Review Response: "Brief Communication: A nonlinear self-similar solution to barotropic flow over varying topography"

We are thankful for the time and effort put into these thorough and thoughtful reviews. It would appear that during our efforts to condense this manuscript into a "brief communication," we have over simplified the presentation. Please allow us to clarify the comments of both reviewers.

1) Let us begin with the major criticism of reviewer 2 regarding the problem formulation. The issue is related to the physical origins of the Ekman suction term ($\vec{\Pi}_E = -h_E\hat{k} \times \vec{u}$) and our imprecise definition of transport function in the original manuscript. Kuehl and Sheremet (2014, JFM) contains the more complete treatment. The transport function is defined through: $h\mathbf{u} = \hat{\mathbf{k}} \times \nabla\psi + \nabla\phi$, which is a generic decomposition. We over simplified this in our write-up to $h\mathbf{u} = \hat{\mathbf{k}} \times \nabla\psi$, which the review correctly identified. Now, $\nabla\phi = -\Pi_E$ which is from standard Ekman suction. Thus, we find $\nabla \cdot (h\mathbf{u}) = \nabla^{\mathbf{2}}\phi = -\nabla \cdot \mathbf{\Pi_E}$, which is the steady state form of the continuity equation (eqn. 1). In the limit of no Ekman layer, the flow must follow isobaths. However, the presence of an Ekman layer allows the flow to cross isobaths. Also, the relationship between h and p is standard hydrostatic, but was not noted as we are considering the vorticity equation.

The text has been updated to clarify this by including the proper definition of ψ, which should address the reviewer concern. "defining an interior transport function ψ through $h\mathbf{u} = \hat{\mathbf{k}} \times \nabla\psi + \nabla\phi$ (where $\nabla\phi = -\Pi_E$ represent Ekman divergence)"

2) Both reviewers have expressed concern over the "special" type of topography considered in this work, $h = h_0 - \alpha x y^{-\gamma}$. This is actually quite a generic topography and we feel this concern is not warranted. Consider the most simplistic model of the ocean slope, a constant slope $h = h_0 - \alpha x$. The next step is to allow that slope to vary linearly in the streamwise direction, $h = h_0 - \alpha x y$. Here we consider even more generic topographic variations, $h = h_0 - \alpha x y^{-\gamma}$. So the topography is not overly idealized, in the sense of a simple oceanic slope. It is a reasonable to ask how well the slope models the actual ocean topography. To this end, Ibanez developed a topography fitting code and has considered several oceanographic regions including the Norwegian coast, west Florida coast and west Keewenaw Peninsula in Lake Superior (Presented at: Ibanez et al. APS DFD 2016, Ibanez et al. APS DFD 2017 and in Ibanez's Masters Thesis), all of which exhibit large regions which fit the generic topography considered. Of particular interest is the

Norwegian coast, in which linearly increasing topography seems to lead to eddy pinch off of the Norwegian coastal current. In the vicinity of the pinch-off, the topography appears to vary cubicly and thus the nonlinear solution may be applicable. The eddies shed from the Norwegian coast appear to be important for deep water formation and thus may have climatological importance. Also note, Kuehl originally found good agreement between linearly varying topography on the northern slope of the Gulf of Mexico.

As this is a brief communication, analysis of ocean topography is not included. We have instead presented such analysis at APS DFD meetings and in prior publications. Thus, we feel the reviewers concerns about the topography are not warranted.

3) Reviewer 1 has expressed concern about the stability of the solution. Fully addressing this concern is on-going research via numerical linear stability analysis and rotating table experimental investigation. We can not fully answer this question at present, but can offer some motivation that the flow will be stable for some set of parameters. First, it is a common lab demonstration during geophysical fluid dynamics courses to illustrate Taylor columns over an isolated "bump" in topography. This is done with a sloping bottom on a rotating platform. I have run this demonstration and observed steady flow along sloping topography, indicating that stable solutions do exist. Also, the solution is evolving downstream, similar to the Blasius boundary layer, and preliminary stability analysis indicates a transition between stable and unstable regions of the flow. Finally, even in unstable regions of the flow, the steady analytic solution provides a means to study wave-mean-flow interaction and advective mixing processes.

We agree with the reviewer that the stability of the solution is an interesting question, but feel is does not diminish the relevance of the steady solution presented. As this is a brief communication, we feel a discussion of this point is not required in the text.

4) Reviewer 1 inquires about consideration of an f-plane versus a beta-plane. This solution considers a topographic beta-plane. It is assumed that the topographic beta terms (indicated between lines 10 and 15) are more significant than the planetary beta affect over the ocean slope. This is a reasonable assumption. Inclusion of a planetary beta-effect would contribute to the $\beta^{(x)}$ and $\beta^{(y)}$ terms.

5) Reviewer 1 has requested clarity concerning the "compressing" and "expanding" jet cases. The manuscript contains a discussion of the domain

of relevance between lines 70 and 85. Compressing jets are those that approach a singularity at $y = \infty$, while expanding jets are those that start from a initial condition singularity at $y = 0$. Physically, this is determined by the exponentials (n or n_y) in the similarity variable, which are related to γ.

We have included a clarifying statement at the end of section 3.2. "Thus, we have adopted the terminology that expanding jet are those with a singularity at the upstream source region ($y = 0$) and compressing jets as those with the singularity at downstream exit region ($y = \infty$)."

6) Pseudo velocity satisfies $g' = u$, while regular velocity comes from the definition of transport.

7) Requiring the y-dependencies balance is just part of the solution procedure. Perhaps more generic cases can be identified in the future.

8) Several typos and formatting issues were identified by the reviewers, which will be corrected. Thank you.

9) As this is a brief communication, we have chosen not to include a discussion of future work. However, we are currently working toward stability analysis of the solutions and rotating table experimental investigation.