

Dear Editor and Reviewers,

The main changes in the revised version of our manuscript are the following. Firstly, we have taken into account the comment (1) of the Referee 2 about the terminology we use. Now we use the term soliton for solutions of the equation for envelope and the term breather for solutions of the compact equation written for the wave train itself (see also our response to the comment (1) below). In addition, we have revised the terminology for the water wave equations. The compact version of the Zakharov equation was derived by Dyachenko and Zakharov in 2011. The envelope version of Zakharov equation was presented by [Dyachenko et al. (2017a)] in the last year. So, the conventional terminology for these versions of Zakharov equation is not established yet. It was suggested in the work [Fedele, F., & Dudykh, D. (2012)] to denote the compact version of the Zakharov equation as compact Dyachenko-Zakharov (DZ) equation. We believe that this nomenclature is better than we used in the previous version of our manuscript. So, the envelope version of the Zakharov equation we now refer to as the Dyachenko-Zakharov envelope equation, or the DZe equation.

We also have extended our study of the soliton space shifts. In accordance with the question (28) of the Referee 2 we performed the additional numerical simulations for the wave steepness: 0.05, 0.1, and 0.15. We have demonstrated how the dependence of the space shifts on  $\Delta\phi$  changes with the value of  $\mu$  - see the figure 10 and also our comment to the question (28).

Then we have found an inaccuracy in the calculation (caused by a misprint in our numerical code) of the origin and sign of  $\Delta\phi$ . After the appropriate correction, all curves depended on  $\Delta\phi$  have been mirrored and shifted a little to the right - see for example the figure 4. We apologise for this inaccuracy and confirm that it does not affect the results of the work. To demonstrate that in the limit of small steepness the dependence of the amplitude amplification becomes very similar to those in the NLS case, we have added simulations for the steepness  $\mu = 0.05$  (compare the curves marked as 4 in the figures 3 and 4).

Finally, we have taken into account the comment (2) of the Referee 2 and now present all results of our numerical simulations in scaled variables - see also our response to the comment (2).

1. The term breathers now is conventional for solitary wave group solutions of the compact Dyachenko-Zakharov (DZ) equation (see the works of Dyachenko, Kachulin and Zakharov cited in the manuscript). We prefer to remain this nomenclature unchanged. However, we agree with the Referee 2, that in case of the envelope Dyachenko-Zakharov (DZe) equation the terms soliton and solitary wave group are more relevant. We have changed the terminology in the whole manuscript and added a short discussion about soliton and breather terminology to the "Introduction" section - see the text marked in red on the pages 1 and 2.

2. Thank you for this question. Now we use the following scaled variables when we present results of the simulations:  $\lambda_0 = \frac{2\pi}{k_0}$  (unit of length),  $T_0 = \frac{2\pi}{\omega_{k_0}} = \frac{2\pi}{\sqrt{g} k_0}$  (unit of time). We measure the velocity of solitons in the unit of group velocity  $V_0 = \frac{\omega_{k_0}}{2k_0}$  corresponding to the characteristic wavelength. The dimensionless wave steepness  $\mu \sim C_0 k_0 / \sqrt{\omega_{k_0}}$ , that is why we measure the wave field amplitude  $C$  in the units  $\sqrt{\omega_{k_0}}/k_0$ .

3. We agree that the suggested references are important and relevant. We have added them (with a short discussion) to the "Introduction" section (page 2) and to the end of the "Conclusions" section (page 15) - see the text marked in red.

4. We have revised all our figures. Now we use different line styles suitable for black and white printing. The style of the coordinate axes was also improved.

5. Done, marked in red on the page 3.

6. Fixed.

7. We have explained the meaning of the parameters  $V$  and  $\Omega$  in details on the page 4 - see the text marked in red. We stressed that  $\Omega$  is nothing more than the new notation for the breather (as well as for the DZe soliton) frequency parameter - see the formula (6). The breather (as well as DZe soliton) solution is determined by two independent parameters: the group velocity  $V$  in the laboratory frame of reference and the frequency  $\Omega$ . Note, that the carrier

wavenumber of the breathers (DZe solitons)  $\tilde{k}$  is not fixed equal to 100. The value of the solitary group carrier wave number  $\tilde{k}$  (and the carrier wave length  $\lambda = 2\pi/\tilde{k}$ ) is defined by the parameter  $V = \frac{1}{2} \sqrt{g/\tilde{k}}$ . The second parameter  $\Omega$  has the value close to  $\frac{\sqrt{g\tilde{k}}}{2}$  (or  $g/4V$  see formula (6)) and implicitly defines the shape and the amplitude of the breather (DZe solitons).

8. We agree with this comment. We have added one additional sentence to the beginning of the section 4 clarifying that we fix  $k_0$  first. Note that now we use scaled variables according to the comment (2).

9. The equation (1) and the DZe equation (11) have the same accuracy. The DZe equation (11) was derived without any assumptions about spectral width of the wave field – see our comment marked in red after formula (13). The DZe equation is valid for any choice of  $k_0$ . The choice of  $k_0$  do not affect to the breather solution of the equation (1). We added the definition of  $\omega_{k_0} = \sqrt{gk_0}$  to page 4 (red text after the formula (10)) and clarified that  $k_0$  is an arbitrary wavenumber.

10. Done, marked in red on the page 5.

11. We have clarified the meaning of  $x_0$  at the page 5, right after the formula (16). However, we prefer to remain our notation  $C_0$  for the soliton amplitude, since in the case of DZe equation the letter  $C$  is conventional notation for the wave field amplitude.

12, 13, 14. Thank you for these comments. We agree that the section 3 in its original version included many technical details which made the text not clear. We have excluded several sentences and now simply write that the group shapes for the given amplitude are different in the frameworks of the NLS equation and DZe equation (as was suggested by the Referee 2). We hope that the section 3 became self-consistent. We also believe that now the expression for  $C_{sk}$  is not necessary since we do not discuss the shift of the soliton carrier wave number anymore.

15. Thank you for this useful comment. We have added this explanation to the end of the section 3 – see the text marked in red.

16. Done, marked in red at the beginning of the section 4.

17. Done, marked in red on the page – see the text marked in red before the formula (17).

18. No, everything is correct. The phase dependence  $\phi(x, t)$  (see the formula (16)) is given by

$$\phi(x, t) = \phi_0 - \frac{4k_0^2}{\omega_{k_0}} U(x - x_0) + \frac{2k_0^2}{\omega_{k_0}} Ut - \frac{c_0^2 k_0^2}{2} t.$$

The time dependence  $\phi(t)$  for the moving soliton ( $x = x_0 + Ut$ ):

$$\phi(x, t) = \phi_0 - \frac{2k_0^2}{\omega_{k_0}} Ut - \frac{c_0^2 k_0^2}{2} t.$$

19. We have added the formula (18) which describes the phase shift in the case of NLS solitons. The sentence after formula 19 was also corrected – see the text marked in red.

20. Done, marked in red on the page 8.

21. We agree with this comment. We have added the formula (22) which clarify that the difference in parameters  $\Omega_1$  and  $\Omega_2$  makes the phase not time invariant.

22. This question is very important. Indeed, we use the simplest definition of the relative phase  $\Delta\phi$  given by formula (23). The expression (23) allows us only to compensate the phase difference that solitons acquire during propagation to the collision area. We agree that the more appropriate choice of the definition of  $\Delta\phi$  is needed – see our comment marked in red at the begging of the section 4.1. We have tried to account phase shifts, changes in the propagation time due to space shifts and some other effects to correct the definition of  $\Delta\phi$ . However, we have no better variant for the expression (23) so far.

23. The scale of the inset picture in the figure 6 was not appropriate to represent the low amplitude radiation. We have improved the inset pictures in the figures 5 and 6 to make the radiation visible.

24 and 25. We have added the expression for the Hamiltonian in  $x$ -space (formula (12)) and clarified at the page 11 that it defines the energy of the wave field (see the formula (24) and the red text around it). In addition, we have clarified that we calculate the total energy loss relative to the total energy of our system (see formula (26) and the red text above), while the individual changes of soliton energies are calculated relative to their individual energies (see the beginning of the section 4.2 and formula (27)). The latter means that in the figure 8 each curve is normalized on different value of soliton energy and thus the sum of the presented functions is not constant.

26. We removed the part of the statement "...the total energy loss due to the radiation is enhanced at large values of the wave steepness..." and hope that now the mentioned sentence is self-consistent – see the text marked in red in the "Conclusion" section.

27. We believe that the problem was caused by the non-clear explanation of which soliton we mark as first and which soliton we mark as second. We have stressed at the beginning of the section 4 and than recall one more time on the page 11 that the soliton 1 is initially located at the left and the soliton 2 is initially located at the right. After collision the solitons swapped their places, so the second soliton in the figure 9 is located at the left. We have checked that the figure 8 is consistent with the figure 9.

28. Thank you for this question. We have performed additional numerical simulations for the wave steepness: 0.05, 0.1, and 0.15. The results are presented in figure 10. We hope that now it is clear how the soliton space shifts curves tend to the small steepness limit. The curves become almost straight, however there are still two separate curves due to the discussed in the manuscript differences between solitons in the NLS and DZe models. We have added the additional discussion of the figure 10 at the end of the section 4.2. – see the text marked in red.

29. Done, marked in red in the "Conclusion" section.

30. Done, marked in red in the "Conclusion" section.

31. We have edited the whole text of the manuscript and improved it significantly.