter case."

In any case, in the same section we have clearly explained the fundamental difference between this sensitivity and that associated with chaotic dynamics, so we believe there can be no misunderstanding.

Having said this, we believe that this dynamical behavior deserves to be highlighted. Moreover, at the end of section 4.1 we have pointed out this: ". . . In our nonautonomous system, the amplitude of the periodic forcing epsilon plays a similar role. This transition to chaos induced by time-dependent forcing appears, therefore, to be directly linked to the existence of regions in phase space in which sensitive dependence to initial data occurs in the limit of periodic solutions. Thus, the chaotic behavior merely due to the time-dependent nature of the forcing can be traced back to the apparently paradoxical property of the autonomous system that was emphasized in Sect. 3.2 This striking observation deserves to be analyzed in greater depth in future studies. . . ." We have therefore not removed section 3.2.

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REVIEWER: Pg 5, line 4: please use the term " time evolution" instead of "time series", as in Pierini 2016, throughout the text.

RESPONSE: Done. Thank you.

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REVIEWER: Pg 5 line 7: (X,Y) is not defined. Is it the initial condition of (Psi1,Psi3) as in eq. 5?

RESPONSE: Yes, but as you noticed, the definition comes later. Thank you very much. In the revised version, we write "depending on the initial point" (p. 5, l. 11).

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REVIEWER: Pg 6 line 5: why sigma> 1 initially? sigma= 1 initially by definition (5).

RESPONSE: The reviewer is absolutely right. Thank you very much. We have removed "initially".

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REVIEWER: Pg 7, Figure 3c. with gamma=1.35 you get chaos. How is it possible that two initially close trajectories never diverge?

RESPONSE: Yes, the red and blue curves are virtually coincident; this is what happens for trajectories leaving from the very restricted cold-color regions of Fig. 2d; on the other hand, this possibility is not surprising in view of the results of Pierini et al. (2016). Of course, these trajectories are nonetheless unstable once they have converged onto the PBA because, as noted in section 4.2 "... they will always pass sufficiently near trajectories that are chaotic, thanks to the mixing properties of the [PBA].".

REVIEWER: This contradicts the statement at the beginning of pg 9.

RESPONSE: We thank the reviewer for this comment. Now we have specified that we

are referring to cases with sigma>1 (p. 9, l. 18-20): "On the contrary, in the chaotic case gamma = 1.35, the sensitivity to initial data for trajectories with sigma>1 always holds, off the attractor as well as on it, in excellent agreement with the chaotic character of the dynamics in the latter case."

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REVIEWER: Pg 9, line 7 and following lines. I understand that you cannot compute with enough accuracy the first Lyapunov exponent (why? I found it strange for such a low dimensional system). If so, you cannot say that "the results prove unequivocally the assumption.."

RESPONSE: We thank the reviewer for this comment. Now we have changed "prove unequivocally" with "are consistent with" (p. 9., l. 25). In the following lines, we explain the typical problems encountered in the computation of the finite-time Lyapunov exponents, but we have not implied that it is impossible to compute them with enough accuracy.

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REVIEWER: Pg 9 line 11: illustrated instead of summarized.

RESPONSE: Done. Thank you.

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REVIEWER: Pg 9 caption of figure 5 and in other parts of the text: "non chaotic".

RESPONSE: Done. Thank you.

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REVIEWER: Pg 9 line 17: 4TDelta=100

RESPONSE: Done. Thank you.

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REVIEWER: Pg 10 line 25. It is a contradictory statement. Please explain it better.

RESPONSE: We thank the reviewer for this comment. Please see p.11, l. 11-13: “Recall that the PBAs of a periodically forced system (e.g., Pierini, 2014, and references therein) are always periodic. This periodicity of the PBAs occurs for both periodic and chaotic systems; the latter are typically referred to as cyclostationary. For the sake of simplicity we will refer below to chaotic and non chaotic PBAs, abbreviated as CPBAs and NPBAs, respectively.”

REVIEWER: Pg 15 eq 10: I do not see the reason to introduce a new symbol for Cmax. There are already a lot of symbols to keep in mind.

RESPONSE: We thank the reviewer for this comment. We have removed the symbol c_max and left only Theta; see the new Eq. (9).

REVIEWER: Pg18, last line: <Theta> -> <Theta>Gamma

RESPONSE: Done. Thank you.

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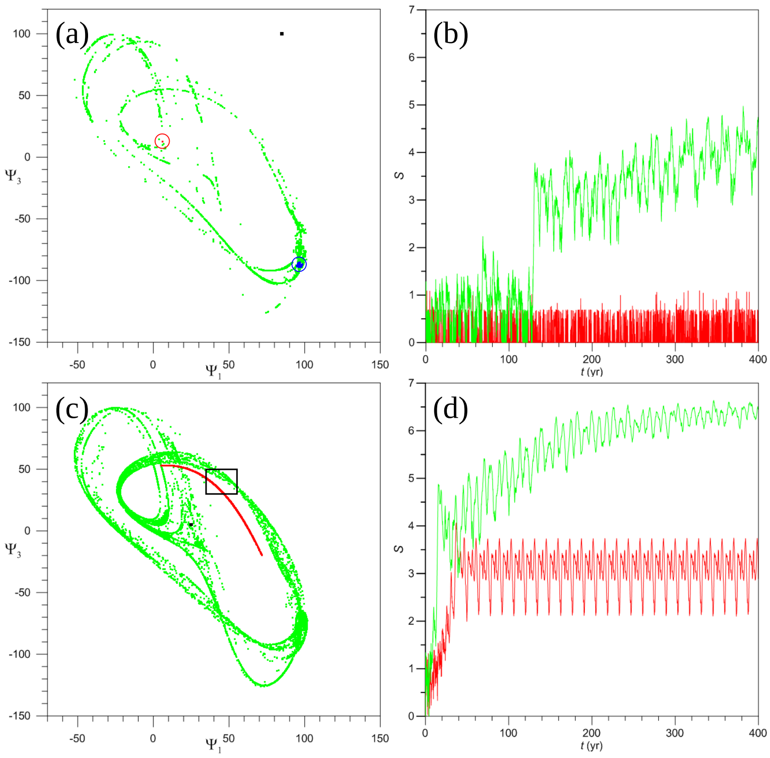

Fig. 1. Figure 4 (modified)

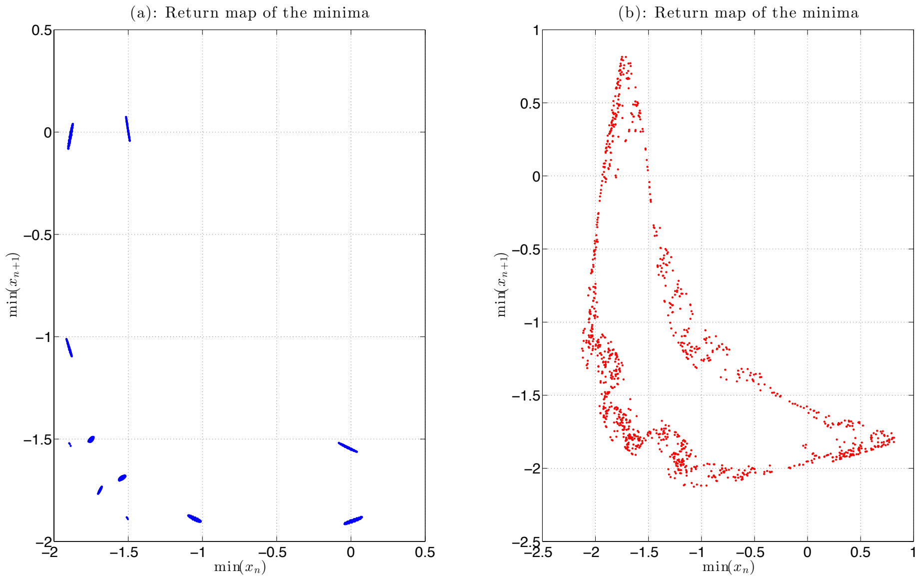

Fig. 2. Figure A3 (new)