Interactive comment on "Simple statistics for complex Earthquakes' time distribution" by Teimuraz Matcharashvili et al.

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Dear reviewer, let us express our sincere gratitude for your work and competent evaluation of our manuscript. In our opinion most of your questions are answered below or we explain our vision of certain questions. Also, let us inform you that as far as the same or almost similar questions are repeated along in the reviewers text we apologize that could not avoid some repetitions in our answers too.

Anonymous Referee #1

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The authors study the southern Californian earthquake catalogue (1975-2017) analyzing the extend of regularity in the time series that is defined by the occurrence of earthquakes with magnitudes above 2.6. For that purpose they introduce the "integral deviation times" (IDT), a simple statistic measure that corresponds to the sum of the deviation times of the earthquake occurrences to regular times steps.

As the authors state, the earthquake time distribution does not follow the patterns of a random process and there are several studies on the determination of the regularity of seismic processes and its changes in time. Yet, with regard to the presented IDT method I have several doubts concerning the appropriateness of that measure. Further, I see some weaknesses in the design of the analysis and clearness of the paper.

At times it looks like you apply a bunch of methods without knowing why and what do you want to show.

We thank reviewer 1, for this remark. We would like to underline that we definitely know why several ("bunch" of) contemporary methods of data analysis have been used in the present work. These well known and often used methods (LZC, RQA, CMSE) are very effective tools, when correctly used, for the task of quantification of dynamics of complex processes: examples can be easily found in a number of articles from different fields. We regret that, as it appears, what we wanted to show using certain data analysis methods was not clear in the previous version of the manuscript. In the corrected version, it is underlined that these methods have been used for the very important for our research task. Namely, it was required to ensure that the simulated random data sequences (here different type of noises and Poisson process data sets), being generally complex and random-like, are still different in the sense of underlying dynamics and that these differences are quantifiable. The problem is that the simulated noise datasets, by the conditions of their generation, should differ by the features of their frequency content. At the same time, it was necessary to know if these data sets are different in the sense of regularity, especially at small differences between spectral exponents. Here, it was necessary to assess the extent of regularity in noise data sets from different point of views, i.e. use analysis methods based on different underlying principles. For our research purposes such testing by standard analysis tools was absolutely necessary step prior to proceed to the analysis of the same simulated data sets by IDT. Next, we needed to be convinced that such a simple statistical method like IDT can discern differences in dynamical features of complex high-dimensional processes (differences in which already have been documented by standard complex data analysis methods). This is why we spent considerable part of our time and carefully compared results of IDT analysis with the results of other, well known and many times critically tested, methods (here LZC, RQA and CMSE). These analyses bring us to the conclusion, that results of IDT are in principal agreement with the results of used standard tools of complex data analysis.

Here, for readers who are not so aware of the details of modern complex data analysis, the following question may arise - if the results of IDT agree with those obtained by some other methods, why do we need to develop a new tool giving similar conclusions? Also, it indeed may be a need of an additional explanation why standard methods have not been used together with IDT calculation for the real (obtained from earthquake catalogue) data sets. At first, we should state that each of methods are developed to test data sets from a certain point of view, e.g. LZC is based on information theory, RQA -on phase space population testing, CMSE – on entropy assessment, etc. Moreover, all the complex data analysis methods (used here and others) to be correctly used necessitate special conditions to be fulfilled both in the sense of quality and length of data sets, as well as in the sense of calculation purposes (e.g. conditions for reliable phase space reconstruction or coarse-grained series construction). Therefore, knowing weak and strong sides of these methods for certain data sets, we additionally wanted to have a testing method based on the very simple statistical and distributional features of complex process (data sets). This was interesting to get in this way the possibility to look at the complex process from a simple new point of view, which will not be complicated by the fundamental principles of method's accomplishment. Such simple vision definitely has its own restrictions and, as in the case of any other data analysis methods, should be used correctly. Anyway, as it follows from our results, proposed calculation method is effective for data sets (especially for short ones), simulated from complex processes as well as original and randomized data sets of earthquake's time distribution. Such test is very important for the usually not perfect quality data sets of the real measurements. It should be also pointed out that the IDT calculation method has no practical restrictions on the length of data sets because of its simplicity. (we mean statistically reasonable length of data sets of at least several tenth of data).

The interpretation of the results could be more detailed and more related to the application, otherwise it is hard to see, what are the findings provided by the paper.

Together with the presentation of a simple and effective method for complex data sequences analysis, in this manuscript we present the results of its application for the time distribution of earthquakes taken from the south Californian catalogue. Main finding of this work is a clear quantitative demonstration that the extent of regularity of earthquakes time distribution is changing over the time. It was shown that, over the period of analysis, we can indicate periods when earthquakes' time distribution became most random as well as those when it is less random. Such a finding for the seismic process, in our opinion, is of immense importance, as far as by many authors seismic process still is regarded as completely random, i.e. not having a quantifiable dynamical structure (unpredictable). Most important is that the extent of randomness never reaches its maximum in periods immediately prior to strongest earthquakes. This points to the increase of determinism in earthquake generation process (at least in temporal domain) and thus makes researches aimed at finding of the precursory markers for strong earthquakes in the complex seismic process, a well-grounded scientific task.

In the following I comment on the mentioned shortcomings in more detail. The motivation of the paper could be stronger. Why is it important to identify changes in the regularity of seismic activity? Do you expect to gain any knowledge for a better understanding of seismic processes? Do you expect to the give better predictions on earthquake occurrence based on changes in regularity? You should also refer to (some of) these questions in your conclusion.

The main general problem targeted in our last researches concerns with the dynamics of seismic process. Exactly, investigation of features of earthquakes time distribution is often posed important task not only for us, but also for many research groups worldwide for last decades [e.g. Davidsen, C. Goltz, 2004; Kawamura. 2007; Kenner, M. Simons, 2005; etc.]. Motivation for the present work was to assess how the extent of regularity in the earthquakes time distribution changes over the considered period of catalogue time span. It needs to be underlined that, in spite of the above-mentioned and many other studies, the problem of how regularity of seismic process is changed still remains unanswered. At the same time, it is clear that without such knowledge the better understanding of seismic processes can not be achieved. Moreover, scientific posing of such general tasks as earthquake prediction or control of seismic processes, will not be look grounded unless basics (at least main) of features of its dynamics in spatial temporal or energetic domains will not be understood.

*Also the provided background (domain)
information could be more precise. Why do you consider only earthquakes with magnitude
above 2.6 and after 1975? Please refer to the magnitude of completeness and
possible changes in the time series due to improvements in recording. You should
also report on the characteristics of seismic activity, e.g occurrence of cluster, foreshocks
and aftershocks accompanying major earthquakes, assumption of ii d (Poisson
process) occurrence for declustered catalogues.*

As it is said in the revised version of manuscript, we aimed to analyze temporal features of the original (natural) process of earthquake's generation. For this purpose, we selected a best quality catalogue of southern Californian seismic activity (Fig.1). Being aware of the problems that can be caused by the inappropriate "bleaching" of complex data sets [e.g. Abarbanel, 1993], and aiming at the analysis of temporal features of seismic process, we would like, when it is possible, to avoid any cleaning, filtering or declustering of catalogue in order to preserve its original time structure. Consequently, we tried to have as possible long period of observation with as possible low representative threshold. For this purpose, according to results of time completeness analysis

(Fig. 3) we decided to be focused on the time period from 1975 onward. Indeed, we see that since the middle of 70th of last century Mc was clearly decreased what finally enabled us to work with the southern Californian earthquake catalogue at the representative threshold M >= 2.6, according to the Gutenberg–Richter relationship analysis (see Fig. 2). We understand that in such catalogue we deal with both, independent as well as dependent (aftershocks or foreshocks) events. Presence of both type of events in the catalogue looked for us quite acceptable in the frame of research task because here we speak about the general features of time behavior of seismic process and also because the physics of generation of dependent and independent events is similar [Davidsen and Goltz, 2004; Martinez, et al. 2005]. In any case to assess possible influence of dependent events on the results of our calculations, we performed analysis at higher representative thresholds M3.6, M4.6 and even for M5.6, when this was possible because of small number of events. According to our analysis dependent events do not essentially influence results of IDT analysis.

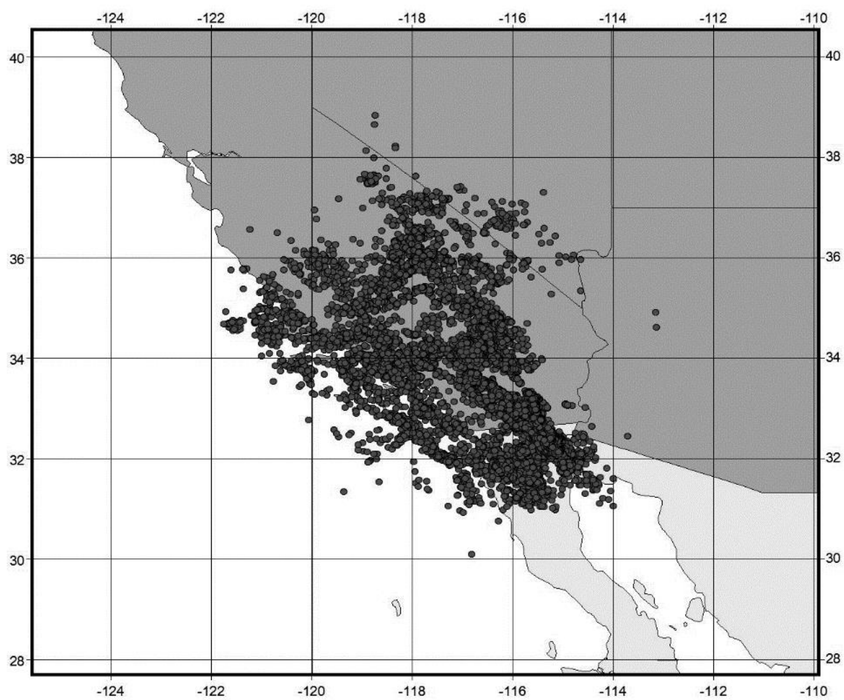


Fig. 1.

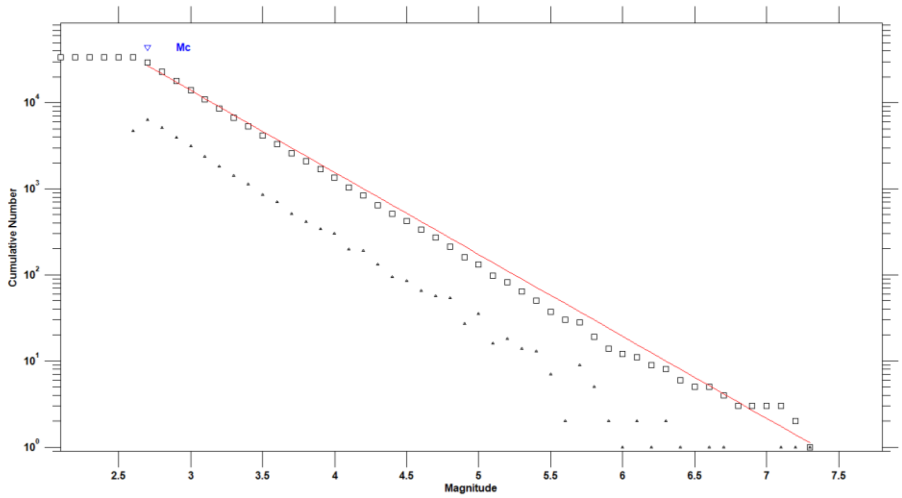


Fig. 2.

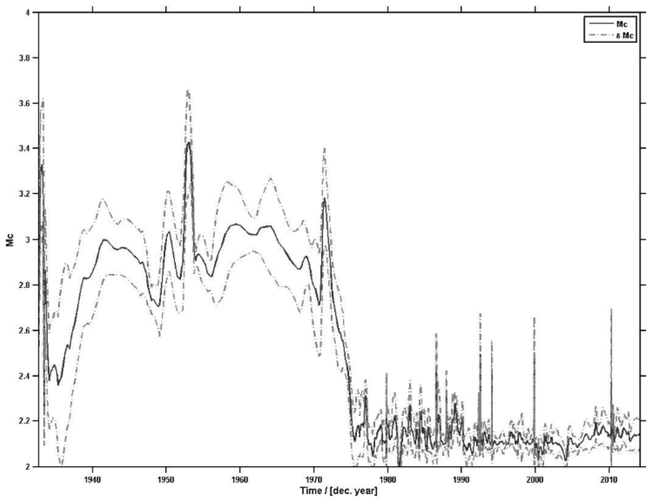


Fig. 3.

It would be also nice to see a plot of (a part) of the time series, that e.g. illustrates the clustering of earthquakes in time.

We analyzed the original (not filtered) catalogue (Fig.1) with obvious natural clustering in different domains. The time clustering in this catalogue is well known and described for many times (also in one of our previous article Matcharashvili et al, in Physica A 433 (2015). Thus, the earthquakes time clustering in Californian catalogue is obvious for our research period and in our opinion there is no need for additional illustrations. This is why we do not show interevent time series, which was possibly meant by the reviewer 1.

Also comment on why did you choose to study the southern Californian catalogue and clarify if there are any issues with induced seismicity.

As mentioned above, we have used southern Californian catalogue for its high quality. Having such best quality catalogue, we do not aimed to go further in the analyses of effects like of induced seismicity. At the same time, testing carried out at increased representative thresholds apparently shows that their influence on IDT calculation results can be regarded as negligible.

As mentioned above, I have some doubts regarding the appropriateness of the IDT measure. On page 2 , line 16-17 you state IDT should approach zero for random sequences, if n goes to infinity. First, please correct the subsequent sentence, which says IDT approaches infinity for large n (I guess, this is a typo). Second, the statement needs to be proofed. Actually I doubt, that it is true.

Sorry, but it is not clear where the typo is. In our text it is said: "the sum of the deviation times should approach zero in the infinite length limit". As we understand in common parlance, this means that IDT approaches zero if number of deviations is large enough.

Generally the question of close to zero IDT values was partly discussed above and here we add some following thoughts. Logically IDT should approach zero for random sequences, if *n* goes to infinity, and empirically for sequences closer to randomness we indeed get IDT closer to zero comparing to less random sequences. Presented in the revised version of manuscript new analysis and results obtained after reviewer's remark confirm this statement. Theoretical basics of IDT will be given in the next article in collaboration with our colleague Prof. Czechowski from Institute of Geophysics, Warsaw, Poland. Thus, in the present work we decided to be restricted by strong empirical argumentation on the certain data sets. As an example of sequences with different extent of randomness we used the series of color noise data sets. In general, there may be many different random sequences and the question about which out of these random sequence is "more random" and which is "less random" is not easy to answer. Therefore, according to our purpose (explained above) and to have strong arguments, why we regard some sequences as more and others as less random, in this research we used well known and accepted methods of complex data analysis like PSR, LZC, RQA and CMSE.

Thus, as it is mentioned in the manuscript, we generated artificial noise data sets, which, as it was shown, are quantifiably different – i.e. represent different types of colored noises (in the revised version we consider 7 simulated data sets instead of 8 in the former version, as far as noise data set with β = 1.932 gave result very similar with β = 1.655). Here it is necessary to emphasize that in order to make the simulated data sets closer to a character of the temporal evolution of seismic process, we used the sequences of positive numbers.

As we explained in the manuscript, as well as here above, we needed to have data sets with reasonable differences in the extent of regularity in order to find out, whether calculation of IDT may be sensitive to the dynamic changes taking place in the analyzed data sets. We agree with reviewer that, as far as we aimed to use IDT for seismic data sets, for method testing purpose, it was indeed more logical to consider also random process, which is often used by seismologists – Poisson process. In the present version of manuscript, we added results for Poisson processes with different lambda values, obtained by used methods. Results of analysis is described in the revised manuscript. From these results, we see that the conclusion drawn from a simulated noise data analysis, that more random process gives closer to zero IDT value, is correct for Poisson processes too. In other words, Poisson process and white noise look similar according to results of PSR, LZC, RQA and CMSE analysis. It is interesting that, by IDT results we see that Poisson process looks even more random than simulated data set, which is closest to a white noise.

Only explanation of why reviewer 1 got IDT for Poisson process "by magnitudes larger than the values calculated for colored noise" is apparently connected with the procedure of norming. In order to avoid possible misunderstandings caused by different "time span of window", IDT should be normed to "time span" (cumulative sum) values or compared to calculation accomplished for the same time span - in case of reviewers example, to IDT from the original catalogue.

For further clarity, regarding IDT values for different noises, in the revised version we add pdf curves in Fig.6. For Poisson process, IDT results is not shown in this figure because it is close to white noise.

Let's assume the earthquakes would follow a Poisson process (purely random), the time series that is defined by the deviation times (DT), will be still highly autocorrelated. E.g. P(DT(i)<0 | DT(i-1)<0) >
P(DT(i)<0 | DT(i-1)>0) I calculated IDT for 100 Poisson processes with n=34020 events and an occurrence rate of 34020 / 22167178. The log value of absolute IDT/n was in 92 cases above 8, which is by magnitudes larger than the values calculated for colored noise in figure 5.

Once you generated Poisson data sets by the condition N=34020 span=22167178, it was more correct to compare the result with real seismic data sets, e.g. Fig.8 (IDT=-14611458375), from which we see that IDT of data set from the catalogue is about three orders of magnitude larger. We emphasize that in this case N and span of these data sets is similar and corrections for differences in the time span is not necessary. On the other hand, when we compare IDT of your Poisson sequences, with the results for color noises, it is necessary to make corrections because of the differences in the time span, i.e. you need to norm the IDT values to the "time span or range", which is different for color noises and Poisson data sets. After norming you will see that from IDT point of view there is no difference between color noises and Poisson sequences (or Poisson process gives somehow smaller IDT values than color noises) and this is logical because we deal generally with random sequences, which may be just slightly different.

In contrast, considering an equidistant time series (deterministic), DT will be zero for each time step and consequently IDT will be 0.

As it was already explained we do not consider the case, when equidistantly distributed over given time interval data set is compared with the sequence of regularly distributed over the same time period markers, this has no sense. This is a prerequisite of the presented method that when it is possible (in the physical world it is practically always), the original sequence and sequence of time markers should follow different features of time evolution. Otherwise we got simply IDT=0 (at least statistically for the set of different time markers with the same distribution features).

In this respect, we repeat the general idea of IDT here. We aimed to analyze the character of EQ time distribution and compare it with the sequences of markers that are distributed over the same time interval according to the predefined distributional features. We are working to develop an analysis tool based on this idea for different time marker sequences (with different distributional features) in our ongoing research. In the present work, in the frame of aforementioned general view, it was logical to start from the comparison of EQ catalog data with the sequence in which time markers are distributed regularly.

In section 2 you explain several techniques for measuring regularity and show the results for applying those techniques to colored noise in section 3.1. This is a nice exercise, but I guess nothing new.

We think that analysis of simulated data sets should not be regarded as a mere exercise, but viewed in the context of targeted research. Indeed, as we mentioned above, we needed to fulfill analysis on simulated complex data sets with predefined different extent of randomness. Only after such analysis and appropriate data selection, we could undoubtedly prove that IDT is able to discern and quantify the changes even in the case, when we deal with short data sets from a complex process. So, this analysis was a necessary part of research aimed to present and launch the new method of IDT. Besides, in our opinion, the results obtained from the careful analysis of different simulated random data sets, given compactly in one article, will be undoubtedly helpful for researchers from different fields for different testing purposes.

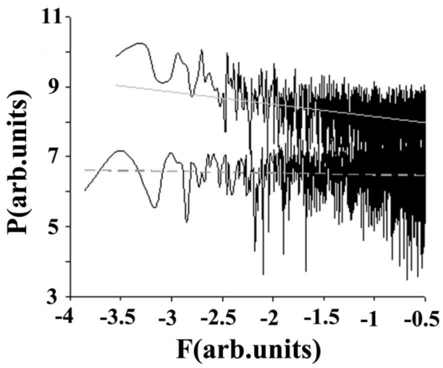
What can you learn from those results and what do they tell you about the seismic time series in southern California?

We mentioned above that analysis on simulated data sets was a necessary step to conclude that IDT is sensitive to dynamical changes in complex data sets and especially even in the case of short data sets. About conclusions on seismic process drawn from the IDT analysis we already described above.

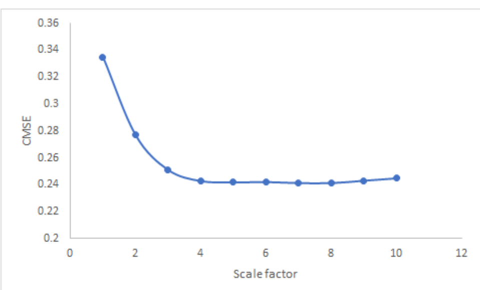
If you include these measures in your study, I would like to see them applied to the seismic time series. E.g. plot the power spectrum (figure 2) for the seismic data (which would be nice anyhow, to get a better impression about the real data) and plot the LZC, DET and CMSE values for the real data in figure 3 and 4. Regarding figure 5 you should also comment on the robustness of your results.

In this work our interests are focused on dynamical changes occurred in small data sets (also obtained from seismic catalogue) and on the development of appropriate for this task analysis method. The reason why we needed to use IDT approach is given above where we explained that LZC, DET and CMSE are not developed for very short data sets used in our research (windows of 100 data span). At the same time, we base our main conclusions on the results obtained for short data sets. Also, we should state again that we work with a specific process of time evolution of earthquake occurrences. In this case we deal with strong trend which usually complicate using of standard data analysis tools. Thus, we do not show results of LZC, DET and CMSE calculation for short and very short sequences.

On the other hand, in order to somehow fulfill the reviewer's interest to the use of complex data analysis tools to seismic process, we present here calculations for the entire length data sets of interevent time sequences: LZC =0.71, %DET=35.



Power spectrum of original (top) and shuffled (bottom) waiting times intervals sequence from the Southern Californian catalogue 1932-2013. (from Matcharashvili et al. Physica A, 2015).



CMSE values versus scale factor for interevent data sequences.

All these results obtained for a whole catalog show just a trivial fact that we deal with a complex seismic process. For reliable quantification of dynamical changes in such processes, especially occurring on the small time scales, we need to use specially developed methods, e.g. like used here IDT test.

Further, it would be helpful to provide some confidence intervals for IDT values of random processes. Actually, I am not sure, if you mix up things, since the IDT values I calculated for random processes are much higher. Do you calculate the sum/integral of deviation times from simulated noise data to regular time steps? Or do you calculate the sum/integral of the simulated noise data? Please, also check and comment on how comparable is the seismic time series to the simulated time series of colored noise.

We thank reviewer for this comment. In the revised version we calculated IDT values for sliding windows of 1000 and 100 data and calculated averaged values. These sequences of IDT values

calculated for 1000 and 100 data windows of simulated data sets then have been compared by paired sample *t* test and significant differences at p=0.01 have been demonstrated. This is mentioned in the revised version.
Noise data sets consisted of positive values and thus generally did not contradict to the physical meaning of the time evolution in original data sets. Additionally, according to the reviewer's suggestion, in the revised version we added also Poisson process data sets as far as this process is often used in the context of seismicity.

In section 3.2 you generate randomized catalogues by shuffling the data, i.e. time and space locations and magnitudes (page. 7, line 20). I do not really understand, what you have done here. Since you do not consider space locations and magnitudes at that point of the paper, what is the effect of shuffling the data. The time steps do not change by shuffling, unless we have a different perception of the meaning of "shuffle". Please be more precise here. Apparently the time steps did change in your shuffled catalogues, otherwise you would receive the same IDT value for all catalogues. What can we learn/conclude from the consideration of the shuffled time series? It is not surprising that a randomized time series behaves more random, than a time series with interdependencies between the events.

As it is said in the revised version, in order to see whether obtained from the original catalogue IDT value is the characteristic of time distribution of natural seismic process or is caused by unknown random effects we started to calculate IDT values for the set of randomized catalogues. Such comparisons are often used in the context of surrogate data testing in complex data analysis. In these artificial catalogues the original time structure of the southern Californian earthquakes distribution was preliminary destroyed. More precisely, the occurrence time of the original events has been randomly shuffled (i.e. earthquakes' time locations have been randomly changed over the more than 42 year of considered period). We have generated 150 of such randomized catalogues and for each of them, IDT values have been calculated for the whole catalogue time span (what was the same as for the original catalogue). Thus, to generate randomized catalogues, we used other method distinct from just shuffling of interevent times. We regret that in the former version of manuscript by mistake it was said that randomization was accomplished in the time, space and energetic domains, which we plan to do in the next works. Randomization based on randomly rearranged occurrence times of earthquakes in the original catalogue was described also in Matcharashvili et al. Physica A 2015.

What can we conclude from comparing the number of events prior (EQp) with those after (EQa) the regular time steps? Is the observed behavior typical for any kind of time series (low/high frequency noise, tendency to cluster, : : :)?

In the present version we consider the differences in the number of earthquakes occurred for the entire observation time, prior and after of the corresponding regular markers. We also decided to carry out additional calculation of summary deviation times separately for each of these groups. The sum of deviation times, normed to the number of corresponding earthquakes prior or after regular markers are essentially smaller in the case of randomized catalogues than for original catalogue. Though this again confirms that in the case of random sequence IDT is closer to zero but finally we agreed with reviewer and decided that in revised version there is no need in such additional arguments.

In figure 8 it looks like
the fraction of EQp to EQa is quite random and could be completely different for similar seismic behavior (e.g. considering earthquakes from 1950 to 1975).

In Fig. 8a of the revised version, we show just an example of the variation of portion of earthquakes occurred prior (grey) and after (black) regular markers for the observation windows. It is clear that this picture will be changed for other time periods (catalogue time span) or areas of location.

The results shown in figure 8 and 9 are not very surprising, Since earthquakes tend to cluster around main shocks (especially after large earthquakes, a large number of aftershock follows). Consequently at times of low seismicity the time steps between EQs are larger and the EQs will tend to occur after the regular time steps, which leads to negative DT values (if DT(i) = T_R(i) – T_EQ(i)) and decreasing IDT.

In our opinion, IDT will not always decrease when the number of earthquakes decrease at relatively seismically quite periods. This will depend on the distribution of events relative to regular markers. More expectable seems that when the number of events (which are functionally connected with independent main shocks) decreases, the probability that these events will be more or less symmetrically distributed on both sides of regular markers will be larger, than in the case of functionally strongly connected (correlated) events. In the last case, asymmetric (relative to regular markers) distribution seems to be more probable. This looks quite logical at least because time intervals between less interconnected earthquakes should be statistically larger than that for correlated events.

At times of
high seismicity (especially after large earthquake) the time steps between the EQs become shorter and EQs will tend to occur prior to the regular time steps, which leads to positive DT values and increasing IDT.

I guess here negative DT and decreasing IDT is meant.

We underline that, here and above, we do not question a trivial fact that time steps between EQs at lower seismicity rate may be longer and that time steps between aftershocks of large earthquakes would be apparently shorter. What we show is that time distribution of earthquakes at lower seismicity is more random than for aftershock activity after strong events. This is quite logical that time evolution of functionally dependent from the main shock aftershocks will be more deterministic- regular, than those not strongly connected with other events. Main result of our work is that now we clearly show that this logical conclusion can be proven quantitatively.

I would need a more in depth analysis and
interpretation of the results, to get any new information. For example, I would like to see

the calculation of the other regularity measures introduced section 2 on the real data set and a comparision with IDT values.

New in this work are two things. First - a demonstration that such a simple statistics like presented here may be useful for complex processes analysis like time evolution of seismic process. Second - the quantitative documentation that the regularity of the time distribution of earthquakes is changing over time and that it is more regular at lower seismic activity than in periods of strong earthquakes occurrences. To our knowledge this is indeed new information. We'd appreciate if the reviewer can suggest references with direct indication of such kind.
Presented analysis is so simple that it can be critically tested by anyone with basic knowledge in statistics (if analysis will be done correctly). As for using standard methods for time distribution of earthquakes we state again that there are two reasons why we did not show results of such analysis. First is the quality of seismic data sets and inappropriateness of these methods for very short sequences. Second is specificity of used time evolution of earthquakes as data sets. It is really not easy to imagine how for such data, with strong trends, methods of complex data analysis can be used unless these data preliminary will be somehow handled (noise reduction, filtering, etc.). But all such procedures will destroy original dynamics of seismic process what we would like to avoid. On the other hand, if we go to the interevent sequences as logical alternative of data sets in the context of time distribution analysis, then we should realize that this is not the same as real time evolution of process. Knowing that such problems may arise we, in the first part of manuscript, tried to analyze effectiveness of method for the set of carefully simulated and tested data sets with known and quantified changes in dynamical structure.

Also you should consider to apply your method on earthquake catalogues of different regions. Considering the results presented in that paper, I have no idea what to expect. I might get a better understanding of the presented IDT approach, if results from other catalogues are compared to the southern California results. You might also study the behavior of IDT in periods of induced seismicity (e.g. Oklahoma).

We definitely have such plans to do analysis on different catalogues in the frame of our future works but do not want to make present article too large. Here we mostly care to present method on the example of trustworthy catalogue and discuss new results on earthquakes time distribution in California.

Some statements would need a statistic test/proof to be more than a subjective judgement.
E.g. page 9, line 6-7: "lower IDT value corresponds to period with decreased sesimic activity".

Apparently, the reviewer means the sentence on page 13, when we comment results presented in Fig. 10 and 11. We again state that in these figures grey vertical lines cross Log E curve exactly at points where in most cases seismic energy release decreases-by about two orders comparing to observed maximums of energy release. The only way to do statistical analysis for this kind of data sets is to compare them with the time evolution and energy release in the randomized catalogues.

In the present version we mention about significant difference between original and time randomized cases.

In figure 9, the IDT values around M6.4 and M7.2 as well as in figure 10 the IDT values around M6.6, M7.3 and M7.2 are quite small compared to the other IDT values.

Indeed, according to our results at decreased energy release, IDT values are smaller.

In fact, large earthquakes are rather close to local minima of IDT values.
Page 10, line 11-13: "close to zero values of IDT can be regarded as random". This needs to be proofed.

In order to prove the fact that "close to zero values of IDT can be regarded as random" we present results in the section 3.1 "Analysis of model data sets" as well as in next section. All this in our opinion is convincing.

"[: : :] they occur in periods of decresed seismic energy release"
This seems to be subjective perception. It is hard to see, but e.g. the energy release for the first and third point is not that small.

We are grateful for this remark. We are sorry for mistake in legends of figures 9-11 in the former version of manuscript. Indeed, it is much more correct to say that the amount of the seismic energy released by the last 10 events of expanding windows decreased; this is shown in the lower curve of Fig.9.
So, the situation in Figs. 9, 10 and 11, indeed looks confusing as far as from one side we present IDT values calculated for expanding windows and on the other side we show seismic energy which is calculated for just the last 10 earthquakes in each expanding windows. It can be assumed that the solution for better visibility here is to come back to the form of energy release presentation like in Fig.8 (where IDT and energy are calculated for the same windows), but in this case we do not see fine structure of changes in energy release against the background of the summary amount (over the whole window) of the energy release. This is why we finally decided that the present form of Figs. 9.10 and 11 is more informative in the sense of better visibility of location of windows, in which the amount of released seismic energy tends to decrease and the process of earthquakes distribution become more random (in the sense already shown for simulated data sets and randomized catalogues) - the curve of IDT values cross the *x* axis. To make situation more convincing and in order to test results obtained for expanded windows, we accomplished additional analysis for the fixed length data windows. In Figs 12 and 13, energy and IDT values are calculated in the same size (100 data) windows. Results obtained for both fixed size and expanded windows shows that decrease in the local amount of seismic energy occurs in windows where IDTs are closer to zero, comparing to other windows.

I agree, that the very small IDT values do not coincide with the large earthquakes, but the chance of coincidence is also quite
small.
It is a good idea to compare the behavior of time series with different threshold magnitudes.
To include more observations for larger magnitudes, you should consider to
increase the considered time span. Since larger earthquakes are easier to detect, time series that start before 1975 can be considered (again, refer to magnitude of completeness).

We give the above explanation why a certain catalogue from 1975 to 2017 is used at M2.6 representative threshold. This catalogue is in complete accordance with our goals in the present research. Anyway, in order to express our gratitude to the reviewers hard work to improve present manuscript, we present here results of our analysis for the south Californian catalogue from 1932 to 2017 at representative threshold M3.5. It is clearly visible that the main conclusions from used catalogue (M2.6) are confirmed for longer time period and higher threshold values. IDT values in the range from 0.1σ to zero were found in the periods with relatively low (two- three orders lower than observed maximums) seismic energy release. At the same time, in the present work we do not intend to highlight these similar results, which definitely would be discussed in our next work in the near future.

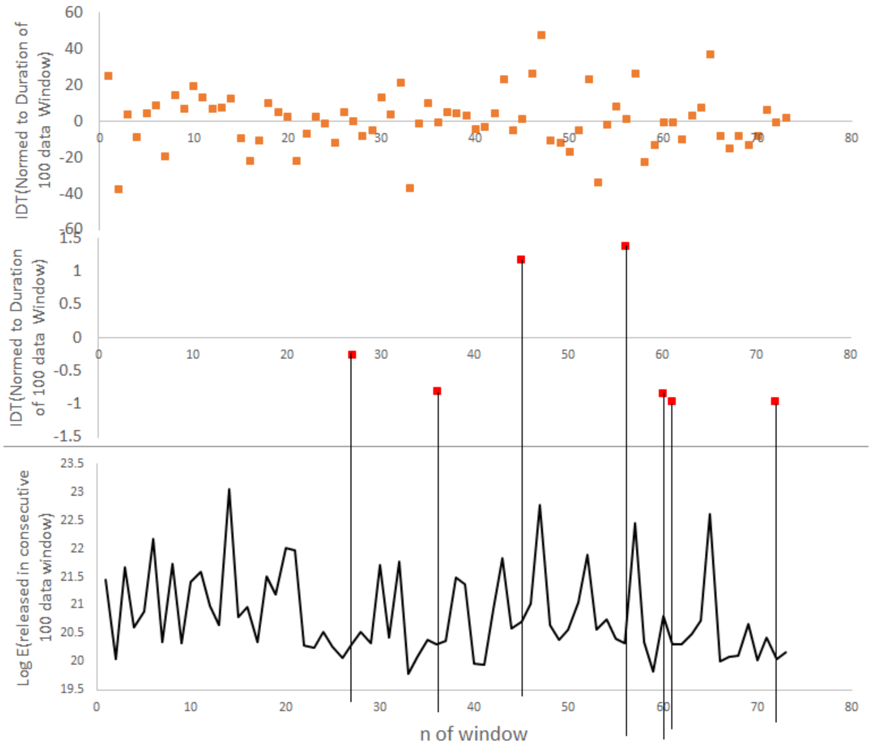


Fig. 5.
Calculated for the non-overlapping 100 data windows (shifted by 100 data), integral deviation times and the released seismic energies (bottom curve). IDT values in vicinity of 0.1σ to zero, are given by red squares. South Californian catalogue 1932-2017, M3.5.

Minor issues:
You should define DT(i). DT(i) = T_EQ(i) – T_R(i) or DT(i) = T_R(i) - T_EQ(i)
Please use scientific format (x*10ˆn) for your numbers. It is quite cumbersome to count the number of digits to be able to compare the provided numbers.

Figure 6: It would be more intuitive to plot a histogram for frequencies, instead of a continuous function. Otherwise explain the meaning of the dots and how you derive the function.

We changed Fig. 6 according to the reviewer's suggestion. Also, we omitted former Fig.7, and added PDF of normed to the window duration time IDT values calculated for consecutive 100 data windows of simulated noise data sequences, shifted by 100, which in our opinion is more informative.

Figure 7: Please use a Y-axis starting with 0. Also, please use intuitive x labels (e.g. SDTa and SDTp).
Done.

You should comment on how you determine the energy release and what is the energy release (relation to magnitude).

We give now reference according to which seismic energy was calculated from magnitudes [Kanamori,1977].

Figure 9: Why do you highlight the points where the IDT curve crosses the abscissa axis? What is the meaning of these points?

We just wanted to make better visible for readers location of crossing points (i.e. situation when IDT comes closer to zero).

When considering shortened time series (e.g. figure 9 – 11), you should take care to also adapt the regular time series to the length and rate of the corresponding time series (otherwise you change your definition of IDT).

We agree with the reviewer about importance of norming. Anyway, in this case, we are mostly interested in the question, whether and when IDT comes closer to zero, what (in this case) is not influenced by the norming procedure.

Page 14, line 7-9: It is very natural that a fraction of points is within one tens of the standard deviation.

Most importantly, these points (IDT values) correspond to windows with lower release of seismic energy.

Language should be improved. Especially, sentences starting with “Exactly” should be replaced with something like “To be (more) specific/precise”, “In detail”, ...

Interactive comment on Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg2017-77,
2018.

Interactive comment on “Simple statistics for complex Earthquakes’ time distribution” by Teimuraz Matcharashvili et al.

Anonymous Referee #2

Received and published: 31 January 2018

GENERAL COMMENTS The authors describes a simple statistical methods to evaluate the time series distribution of earthquakes picked up from the Californian Earthquake Data Center.

SPECIFIC COMMENTS They limit the study since 1975, why? The Catalog reports data since at least 1932. They select the earthquake’s magnitudes greater than 2.6, moreover they do not make distinctions between depths of hypocenter.

As it is underlined in the revised version of manuscript, we aimed to analyze temporal features of original earthquakes generation process. For this, we selected the best quality catalogue of southern Californian seismic activity (Fig. 1). Knowing problems, which can be caused by inappropriate “bleaching” of complex data sets [e.g. Abarbanel, 1993], in this work aiming at the analysis of temporal features of the original seismic process, we needed to avoid procedures like cleaning, filtering or declustering. Otherwise it would be impossible to preserve original time structure of earthquakes distribution. This, together with the necessity to have as possible long data sets, forced us to select as possible long period of observation with as possible low representative threshold. Such compromise, when catalogue is long enough and completeness threshold is as low as possible, according to results of time completeness analysis, seemed to be possible from 1975. Indeed, in Fig. 3, we see that since the middle of 70th of the last century Mc clearly decreased, what finally enabled us to work with southern Californian earthquake catalogue with magnitude of completeness M=2.6, according to the Gutenberg–Richter relationship analysis (see Fig. 2). We understand that in such catalogue we deal with both independent, as well as dependent (aftershocks or foreshocks) events, but in the frame of aims, targeted in the present work this is quite acceptable, because we speak about general temporal behavior of seismic process and because, as it is known, physics of generation of dependent and independent events is similar (See e.g. [Davidsen, Goltz, Geophys. Res. Lett.31(2004), pp. L21612.; P. Bak, C. Tang, K. Wiesenfeld, Phys. Rev. A 38(1) (1988), pp.364–374]).

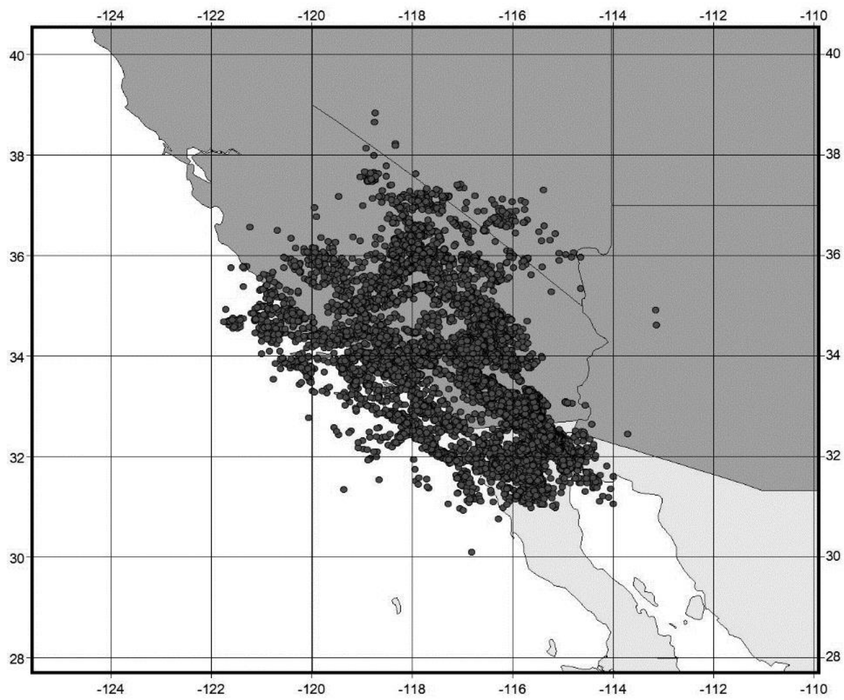

-124
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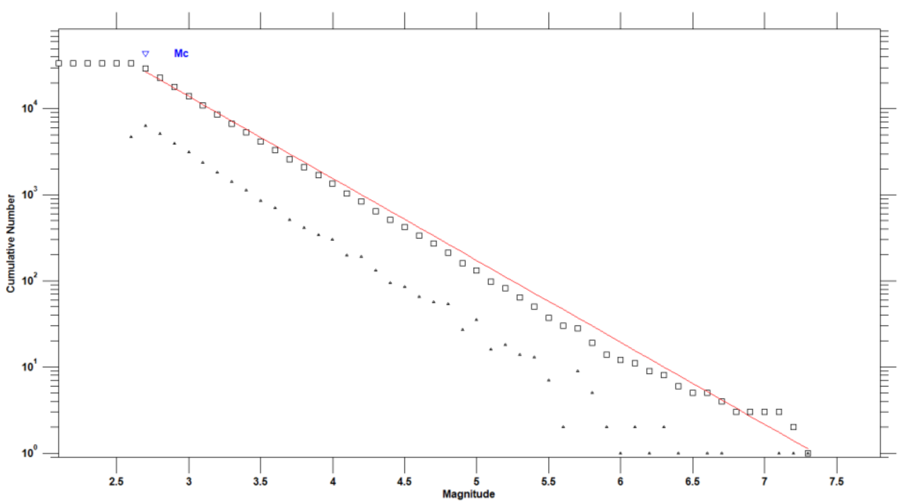
Fig. 1.


Mc
Cumulative Number
Magnitude
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3
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4
4.5
5
5.5
6
6.5
7
7.5

Fig. 2.

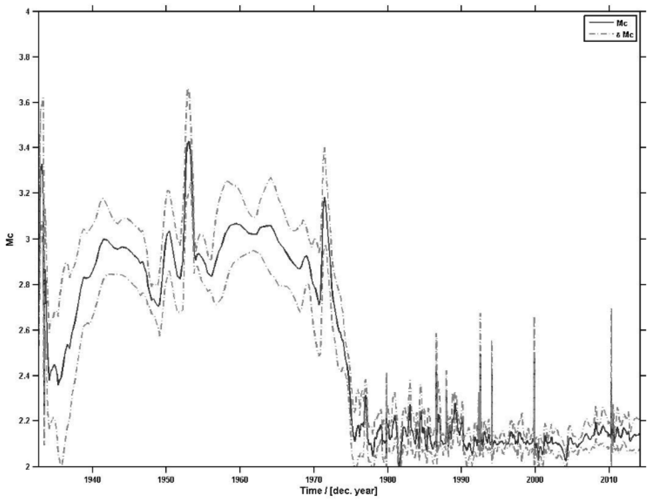

Fig. 3.

We would like further underline that, in any case to assess the possible influence of dependent events on the results of our calculations, we performed analysis at higher representative thresholds M3.6, M4.6 and even for M5.6. According to our analysis dependent events do not essentially influence results of IDT analysis.

Reviewer is correct saying that author of manuscript “do not make distinctions between depths of hypocenter”. From above said it should not be surprising that we do not wanted to differentiate entire process by hypocenters depths and thus change the time structure of original earthquake occurrences. As we pointed above, from the same logic we do not make any catalogue cleaning, declustering, etc. Again, this was quite logical for the targeted research purpose, aiming at the analysis of temporal features of the original (natural) seismic process. This goal to be correctly achieved necessitates avoiding artificial distortion of original dynamical features of earthquakes time distribution, what usually is impossible by any cleaning or filtering of catalogue (especially of such high quality as used in our work south Californian earthquake catalogue). We base our analysis on the often practice, when [see e.g. P. Bak, in (How Nature Works: The Science of Self-Organized Criticality,1996); Christensen et al.(in Proc.Natl. Acad. Sci. U.S.A. 99, 2509, 2002); Corral (in Phys.Rev.Letters, 2004); Corral (in Phys. Rev. E 68, 035102(R) 2003); etc.] seismic processes in catalogue is regarded as a whole, irrespective of the details of tectonic features, earthquakes location or their classification as mainshocks or aftershocks. Thus, we logically abandoned also differentiation of earthquakes according to depths of hypocenters.

In fact, answers to the almost all questions of reviewer 2, are already done in one of the famous articles of Alvaro Corral (in Phys. Rev. E 68, 035102(R) 2003) where it is said that view similar to used in our analysis “.. follows one of the key guidelines of complexity philosophy, which is to find descriptions on a general level; the existence of general laws fulfilled by all the earthquakes unveil a degree of unity in an extremely complex phenomenon”.

The authors
don’t even identify the spatial region, they simply took the data in the archive taken without criticism. They don’t select the main shock from aftershocks.

Answers to these remarks, see above.

So the statistical
description and the results are affected by these undefined choices.

Here we completely agree with the statement of reviewer 2. Indeed, our results obtained by analyzes accomplished by the carefully tested IDT method, express features of earthquakes’ time distribution in the original catalogue, in which the temporal structure of seismic process (as possible) is not distorted by the some, not always well grounded, procedures. Unfortunately, blind inclinations of some researchers to change reality in accordance with their personal preferences or to make “defined choices”, especially when we deal with complex process, often lead to unscientific and incorrect conclusions. Thus, YES, we agree that results really are affected by the features of natural earthquake's time distribution. Moreover, our results reflect features of this natural (as possible untouched) seismic process. This is why they are new and important, as they show changing in time extent of regularity and periods, when seismic process is most random-like.

The Conclusions are trivial.

We would sincerely appreciate reviewer 2, if he/she could provide in depth explanation why our results can be regarded as trivial. In the report of reviewer 2, we do not see any documentation indicating that our findings are something well known or not deserving any attention. Especially we’d be glad to get references, in which it is shown convincingly that the extent of randomness in earthquake time distribution is changing over time and that there are better methods applicable to short periods, when the seismic process is closer to randomness.

CONCLUDING REMARK The goals of the work are not well motivated; it seems to be a mere statistical exercise.

It is said in manuscript that the motivation for the present work was to assess how the extent of regularity in the earthquakes time distribution changes over the considered period of catalogue time span. This problem, in spite of wide scientific interest [e.g. Davidsen, C. Goltz, 2004; Kawamura. 2007; Kenner, M. Simons, 2005; etc.] still remains unanswered. At the same time, it is clear that without such knowledge the better understanding of seismic processes can not be achieved. Moreover, scientific posing of such general tasks as earthquake prediction or forecast, will not look well-grounded unless basic features of seismic process dynamics in spatial, temporal or energy domains will not be understood.

We think that analysis of simulated data sets, carefully accomplished in our research, should not be regarded just as “statistical exercise”. The matter is that one needs to fulfill analysis by suggested IDT method on simulated complex data sets with (predefined) different extent of randomness in order to apply the method to seismic data sets with unknown complex structure. Only after such comparative analysis (calibration) and appropriate data selection we could undoubtedly prove that IDT approach is able to discern and quantify the changes in the complexity level of the process even in the case when we deal with short data sets from a complex process like seismicity. So, this analysis was a necessary part of research aimed to present and launch the new method of IDT.

Besides, in our opinion results obtained from the careful analysis of different simulated random data sets, given compactly in one article, will be undoubtedly helpful for researchers from different fields for different testing purposes.

So the paper needs a deep afterthought.

We corrected our manuscript significantly. The revised version of manuscript contains a result of additional work and testing of data sets.

Simple Statistics for Complex Earthquakes' Time Distribution

T. Matcharashvili[1], T. Hatano[2], T. Chelidze[1], N. Zhukova[1]

1. M. Nodia Institute of Geophysics, Tbilisi State University, Tbilisi, Georgia

2. Earthquake Research Institute, the University of Tokyo, Tokyo, Japan

Abstract

Here we investigated a statistical feature of earthquakes time distribution in southern Californian earthquake catalogue. As a main data analysis tool, we used simple statistical approach based on the calculation of integral deviation times (IDT) from the time distribution of regular markers. The research objective is to define whether and when the process of earthquakes time distribution approaches to randomness. Effectiveness of the IDT calculation method was tested on the set of simulated color noise data sets with the different extent of regularity as well as for Poisson process data sets. Standard methods of complex data analysis have also been used, such as power spectrum regression, Lempel and Ziv complexity and recurrence quantification analysis as well as multi-scale entropy calculation. After testing the IDT calculation method for simulated model data sets, we have analyzed the variation of the extent of regularity in southern Californian earthquake catalogue. Analysis was carried out for different periods and at different magnitude thresholds. It was found that the extent of the order in earthquakes time distribution is fluctuating over the catalogue. Particularly, we show that the process of earthquakes' time distribution becomes most random-like in periods of relatively decreased local seismic activity.

Introduction

Time distribution of earthquakes remain one of the important questions in nowadays geophysics. At present, the results of theoretical research and the analysis of features of earthquakes' temporal distributions from different seismic regions with different tectonic regimes carried worldwide can be found in [e.g. Matcharashvili et al. 2000; Telesca, et al. 2001, 2012; Corral, 2004; Davidsen, and Goltz, 2004; Martınez et al. 2005; Lennartz, et al. 2008; Chelidze and Matcharashvili, 2007; etc.].

Such analyses among others often aims to the assessment of the strength of correlations or the extent of the determinism/regularity in the earthquakes time distribution. One of the main conclusions of such analysis is the understanding that earthquake generation in general does not follow the patterns of random process. Exactly, well established clustering, at least in time (and spatial domains), suggests that earthquakes are not independent completely and that seismicity is characterized by slowly decaying correlations (named long-range correlations): such behavior is commonly exhibited by non-linear dynamical systems far from the equilibrium [Peng et al., 1994, 1995]. Moreover, it was shown that in the temporal and spatial domains earthquakes' distribution may reveal some features of a low-dimensional, nonlinear structure, while in the energy domain (magnitude distribution) it is close to a random-like high dimensional process [Goltz, 1998; Matcharashvili, et al. 2000]. Moreover, according to present views, the extent of regularity of the seismic process should vary in time and space [Goltz, 1998; Matcharashvili et al. 2000; Matcharashvili et al. 2002; Abe&Suzuki, 2004; Chelidze and Matcharashvili, 2007; Iliopoulos et al. 2012].

At the same time, the details of how the extent of randomness (or non-randomness) of seismic process changes over the time and space still remain unclear. In the present work, on the basis of southern California earthquake catalogue, we aimed to be focused on this question and analyzed earthquake time distribution to find where it is closer to randomness.

Data and used Methods

Our analysis is based on the southern Californian earthquakes catalogue available from http://www.data.scec.org/ftp/catalogs/**.** As far as we aimed to analyze temporal features of original earthquakes generation, we tried to have as long as possible period of observation with as low as possible representative threshold. For this purpose, according to results of time completeness analysis (not shown here) we decided to be focused on the time period from 1975 to 2017 since in the middle of 70th of last century Mc clearly decreased. Southern Californian (SC) catalogue for the considered period is complete for M = 2.6, according to the Gutenberg–Richter relationship analysis.

In general, presently we are developing an approach aiming to discern features of the complex data sets (in this case EQ time distribution) by comparing them with data sets with the predefined dynamical structures. In the present work in the frame of this general idea we started from the simplest case, comparing natural time distribution of earthquakes in SC catalogue with the time distribution of regular markers, according to the scheme shown in Fig. 1.

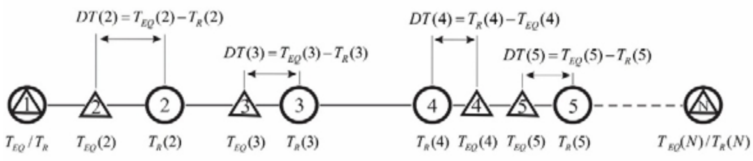

Fig. 1. Explanation of the used approach. Triangles - time locations of original earthquakes ($T_{EQ}(i)$), circles – time locations of regular markers ($T_R(i)$). $DT(i)$ denotes the difference between the time of earthquake occurrence ($T_{EQ}(i)$) in the catalogue and the time point of the regular marker ($T_R(i)$).

Namely, knowing duration of the whole period of observation in considered catalogue (22167178 minutes, from 01.01.1975 to 23.02.2017) and the number of earthquakes (34020) with the magnitude above a representative threshold (M2.6), we calculated the time step between consecutive regular markers (651.6 min), what in fact is the mean time of earthquakes occurrence for the considered period. Then, for each of earthquakes in the catalogue we calculated difference between original event occurrence time and time point of the regular marker. We denoted as $DT_{(i)}$ the time interval (delay or deviation time) between occurrence of original earthquake $T_{EQ(i)}$ and corresponding i-th regular marker $T_{R(i)}$. It is clear that original earthquake (EQ_i) may occur prior or after of corresponding regular marker (R_i), so by $DT_{(i)}$ with minus or plus sign we understand earthquakes occurred prior or after regular markers accordingly. Summation of deviation times may provide interesting knowledge about character of distribution of earthquakes comparing to regular time markers. Here we mention that, alternatively, the same can be viewed as a summation of differences (deviations) between observed waiting times and mean occurrence time over considered period.

In any case, logically, for any random sequence, the sum of the deviation times should approach zero, when $n \to \infty$. Thus, the main assumption is that the integral of deviation times (IDT) or the sum $\sum_{1}^{N} DT(i) \to 0$, when the time distribution of events is random-like. From this point of view the used approach looks close to Cumulative Sums (Cusum) test, where for a random sequence, the sum of excursions of the random walk should be near zero [Rukhin et al, 2010].

Prior to use for the seismic process, we needed to test whether IDT calculation can be sensitive to dynamical changes occurred in complex data sets with known dynamical structures. We started from the analysis of model data sets with a different extent of randomness. Exactly, we used simulated noise data sets of different color with power spectrum function *($1/f^{\beta}$)*, where scale exponent (β) varied in range from 0 to 2. These noises, according to generation principles, logically have to be different, but for purposes of our analysis we needed to have strong quantitative assessments of such differences. This is why at first, these noise data sets have been investigated by several data analysis methods, often used to assess different aspects of changes occurred in dynamical process of interest. Exactly, power spectrum regression, Lempel and Ziv algorithmic complexity calculation, as well as recurrence quantification analysis and Multi-scale entropy calculation method have been used for simulated model data sets. All these popular methods of time series analysis are well described in number of research articles and we will just briefly mention their main principles.

Power spectrum regression exponent calculation enables to elucidate scaling features of data set in the frequency domain. By this method a fractal property is reflected as a power law dependence between the spectral power ($S(f)$) and the frequency (f) by spectral exponent β:

$$S(f) \sim \frac{1}{f^{\beta}} \ .$$

β often is regarded as a measure of the strength of the persistence or anti-persistence in data set. As easily calculated from log-log power spectrum plot, β is related to the type of correlations, present in time series [Malamud, 1999; Munoz-Diosdado, 2005; Stadnitski, 2012]. For example, $\beta = 0$, corresponds to the uncorrelated white noise and processes with some extent of memory or long-range correlations are characterized by nonzero values of spectral exponents.

Next, we proceeded to the Lempel and Ziv algorithmic complexity (LZC) calculation [Lempel & Ziv, 1976; Aboy et al. 2006; Hu&Gao, 2006], which is a common method for quantification of the extent of order (or randomness) in data sets of different origin. LZC is based on the transformation of analyzed sequence into new symbolic sequence. For this, original data are converted into a 0, 1, sequence by comparing to a certain threshold value (usually median of the original data set). Once the symbolic sequence is obtained, it is parsed to obtain distinct words, and the words are encoded. Denoting the length of the encoded sequence for those words, the LZ complexity can be defined as

$$C_{LZ} = \frac{L(n)}{n}$$

where $L(n)$ is the length of the encoded sequence and n is the total length of sequence [Hu&Gao, 2006]. Parsing methods can be different [Cover & Thomas, 1991; Hu&Gao, 2006]. In this work, we used scheme described in Hu&Gao [2006].

Next, in order to further quantify changes in the dynamical structure of simulated data sets, we have used recurrence quantification analysis (RQA) approach [Zbilut and Webber, 1992; Webber and Zbilut, 1994; Marwan et al., 2007]. RQA is often used for analysis of different types of data sets and represents a quantitative extension of Recurrent Plot (RP) construction method. RP, itself, is based on the fact that returns (recurrence) to the certain condition of the system (or state space location) is a fundamental property of any dynamical system with quantifiable extent of determinism in underlying laws [Eckman et al., 1987]. In order to successfully fulfill RQA calculations, at first the phase space trajectory should be reconstructed from the given scalar data sets. It is important to test the proximity of points of the phase trajectory by the condition that the distance between them is less than a specified threshold ε [Eckman et al., 1987]. In this way, we obtain two-dimensional representation of the recurrence features of dynamics, which is embedded in a high-dimensional phase space. Then a small-scale structure of recurrence plots can be quantified [Zbilut and Webber, 1992; Webber and Zbilut, 1994; Webber and Zbilut, 2005; Marwan et al., 2007; Webber et al., 2009; Webber and Marwan, 2015]. Namely, RQA method enables to quantify features of a distance matrix of recurrence plot, by means of several measures of complexity. These measures of complexity are based on the quantification of diagonally and vertically oriented lines in the recurrence plot. In this research we calculated one of such measures: the percent determinism (%DET), which is defined as the fraction of recurrence points that form diagonal lines of recurrence plots and which shows changes in the extent of determinism in the analyzed data sets.

An additional test, which we used to quantify the extent of regularity in modeled data sets, is the composite multi-scale entropy (CMSE) calculation [Wu et al. 2013 *a*]. CMSE method represents expansion of multi-scale entropy (MSE) [Costa, et al. 2005] method, which in turn originates from the concept of sample entropy (SampEn) [Richman&Moorman, 2000]. SampEn is regarded as an estimator of complexity of data sets for a single time scale. In order to capture the long-term structures in the time series, further Costa et al. [2005] proposed the above mentioned multi-scale entropy (MSE) algorithm, which uses sample entropies (SampEns) of a time series at multiple scales. At the first step of this algorithm, often used in different fields, a coarse-graining procedure is used to derive the representations of a system's dynamics at different time scales; at the next step, the SampEn algorithm is used to quantify the regularity of a coarse-grained time series at each time scale factor. Main problem of MSE is that, for a shorter time series, the variance of the entropy estimator grows very fast as the number of data points is reduced. In order to avoid this problem and reduce the variance of estimated entropy values at large scales, a method of the composite multi-scale entropy (CMSE) calculation was developed by Wu and colleagues [Wu et al. 2013 *a*].

Results and discussion

Analysis of model data sets

As we mentioned in previous section, first, we needed to ascertain whether calculation of IDT values is sensitive to dynamical changes, occurred in analyzed data sets. To this end, we decided to generate artificial datasets of one and the same type, for example noises, which according to the generation procedure should be measurably different in the frequency content, representing a different types of color noises. We have started from the analysis of 34020 data length sequences of these noise data sets. For clarity we add here that to test the robustness of results, the same analyses were performed on much longer data sets, but here we show results for simulated noise data sets, which are of the same length as the original data sets from the used seismic catalogue. The noise data sets have been generated according to concepts described in Kasdin [1995], Milotti [2007] and Beran et al [2013]. As a metrics for these data sets we have used the mentioned above power spectrum exponents (β) also referred to as the spectral indexes [Schaefer et al. 2014]. Exactly, we have analyzed seven of such data sets having spectral exponents: 0.001, 0.275, 0.545, 0.810, 1.120, 1.387, 1.655. Values of β are often used as a metric for the fractal characteristics of data sequences [Shlesinger, 1987; Schaefer et al.2014]. In our case different spectrum exponents of simulated noise data sets indicate that they are different by the extent of correlations in the frequency content [Schaefer et al.2014]. Indeed, the first noise set, with the $\beta = 0.001$ (Fig.2, *a*), was the closest to the white noise and the last one with the $\beta = 1.655$ (Fig.2, *b*), manifested the features closer to colored noises of red or Brownian type, with a detectable dynamical structure. In addition to this, taking into account that we aimed to analyze seismic data sets, we regarded as logical to consider also the random process, which is often used by seismologists – a Poisson process. We generated the set of 34020 data long sequences of Poisson process. It was quite expectable that spectral exponent of such sequences is close to that of a white noise.

For further analysis, in order to differentiate simulated (noise and Poisson process) data sets by the extent of randomness, we used algorithmic complexity (LZC) and recurrence quantification analysis methods as well as testing, based on multi-scale entropy (MSE) analysis.

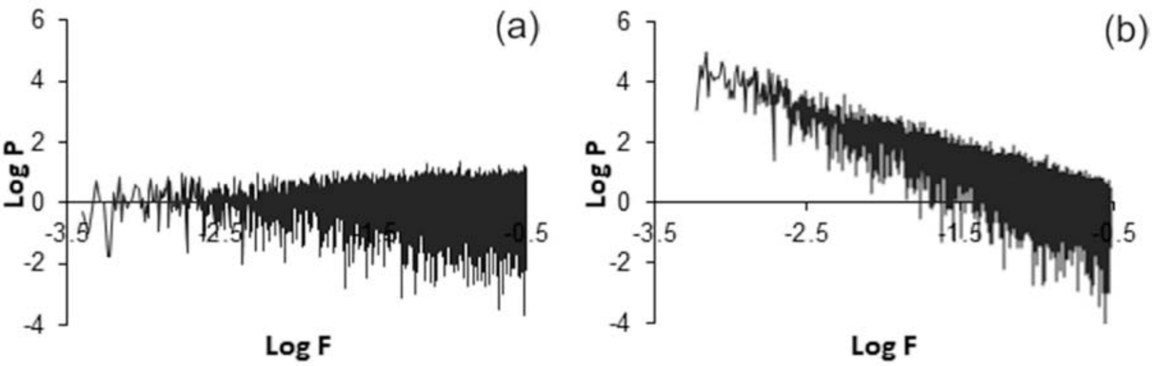

Fig. 2. Typical plot of the power spectrum of simulated noise data sets with different spectral regression, a) β=0.001 and b) β= 1.655.

In Fig. 3, we show results of LZC and %DET calculation; namely here are presented averages of values calculated for consecutive 1000 data windows shifted by 100 data. Both methods, though based on different principles, help to answer question, how similar or dissimilar are considered data sets by the extent of randomness. We see that, Lempel and Ziv complexity measure decreases from 0.98 to 0.21, when β noises increases. It means that the extent of regularity in simulated data sets increases. The same conclusion is drawn from RQA: the percentage of determinism increases from about 25 to 96.5, when the spectral exponent increases. For both LZC and RQA measures, differences of compared neighbor groups in figures are statistically significant at p=0.01. Thus, according to Fig. 3, the extent of regularity in the simulated noise sequences clearly increases from the close to white (β=0.001) to close to Brownian (β=1.655) noise. For the Poisson process data sequences, LZC measure reaches 0.97-0.98 and %DET is in the range 25-26, i.e. these values are close to what we obtain for white noises.

Thus, the results of LZC and %DET calculations confirming results of power spectrum exponents calculations, show that considered color noise data sets are different from white noise and Poisson process by the extent of regularity.

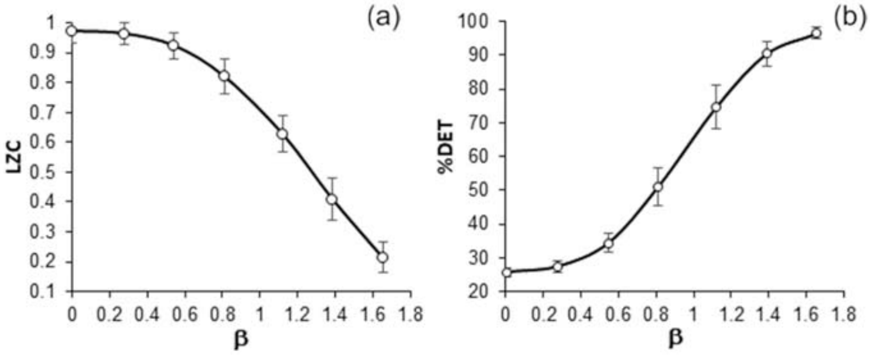

Fig. 3. LZC and %DET values calculated for seven noise data sets with different spectral indexes.

Additional multi-scale, CMSE, analysis also shows (Fig. 4) that the extent of regularity in a model noise data sets increases, when they become "more" colored (from β= 0.001 to β=1.655). We see that for small scales (exactly for scale one and partly for scale two), noise data sets reveal decrease in the entropy values for simulated data sets, when spectral indexes rise from β= 0.001 to β=1.655.

This is logical for simulated data sets, where the extent of order, according to the above analysis, should slightly increase. At the same time, while at larger scales the value of entropy for the noise data set with β=0.001 continues monotonically decrease, like for the coarse-grained white noise time series [Costa et al. 2005], the value of entropy for *1/f* type processes with β values close to pink noises (0.81, 1.12) remained almost constant for all scales. As noticed by Costa et al. [2005] this fact was confirmed in different articles on multi-scale entropy calculation [see e.g. Chou, 2012; Wu, et al. 2013 *a, b*]. Costa and coauthors explained this result by the presence of complex structures across multiple scales for *1/f* type of noises. From this point of view, in color noise sets, closer to a Brownian type process, the emerging complex dynamical structures should become more and more organized. Apparently, these structures are preserved over multiple scales including small ones. This is clearly indicated by the gradual decrease of calculated values of entropy for sequences with β=1.12, β= 1.387 and β= 1.654 at all considered scales (see Fig. 4). Poisson process data sets (not shown in figure) in the sense of results of multiscale analysis are close to a white noise sequence with β =*0.001*.

Thus, CMSE analysis additionally confirms that used in this research complex model data sets are characterized by quantifiable dynamical differences.

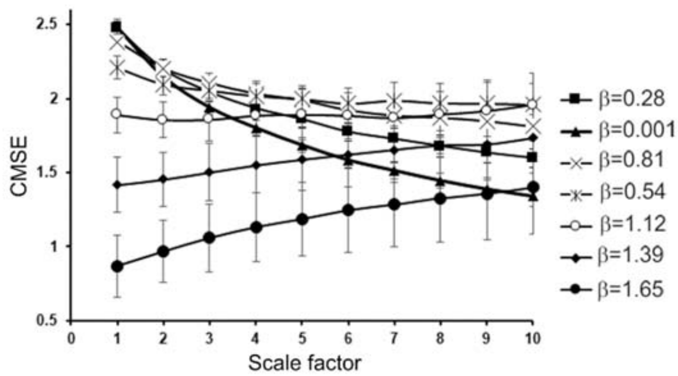

Fig. 4. **C**MSE values versus scale factor for simulated data sequences with different spectral indexes.

Once we had data sets with the quantifiable differences in their dynamical structures, we started to test the ability of IDT calculation to detect these differences.

For this, we created cumulative sum sequences of seven noise data sets and data sets of Poisson process and regarded them as models of time occurrences of consecutive events. We treated these, 34020 data long, sets for time occurrence sequence of real earthquakes and calculated IDT values for different windows. Taking into consideration that cumulative sum (or time span in the case of seismic catalogue) of windows may be different (excluding the case when data sets have been specially generated so that cumulative sum to be equal) we normed obtained IDT values to the span of window. Results of calculation are presented in the upper curve (circles) in Fig. 5*a*. Here also we present results of similar calculations on the same data sets performed for shorter windows (see squares, triangles and diamonds in Fig. 5*a*).

As we see, absolute values of normed to window span IDTs, calculated for the model data sets indicate stronger deviation from zero, when the extent of order in simulated noise data sets increases (according to the above analysis). Average values of IDTs calculated for data sets with spectral exponents closer to Brownian noise significantly differ from white noise at p=0.01 (Fig. 5 a). It needs to be pointed that comparing to results obtained by the used above methods, IDT calculation looks even more sensitive to the dynamical changes occurred in the simulated data sets; note more than 1.5

order difference between sequences with β=0.001 and β=1.654 for the entire length of data sets (circles in Fig.5a).

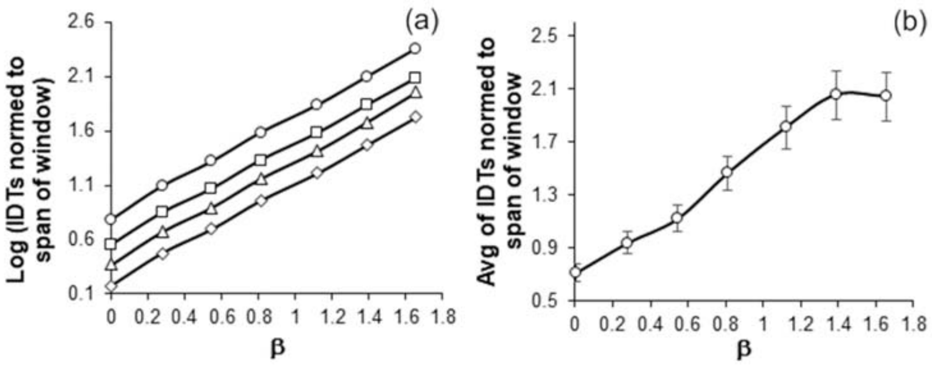

Fig. 5. a) Logarithms of, normed to the span of window, absolute values of IDT calculated for different length (circles-34020, squares-20000, triangles-10000, diamonds-5000 data) of windows of simulated noise data sets with different spectral indexes, b) averages of IDT values calculated for 100 data windows of normed to the span of window simulated noise data sets with different spectral indexes.

It needs to be pointed, that according to IDT calculations, Poisson process looks more random than white noise. Indeed, logarithms of normed to window span IDT values calculated for random Poisson process data sets were lower than for white noise (0.38, 0.1, 0.04, 0.03 for 30000, 20000, 10000 and 5000 data sets accordingly). For the further analysis, it is important to mention that results of above calculations do not practically depend on the length of used data sets. Not the less important is that, as it is shown in Fig. 5*b*, differences found for longer windows is preserved for the short, 100 data long sequences. For 100 data windows difference between white noise, as well as Poisson process, data sets and colored noises is statistically significant at p=0.01. Taking into consideration the importance of results of IDT calculations for short (100 data) windows, we additionally present reconstructed PDF curves fitted to the normal distribution according to real calculations (different marks in Fig. 6). From this figure we see that IDT values goes closer to zero when the extent of order decreases. Besides, it also becomes clear that even for short data sets IDT calculation is useful to detect differences in considered data sets.

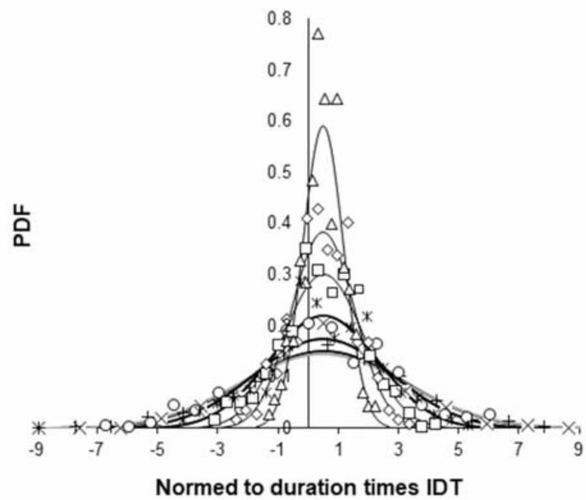

Fig. 6. PDF of, normed to the window length, IDT values calculated for consecutive 100 data windows of simulated noise data sequences, shifted by 100 data. From top to bottom black curves correspond to β 0.001(triangles), β 0.275(diamonds), β 0.545(squares), β 0.810(asterisks), β 1.120(circles), β 1.387(plus signs), and grey curve corresponds to β 1.655(cross signs).

Thus, based on the analysis of specially simulated data sequences we conclude that IDT calculation method is effective in detecting small changes occurred in, even short, complex data sets with different dynamical structures.

Analysis of earthquakes time distribution in south California catalogue

In this section we proceeded to the analysis of original data sets drawn from the south California seismic catalogue using IDT calculation approach.

As it was said above, for random sequences, the sum of the deviation times should approach zero in the infinite length limit. Results of presented in the previous section analysis confirms this on the example of model random (or random-like) data sets with different extent of regularity (or randomness).

In the case of real earthquake generation process, which according to present views can not be regarded as completely random [Goltz, 1998; Matcharashvili et al. 2000, 2016; Abe&Suzuki, 2004; Illiopoulos et al. 2012], we should assume that the integral of deviations times (IDTs) for the periods with the more random-like earthquake's time distribution will be closer to zero, compared to the less random ones.

To show this, we used seismic catalogue of south California, the most trustworthy data base for analysis like targeted in this research. Aiming at the analysis of temporal features of seismic process, we intentionally avoided any cleaning or filtering of catalogue in order to preserve its original temporal structure. Therefore, according to a common practice [see e.g. Christensen et al. 2002; Corral, 2004], we regard the seismic processes in this catalogue as a whole, irrespective of the details of tectonic features, earthquakes location or their classification as mainshocks or aftershocks.

It was quite understandable that, for such catalogue we logically should expect time clustering of interdependent events: fore- and aftershocks. This, in the context of our analysis, apparently could lead to the considerable amount of events occurred prior to corresponding regular markers (probably mostly aftershocks). Thus, it was interesting to know how the number of events occurred prior or after regular markers and especially result of IDT calculation (which directly depends on the number of events occurred prior and after regular markers), is related with the time locations of such interdependent events.

Here we underline the fact that both IDT values as well as the portion of events occurred prior or after regular markers (as defined in the methods section) would strongly depend on the position and length of certain time window in catalogue. Thus, we focused on the whole duration period of considered catalogue (at the representative threshold M2.6). We found that in this catalogue 55% of all earthquakes occurred prior and 45% after the regular time markers. To elucidate the role of dependent events on this ratio we analyzed catalogue for higher representative thresholds. At increased to M3.6 representative magnitude threshold we found that the portion of earthquakes occurred prior to markers decrease (33% of all earthquakes). This provided argument in favor of assumption that the prevalence of earthquakes, which occur prior to markers may indeed be related with low-magnitude dependent events (supposedly mostly aftershocks). At the same time, further increase of representative threshold convinces that low magnitude dependent events in catalogue may not be the sole cause influencing the amount of earthquakes occurred prior to markers. Indeed, the portion of events occurred prior to markers increased to 42% at the representative threshold M4.6. Most noticeable is that at highest considered representative threshold, M5.6, we observe further increase of portion of earthquakes, occurred prior to regular markers; to the level observed for M2.6 threshold (55% of all events). Thus, it seems unlike that ratio between events that occurred prior or after regular markers may be related only with dependent events (aftershocks).

Next, we calculated IDT values for entire observation period at different representative thresholds. It was found that IDT value calculated for the entire observation period of considered southern Californian earthquake catalog (at representative threshold M2.6), equals: -14611458375 minutes (as mentioned above sign ‘minus’ here denotes the direction of summary deviation along time axis). We compare this value to the IDT values at larger representative thresholds. Taking into account that increase of threshold may somehow change the analyzed period of catalogue, below we show normed to the corresponding time span of catalogue values of IDT. Namely, for M2.6 catalogue the normed value of IDT= -659.15. Increasing the threshold to M3.6, M4.6, and M5.6 leads to following IDT values: 71.7, 6.7, -0.87 accordingly. Two important things can be underlined here: first, the increase of the magnitude threshold makes the time distribution of remained EQs more random and second, according to our conjecture to the more random EQ distribution should corresponds the closer to zero IDT value, what indeed is shown above.

Interesting fact is that decreased probability of dependent events, at increased representative threshold, do not necessarily causes proportional increase of the number of occurred after regular markers events, though absolute values of IDT drastically decreases. These results also indicate that the ratio between events, occurred prior or after regular markers, found for the entire span of SC catalogue, as well as the IDT value, should not be reduced due only to the distributional features of dependent events.

Further we needed to clear up whether the ratio of events occurred prior or after regular markers and especially obtained IDT value, are characteristics of time distribution of earthquakes in the SC catalogue or they reflect influence of some unknown random effects, which are not directly related with the seismic process. For this we started to calculate IDT values for the set of randomized catalogues. In these artificial catalogues the original time structure of the southern Californian earthquakes distribution was preliminary destroyed. Exactly, occurrence times of original events’ have been randomly shuffled (i.e. earthquakes’ time locations have been randomly changed over the entire time span of more than 42 years). We have generated 150 of such randomized catalogues and for each of them, IDT values have been calculated for the whole catalogue time span (which was the same as for the original catalogue).

It was found that for the whole period of observation, in 58% of all time-randomized catalogues prevailed earthquakes, occurred prior to corresponding regular markers. At the same time, unlike the original catalogue, where 55% of earthquakes occurred prior to corresponding regular markers, in randomized catalogues the portion of such earthquakes, occurred prior to markers, varied in the wide range (from 51% to 92%). Thus, in spite of some similarity, by the portion of events occurred prior and after of regular markers original and time randomized catalogues are still different.

Next, comparing the averaged IDT value of randomized catalogues (calculated from IDTs of 150 randomly shuffled catalogues) we found that it is also with minus sign (-159755608 min). This was expectable since in 58% of cases of randomized catalogues, prevailed earthquakes occurred prior to regular markers. Thus, comparing the average of integral deviation times, calculated for the entire length of randomized catalogues, with the IDT value of the original SC catalogue, we see that the last one is two orders of magnitude larger. The difference between IDT of the whole original catalogue and the average IDT of randomized catalogues was statistically significant, with Z score =11.2, corresponding to p=0.001 [Bevington and Robinson, 2002; Sales-Pardo et al. 2007].

The difference between IDT values calculated for original and time randomized catalogues is further highlighted in Fig. 7, where normed to the windows span IDT values are presented. We see, that in all cases normed to windows span IDTs are clearly smaller than for the original catalogue (6.59E+02). It is interesting that in at least 30% of cases IDTs calculated for randomized catalogues are more than two order smaller than IDT for original catalogue.

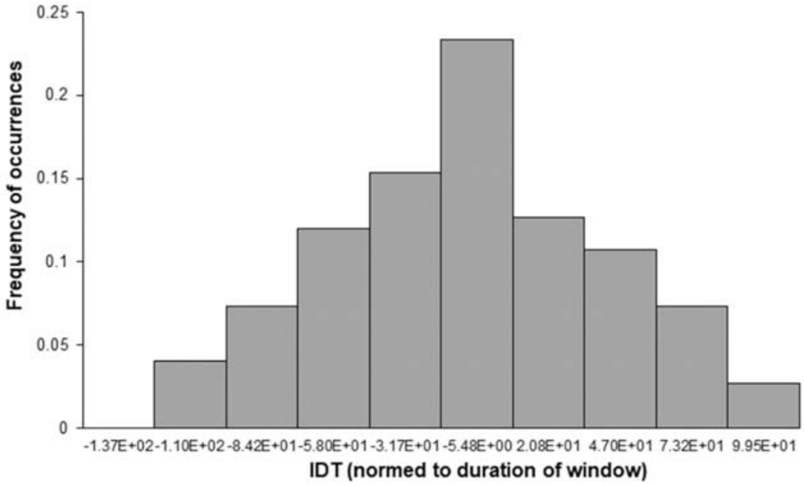

Fig. 7. Frequency of occurrences of normed to the span of window, integral deviation time values, calculated for each of 150 randomized catalogues for the whole period of duration.

From this analysis two important conclusions can be drawn: *i*) IDT value, calculated for the considered period of south Californian earthquake catalogue, expresses the internal time distribution features of the original seismic process, and *ii*) random-like earthquake time distribution lead to lower (closer to zero) IDT values comparing to the whole original catalogue.

All above results obtained for simulated data sets as well as for the time distribution of earthquakes in the original catalogue shows undoubtedly that the time distribution of earthquakes in south California for the entire considered period should be regarded as a strongly non-random process (IDT is larger than for randomly distributed in time earthquakes). Therefore, result of this simple statistical analysis is in complete agreement with our earlier results, obtained by contemporary nonlinear data analysis methods, as well as with the results of similar analysis reported by other authors, which used different methodological approaches [see e.g. Goltz, 1998; Matcharashvili et al. 2000, 2016; Abe&Suzuki, 2004; Telesca et al. 2012; Illiopoulos et al. 2012].

Thus, we found that for the whole period of considered catalogue, prevailed earthquakes occurred prior to corresponding regular markers (see also the last point in the upper curve of Fig. 8*b*). At the same time, as also was mentioned, the number of earthquakes occurred prior or after corresponding regular markers may change depending on the time span of analyzed catalogue. The same can be said about the values of the integral deviations times. In order to investigate the character of the time variation of IDT values of SC catalogue in different periods we fulfilled calculation for the expanding time windows. Exactly, we have calculated IDT values starting from the first 100 data (earthquakes), expanding initial window by the consecutive 10 data to the end of catalogue. In Fig. 8, we see that the number and the time location of earthquakes (relative to regular markers), undergoes essential change, when the length of analyzed part of catalogue (analyzed window's length) gradually expands to the end of catalogue (in our case from 01.01.1975 to 23.02. 2017).

As it is shown in Fig. 8*a*, in the most of the analyzed windows the majority of earthquakes occurred after regular markers, although there are windows with opposite behavior. So far as in the most windows prevail earthquakes, which occurred after regular markers, it is not surprising that calculated for consecutive windows integral deviation times are mostly positive. This is clear from Fig. 8*b*, (upper curve), where we see windows with negative IDTs too. Thus, the values of IDTs, calculated for extended windows in different periods vary in a wide range, increasing or decreasing and sometimes coming close to zero.

Here we point again, that based on results of above analysis, accomplished for simulated data sets and randomized catalogues, we suppose that when IDT value approaches zero, the dynamical features of originally nonrandom seismic process undergoes qualitative changes and becomes random-like or at least is closer to randomness. In other cases, when IDT value changes over time, but is far from zero, we apparently observe quantitative changes in the extent of regularity of nonrandom earthquakes time distribution.

From this point of view it is interesting that earthquakes' time distribution looks more random-like for the relatively quiet periods, when the amount of seismic energy calculated according to Kanamori, [1977], decreases comparing to values, released in the neighbor windows prior or after strongest earthquakes. This is noticeable, in the lower curve of Fig. 8*b*, where we present cumulative values of seismic energy, calculated for consecutive windows, expanding by 10 events to the end of catalogue. We see that in south California, from 1975 to 2017, the strongest earthquakes never occurred in periods, when IDT curve comes close to zero value or crosses abscissa line. To avoid misunderstanding because of restricted visibility in Fig. 8*b*, we point here that M6.4 earthquake has occurred in the window 256, from the start of catalogue, and IDT curve crossed abscissa later, in the window 265, from the start of catalogue, i.e. 100 events later.

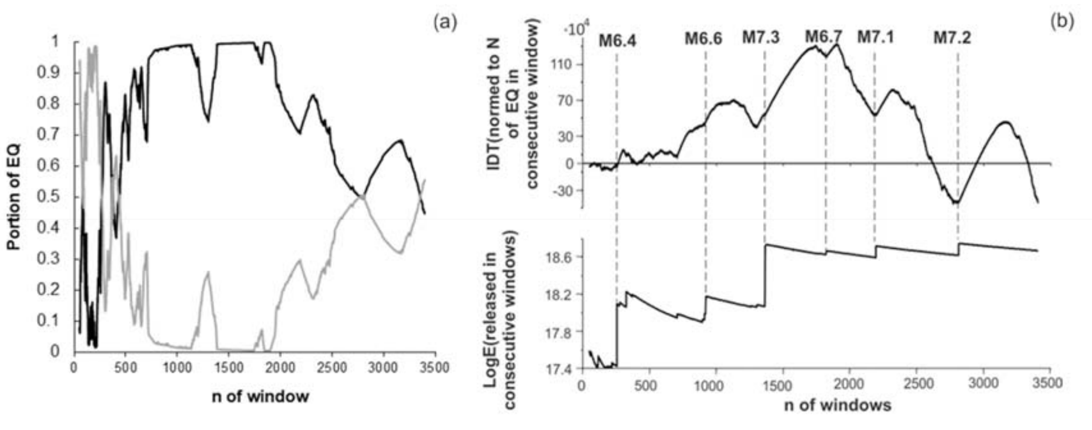

Fig. 8. Calculated for extending by 10 consecutive data windows in the South California earthquake catalogue, a) portion of earthquakes occurred prior (grey) and after (black) regular markers in each window, b) normed to the number of EQs integral deviation times (top) and cumulative seismic energies (bottom)

Results in Fig. 8, also provide interesting knowledge about relation between IDT and the amount of released seismic energy. As we see, three strongest earthquakes in southern Californian earthquake catalogue (1975-2017) occurred on the rising branch of IDT curve close or immediately after local minima. This local decrease of IDT values, possibly, points to the decreased extent of regularity (or increased randomness) in the earthquakes temporal distribution in periods prior to strongest earthquakes in California.

In order to avoid doubts related to the fixed starting point in the above analysis, we have carried out the same calculation of IDT values for catalogues, which started in 1985, 1990, 1995, 2000 and 2005. As we see in Fig. 9, analysis carried out on shorter catalogues, confirm the result obtained for the entire period of observation (1975-2017) and convinces that the curve of IDT values crosses abscissa at periods of relatively decreased seismic energy release. The case of M6.4 earthquake in Fig. 8*b*, is not an exception, as we explained above.

For better visibility of changes in the process of energy release, in Fig. 9 (bottom) we show increments of seismic energy release calculated for only the last 10 events in each consecutive windows, opposite to Fig. 8, where we presented energies released by all earthquakes in each window. This was done to make more visible the fine structure of changes in energy release in the expanding (by consecutive 10 events) part of windows, which otherwise is hidden by the strong background level of the summary energy release in the whole window. At the same time, we should not forget that IDTs in Fig.9 are calculated for the entire length of windows and that real evolution of energy release looks similar to presented in Fig.8 *b*.

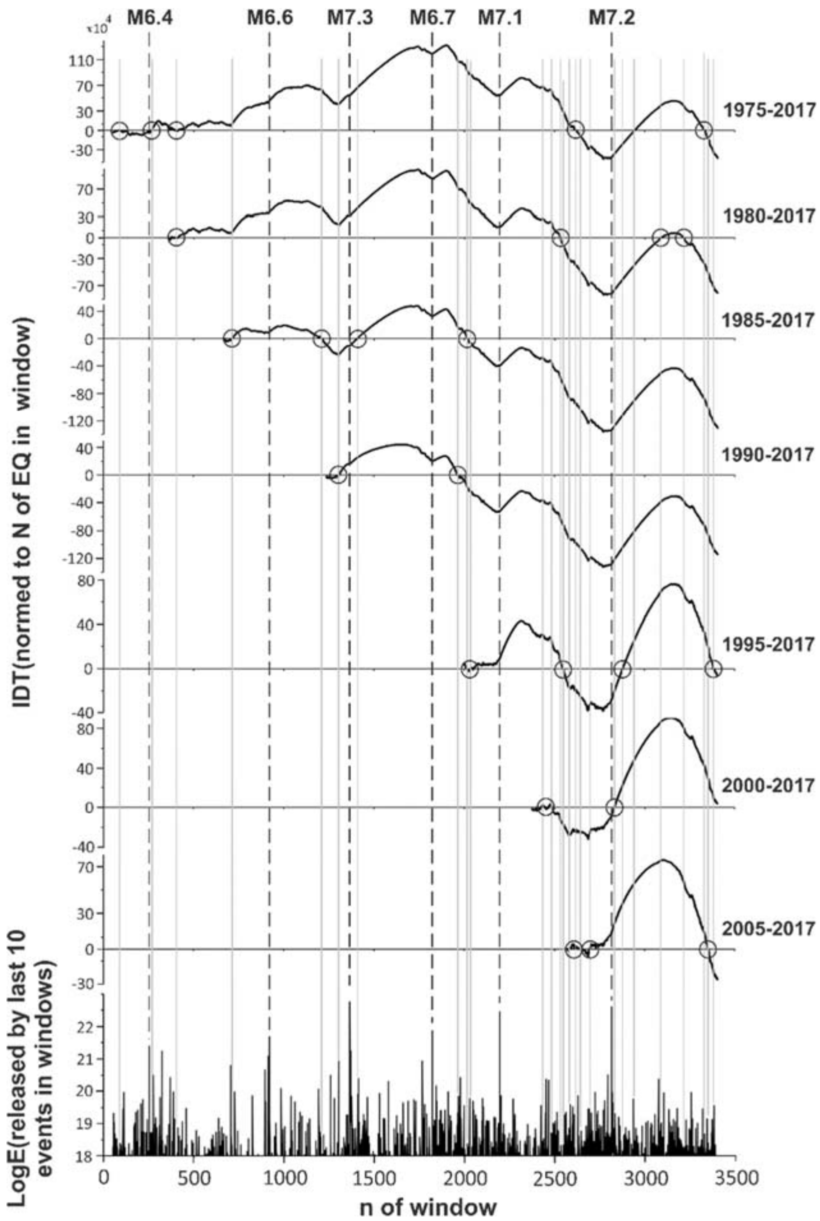

Fig. 9. Calculated for the expanding (by consecutive 10 data) windows, integral deviation times (top 7 curves) and the increments of seismic energies released by 10 last events in consecutive windows (bottom curve) obtained from the south

California earthquakes catalogue (above threshold M2.6). By the grey circles and grey vertical lines we show, where IDT curves cross abscissa axis. Dashed lines show the occurrence of largest earthquakes in the catalogue.

Thus, we see that shortening the time span of the analyzed part of catalogue does not influence obtained results.

We above already discussed influence of increased representative threshold on the calculated for entire catalogue span IDT value. Now, it was necessary to check, how the change of representative threshold will influence obtained results for expanding windows. This was a very important aspect of our analysis, because there is a well known point of view that the time distribution of large events (considered as independent ones), coupling between which is exception rather than a rule and medium/small earthquakes (for which time distribution may be governed or triggered by the interaction between events) is significantly different [Lombardi and Marzocchi, 2007].

To see how the results of integral deviation times analysis may be influenced by considering smaller or stronger earthquakes we carried out analysis of south California catalogue for earthquakes above M3.6 and M4.6 thresholds. Analysis (see results in Figs. 10 and 11) has been accomplished in a manner, similar to the scheme for the threshold M2.6, i.e. for the entire available period 1975-2017 and for shorter periods (from 1985, 1990, 1995, 2000, 2005 to 2017). Further analysis by the same scheme for higher (e.g. M5.6) threshold magnitudes was impossible because of the scarce number of large earthquakes in the considered seismic catalogue (just 29 earthquakes above M5.6). At the same time we point out that even for M5.6 representative threshold, for the entire period 1975-2017, the results obtained for two or three available windows (29 events occurred in windows, expanding by 9 or 10 data) agree with the above results and show that a lower IDT value corresponds to period with decreased seismic energy release.

Thus, we conclude that the increase of magnitude threshold (Figs. 10 and 11) practically do not change the results, found for a lower representative threshold. This means that increasing representative threshold we still deal with the catalogue, in which relatively small and medium size events prevail. Therefore, conclusions drawn from the analysis for original representative threshold (M2.6) remain correct for the case, when we consider a catalogue with relatively stronger events; it seems that there is no principal difference in the character of the contribution of smaller and stronger events to the results of IDT calculation. Comparison with the results obtained for time randomized catalogues confirm this conclusion.

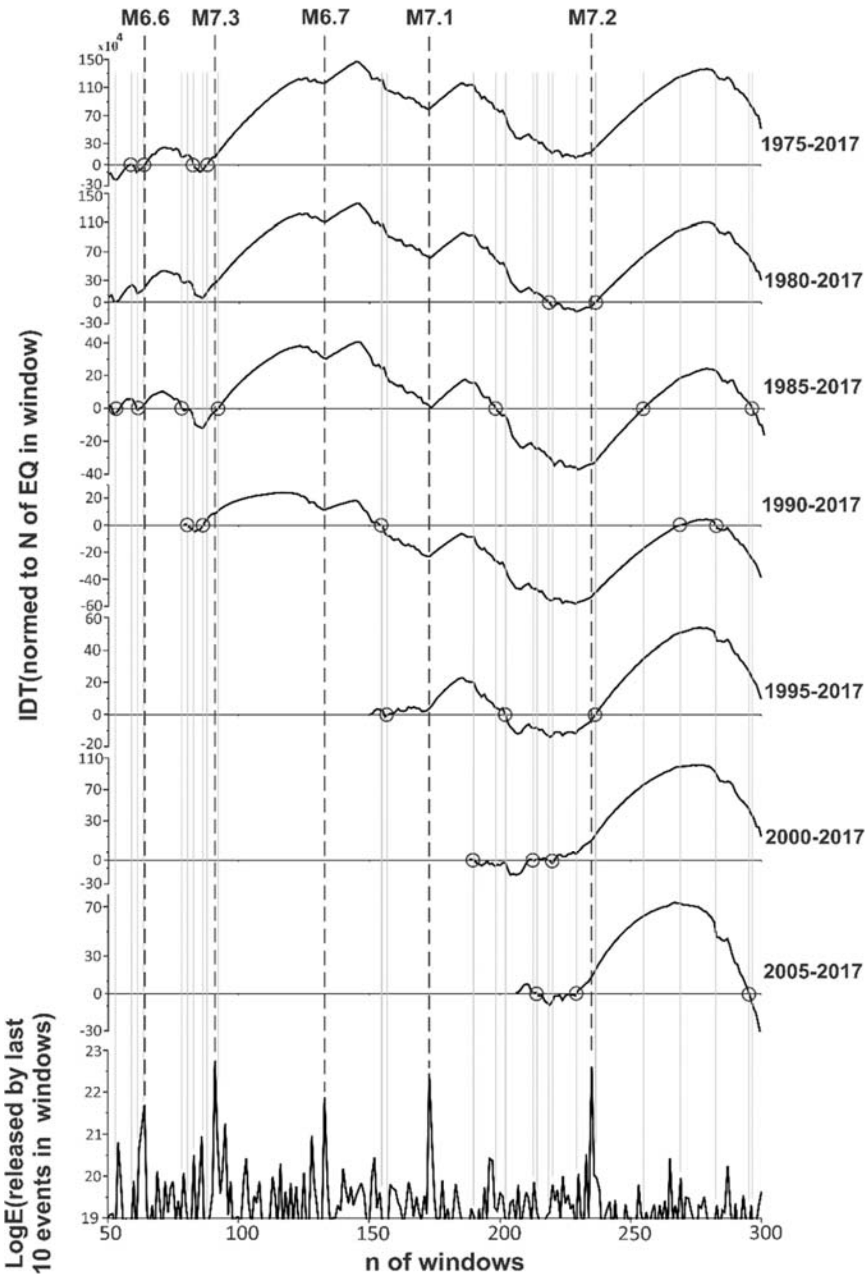

Fig. 10. Calculated for the expanding (by consecutive 10 data) windows integral deviation times (top 7 curves) and increments of seismic energies released by 10 last events in consecutive windows (bottom curve) obtained from the South California earthquakes catalogue (above threshold M3.6). By the grey circles and grey vertical lines we show, where IDT curves cross abscissa axis. Dashed lines show the occurrence of largest earthquakes in the catalogue.

Because of mentioned above unclearness in Figs. 9-11, when we calculated IDTs for the expanding windows and discuss results for the energy release occurred in the last 10 data windows, we accomplished additional analysis on the sliding windows with fixed number of events. In detail, in the South Californian earthquake catalogue we have calculated IDT values for non-overlapping windows of 100 consecutive events, shifted by 100 data (Figs. 12, 13). We have used short sliding windows of 100 data for two reasons: i. to have good resolution of changes occurred in the time distribution of earthquakes and because, ii. even relatively short, 100 data span windows also provide good enough discrimination in the IDT values, as it is shown in Figs. 5*b* and 6.

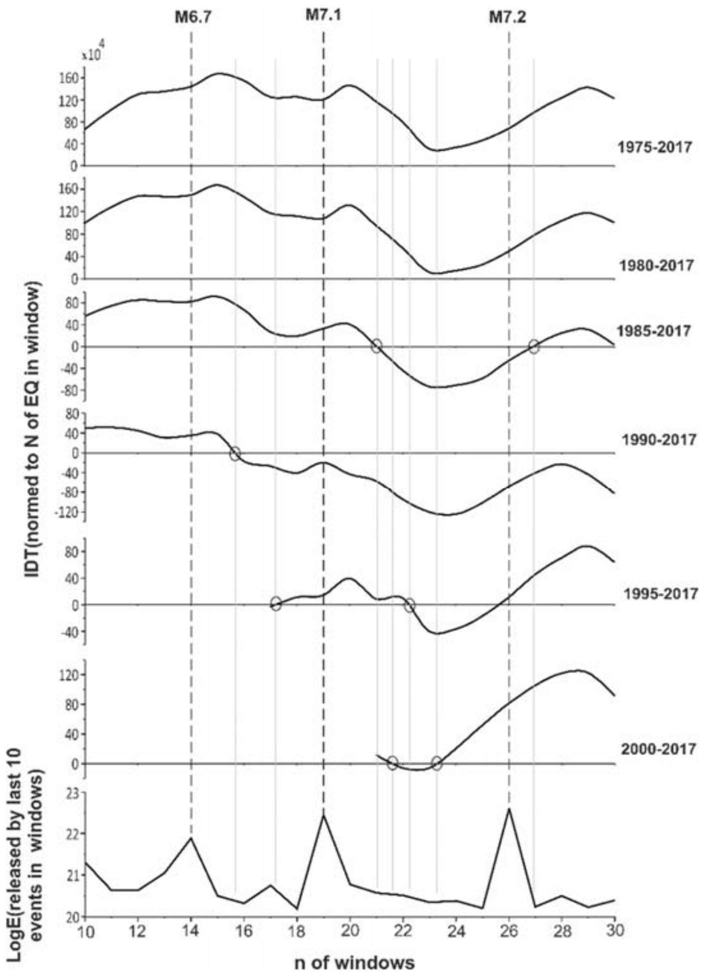

Fig. 11. Calculated for the expanding (by consecutive 10 data) windows integral deviation times (here we have only top 6 curves, because for the 7th curve, corresponding to the period 2005-2017 the number of events at the threshold M4.6 is small) and the increments of seismic energies released by 10 last events in consecutive windows (bottom curve) obtained from the south California earthquakes catalogue (above threshold M4.6). By the grey circles and grey vertical lines we show, where IDT curves cross abscissa axis. Dashed lines show the occurrence of largest earthquakes in the catalogue.

In Fig. 12 (top), we see that for the entire period of analysis there are dozens of IDT values that are not far from one tenth of the standard deviation (σ) from zero (given by black circles). Most importantly, among them 8 of IDT values are within of 0.01σ to zero. These values of IDT (shown in black in the middle figure), can be regarded practically equal to zero. According to the above results on simulated and original data sets, the seismic process in the windows with close to zero IDT values can be regarded as random. If we compare occurrence of these practically zero IDT values with the amount of seismic energy released in consecutive windows (bottom graph in Fig. 12) it becomes clear that they occur in periods of essentially decreased (comparing to observed maximums) seismic energy release. Similar conclusion we draw from the analysis of catalogues for earthquakes above M3.6 threshold (Fig.13). Because of the restricted number of strong events in the catalogue, further increase of the threshold magnitude was impossible for the case of 100 data non-overlapping sliding windows.

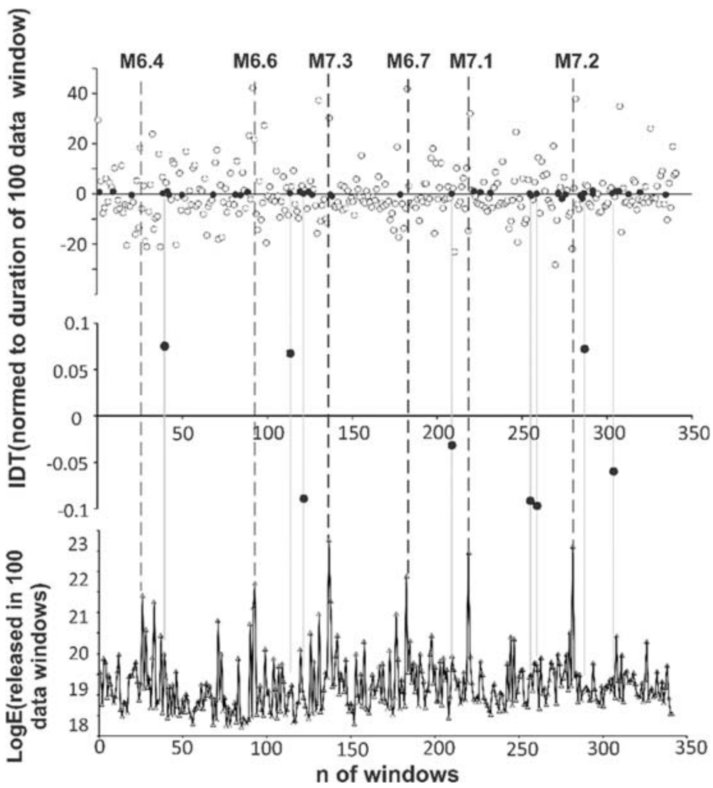

Fig. 12. Calculated for the non-overlapping 100 data windows (shifted by 100 data), integral deviation times (circles in the upper and middle curves) and the released seismic energies (triangles in bottom curve). IDT values in vicinity of 0.1σ to zero are given by black circles in the top figure. IDT values in vicinity of 0.01σ to zero are given by black circles in the middle figure. By grey lines, we show location of closest to zero IDT values relative to the released seismic energy. Dashed lines show the occurrence of largest earthquakes in the south California catalogue (above threshold M2.6).

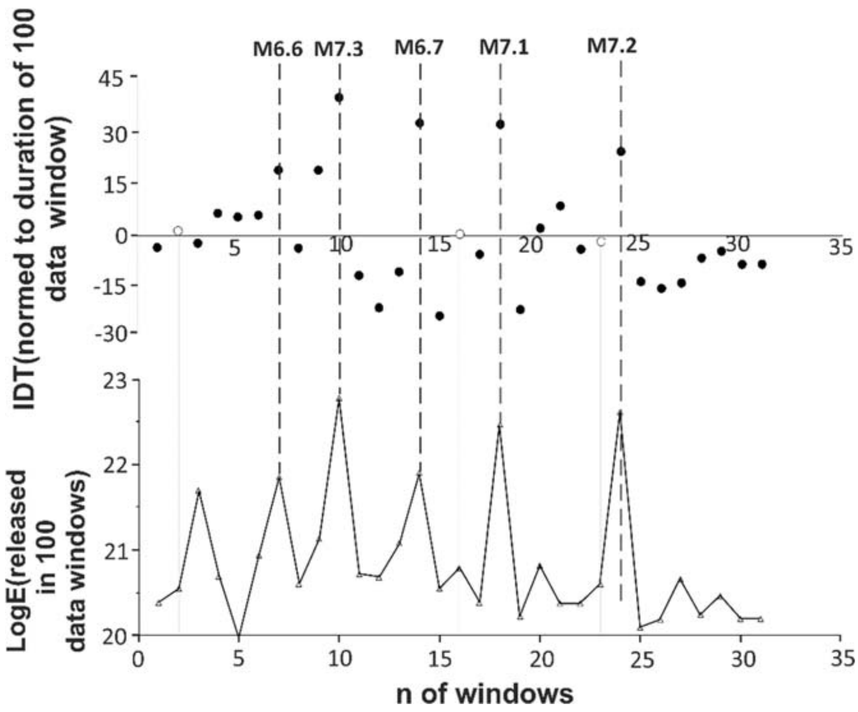

Fig. 13. Calculated for the non-overlapping 100 data windows (shifted by 100 data), integral deviation times (circles in the upper curve) and the released seismic energies (triangles in bottom curve). IDT values in vicinity of 0.1σ zero, are given by open circles in the top figure. By grey lines, we show location of closest to zero IDT values relative to the released seismic energy. Dashed lines show the occurrence of largest earthquakes in the South California catalogue (above threshold M3.6).

Results, obtained for non-overlapping sliding windows of fixed length also confirm the results for expanding windows.

Simple statistical approach used here shows that the extent of randomness in the earthquakes time distribution is changing over time and that it is most random-like at periods of decreased seismic activity. The results of this analysis provide additional indirect arguments in favor of our earlier suggestion that the extent of regularity in the earthquakes' time distribution should decrease in seismically quiet periods and increase in the periods of strong earthquakes preparation [Matcharashvili, et al. 2011; Matcharashvili, et al. 2013].

Summary

We investigated earthquakes time distribution in the Southern Californian earthquake catalogue by the method of calculation of integral deviation times relative to regular time marks. The main goal of research was to quantify, when the time distribution of earthquakes become closer to the random process. Together with IDT calculation, standard methods of complex data analysis such as power spectrum regression, Lempel and Ziv complexity and recurrence quantification analysis, as well as multi-scale entropy calculation have also been used. Analysis was accomplished for different time intervals and for different magnitude thresholds. Based on a simple statistical analysis results, we infer that the temporal distribution of earthquakes in Southern Californian catalogue is the most random-like at the periods of decreased local seismic activity.

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