Reply to rev. 1

We would like to thank S.E. Cohn for his review on our paper and for giving us the opportunity to improve our paper.

We have improve the description of the numerical experiments with some details on the implementation used: finite difference for the spacial discretization, a fourth order Runge-Kutta for the time scheme and we have specified the numerical setting (time step, numerical value of the diffusion coefficient). The ensemble size has been increased to 6400 in order to limit the sampling noise, and a single ensemble of normalized error has been generated then used with appropriate initial error magnitude – this reduces the sampling fluctuations when comparing the numerical results from a method to another.

In order to investigate the limitation of the tangent-linear covariance dynamics the manuscript incorporates new results (even if further research are still needed to investigation more completely the limitations of the parametric formulation as highlighted in the manuscript):

A study of the mean predicted by the parametric model and estimated from the ensemble has been introduced in order to illustrate the ability of the PKF to provide an estimation of the true mean state when small non-linearities are present: see Fig. 4 and the new section 4.3.1. A long term experiment has been introduced to determine if there is an exponential growth of the error that could be a side effect of the tangent-linear approximation: see Fig. 8 and the new section 4.3.3. The discussion of the results has been put in a new section 4.3.4.

We copied your commentary in italics below, we reply in normal blue font

Major comments:

two errors:
1) *"First, there is an error passing from Eq. (27b) to Eq. (28b): a coefficient 2 has crept into the third-from-last term on the right side of Eq. (28b) which does not belong there, and it is repeated in the final Eq. (29c). If this is just a typo, it simply needs to be corrected. But if this error has also made its way into the computer code, then the numerical experiments will need to be re-run."*

This has been corrected, thank you very much.

2) *"Second, the Gaussian initial covariance function, Eq. (30), is not appropriate for the geometry of the numerical experiments. Since the domain is periodic, the distance |x-y| should be replaced by a distance function that reflects this periodicity, such as the great-circle distance or chordal distance. As it stands, the covariance function has a slight first-derivative discontinuity at |x-y| = D/2, and this introduces spurious odd-order terms in the series expansion of the correlation function that have been neglected. Although this might be a small effect initially, since the initial correlation length L was taken to be small, the numerical experiments showed that the correlation length grows by nearly an order of magnitude. The numerical experiments will need to be repeated with an appropriate initial covariance model."*

For the experiment considered in the manuscript, only the initial covariance Eq.(30) (from the previous version) is needed. Since the length-scale considered is relatively small considering the length of the domain, the Gaussian correlation applies here. However, we agree with the referee that this is not strictly a valid correlation function, and a chordal distance has been introduced in accordance with the geometry of the domain. This modification does not change the results but is theoretically better.

Minor comments and typos:

1. *It is mentioned a few times that "operator splitting" (p.2 l.25) or "time splitting" (p.6 l.26) is used in the derivation. Actually, the authors are simply carrying out the derivation term-by-term without any approximation introduced by doing so. The authors' use of this terminology is not at all standard; usually it means that a time discretization error is introduced in a numerical method. I suggest removing the terminology altogether.*
We agree with the referee that the terminology "time-splitting" could introduce a confusion with the classical numerical time-splitting. In order to avoid this confusion, we have introduced the following lines in section 3.1, where the splitting method is mentioned:

"The splitting strategy is a theoretical method to deduced the so-called infinitesimal generator of an evolution equation, by taking advantage of the Lie-Trotter formula to separate each processes (or appropriate arrangements of the processes). This strategy should not be confused with the numerical time-splitting which introduces numerical errors (Sportisse, 2007)."

2. P.7 l.13: I would change "Hilbert space" to the more general "function space" since no Hilbert space apparatus has been introduced in the paper.
Yes the Hilbert structure is not important here and it has been replace by "function space" as suggested by the referee.

3. In the title itself, the apostrophe after Burgers should be removed: the possessive is not correct here.
The typos is now corrected.

4. P.4 l.16: recipes → recipe
The typos is now corrected.

5. P.6 l.12: te → the
The typos is now corrected.

6. Eq. (25): One appearance of δx 2 in the second term, and one in the third term, should be removed.
The Taylor expansion Eq.(25) has been corrected.

7. P.10 l.8: express → expressed
The typos is now corrected.

8. Eq. (29): The subscript x on the symbol V should be used consistently.
The subscript x has been removed from Eq.(29).

9. P.13 l.33: 8.8 of → 8.8 times
The typos is now corrected.

10. P.14 l.9: 5.5 of → 5.5 times
The typos is now corrected.

11. P.14 l.12: 7.5 of → 7.5 times
The typos is now corrected.

12. P.18 l.13: variance field → normalized variance field
The typos is now corrected.

13. P.18 l.21: third term → third order term
The typos is now corrected.

14. P.19 l.1: fourth term → fourth order term
The typos is now corrected.

15. P.19 l.8: third order term → fourth order term
The typos is now corrected.