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Interactive comment on "Phase-dependent dynamics of breather collisions in the compact Zakharov equation for envelope" *by* Dmitry Kachulin and Andrey Gelash

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Dear Editor and Reviewers,

We agree, that the mentioned by the Referee1 works [1,2] are important in the studies of the properties of the deep water surface waves equations and we have added the corresponding short discussion to the section "1 Introduction" (see the corrected paragraph marked in red). However, we do not consider it appropriate to study peakon-type breathers in our work for the following reason.

First we note that in our work we study compact Zakharov equation for envelope writ-

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Discussion paper

ten using "super compact" complex normal variable $c(x)$ (see the equation 1 in our manuscript). Meanwhile in the works [3,4,5] the compact Zakharov equation is written in different complex normal variable $b(x)$. In our response we use the notations from the papers [3,4,5], i.e. variable $b(x)$ and stress that the same arguments are also valid for the "super compact" variable $c(x)$.

The compact Zakharov equation is derived in the assumption that all waves are unidirectional [3,4,5]. This means that the Fourier spectrum of the normal variable $b(x)$ has only positive wavenumbers. Thus the dynamic equation includes projection operator $\hat{P}^+$ to the upper half plane. Eigenvalues of the operator $\hat{P}^+$ in the Fourier-space are presented by the step-function $\theta(k)$:

$$\begin{array}{llll} \theta(k) = & 1, & if & k > 0 \\ \theta(k) = & 0, & if & k \leq 0. \end{array}$$

The dynamic equation for the envelope $B(x,t)$ of the wave train with the carrier wave number k_0:

$$b(x,t) = B(x,t)e^{ik_0x - i\omega t}, \tag{1}$$

must include the operators $\hat{\theta}(k_0+k)$ acting as $\theta(k_0+k)$ in nonlinear parts of the equation to cancel Fourier modes with the wavenumbers $k < -k_0$.

Unfortunately, the early papers of Dyachenko and Zakharov [3,4] have misprints. While the derivation of the compact Zakharov equation in the Fourier space is correct, in the final explicit form of the equation in x-space the projector $\hat{P}^+$ was missed (it was a typographical mistake). In the later papers (see, for instance [5]) this mistake was corrected.

In the paper mentioned by the Referee1 [1] the envelope equation has no operators of the form $\hat{\theta}(k_0 + k)$. Therefore, this equation is coincide with the compact Zakharov equation for envelope only in the case of the narrow spectral bandwidth $\frac{\Delta k}{k_0} < 1$. This is consistent with the fact that one can extract the NLS and the Dysthe equations both from the Zakharov equation for envelope [6] and from the envelope equation written in the paper [1].

However without using of the projection operator the equation written in the work [1] and equation studied in our work differ significantly at large spectral widths of the solutions i.e. in the particular case of peakons. Indeed, as was found in the work [1], peakons have large spectral width – see the sentence at the page 652 "Figure 7(a) shows the numerically converged peakon obtained via the Petviashvili scheme using $N \sim 1.5\times10^6$ Fourier modes...". In our preliminary simulations that we have made with compact Zakharov equation (including the projection operator) using the Petviashvili method we do not observe the bifurcation of breathers to peakons even at high wave steepness. Our numerical Petviashvili scheme allows to find very narrow breathers having only three waves under the envelope. The wave steepness of such solutions reaches ≈ 0.6. As we understand the results of the work [1] the bifurcation of breathers to peakons was observed already at significantly smaller values of wave steepness. Thus we believe that the question about peakon-type solutions in the compact Zakharov equation for envelope of unidirectional waves demands separate detailed study.

In our work we focus on the breather solutions that were recently successfully reproduced in the water wave tanks – see the works [7,8]. We stress, that we have found the fundamental differences of the breather interactions comparing to the simple NLSE soliton dynamics. We observe not only radiation and magnification of amplitude amplification, but also the energy exchange between breathers and nontrivial behaviour of the breather space shifts. All the mentioned effects turned out to be phase-dependent and is to be studied in laboratory experiments.

Answer to the question "Why does it take several collisions before solitons radiate energy?"

Indeed, previously Dyachenko, Kachulin and Zakharov shown in the work [3], that multiple breather collision in the compact Zakharov equation leads to the energy losses.

However in the work [3] only one particular case of breather phases was studied and the steepness of the breathers in numerical experiments was small, about 0.08. Thus, the minor energy radiation becomes clearly visible only after the 100 breather collisions. In our work we study the complete dependence of energy looses from the relative breathers phase in the case of lager breather steepness 0.15. We have found that the level of radiation reaches 3% of the total breather energy at certain phase synchronisation $\Delta\theta \approx 0$ (see the figure 7 in our manuscript), so that incoherent radiation is distinguishable even after a single interaction of breathters – see the last snapshot in figure 5.

We have added the corresponding short discussion to the section "5 Conclusions" (see the corrected paragraph marked in red).

Yours sincerely,

Authors

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