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Discussions

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# *Interactive comment on* "Wave propagation in the Lorenz-96 model" *by* Dirk L. Van Kekem and Alef E. Sterk

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We would like to thank the Referee for reading our manuscript and for providing constructive remarks and suggestions.

**Comment 1**

The famous Lorenz-63 model is a Galerkin projection of a fluid dynamical model describing Rayleigh-Bénard convection and hence has a clear physical interpretation. In contrast, the Lorenz-96 model does not have such an interpretation. In fact, Lorenz (2006) writes:

"The physics of the atmosphere is present only to the extent that there

are external forcing and internal dissipation, simulated by the constant and linear terms, while the quadratic terms, simulating advection, together conserve the total energy [...]"

The interpretation of Lorenz is that the variables "[...] may be thought of as values of some atmospheric quantity in $K$ sectors of a latitude circle." (In our paper, we denote the dimension by $n$ instead of $K$.) This is also the interpretation we have adopted in our paper. Since the Lorenz-96 model was not derived from physical principles the sign of the forcing parameter $F$ has no physical interpretation either.

The Hovmöller diagrams of the Lorenz-96 model reveal both travelling waves and stationary waves which have also been observed in physical models, such as the low-order shallow water model studied in Sterk et al. (2010). However, we do not think that it is possible to provide any stronger link with "reality" than that.

It is of course a legitimate question why one would be interested in a model without a physical interpretation. Despite its physical interpretation the Lorenz-63 model has two drawbacks. Firstly, it consists of only three ordinary differential equations. Secondly, for the classical parameter values the model has Lyapunov exponents $0.91$, $0$, and $-14.57$, which makes the model extremely dissipative. Such properties are atypical for atmospheric models. The Lorenz-96 model is a simple nonlinear model with a chaotic regime. Its dimension $n$ can be chosen arbitrarily large, and for suitable values of the parameters the Lyapunov spectrum resembles the spectra found in models obtained from discretized partial differential equations. For these reasons the Lorenz-96 model is used as a test model for a wide range of geophysical applications.

It is an interesting question how the parameter $n$ should be interpreted. It could indeed be interpreted as the resolution of discretization (a larger $n$ implying a finer grid). In fact, some authors interpret the Lorenz-96 model as a discretized partial differential equation; see, for example, Reich & Cotter, "Probabilistic Forecasting and Bayesian Data Assimilation", Cambridge University Press, 2015. However, our results show that

spatiotemporal properties of waves do not always converge as $n \to \infty$. This is also discussed in the context of routes to chaos in our paper arXiv:1704.05442 (v3). Firstly, this makes the interpretation of the Lorenz-96 model as a discretized PDE questionable. Secondly, this implies that the parameters $n$ and $F$ should be chosen carefully when using the Lorenz-96 model for testing purposes. For example, when using the model for experiments in the setting of extreme events, the spatiotemporal properties have a large impact on return times and hence the predictability and statistics of extremes. We will explain this more clearly in the revised manuscript.

The lack of convergence of the dynamics with $n$ could be caused by the fact that the coefficients of all terms in the model are 1. We consider it to be an interesting mathematical problem to investigate whether the coefficients of the model can be changed in such a way that dynamical properties do converge as $n \to \infty$. However, we also feel that this question is beyond the scope of the present manuscript.

**Comment 2**

We agree with the referee that there is some overlap with the present manuscript with our paper arXiv:1704.05442 (v3):

- The eigenvalues of the trivial equilibrium $(F, \ldots, F)$.

- Theorem 1 is a concise summary of Theorems in arXiv:1704.05442 (v3).

- The bounds on the wave number for $F > 0$.

We feel that this overlap is needed in order to (1) give a complete and coherent overview of waves in the Lorenz-96 model and (2) to emphasize the difference between the cases $F > 0$ and $F < 0$. In particular, for $F < 0$ the spatiotemporal properties of waves in the Lorenz-96 model depend on the remainder of $n$ upon division by 4. Moreover, for $F < 0$ the spatial pattern of the wave is determined by the structure of the equilibrium solution that is born through one or two pitchfork bifurcations. To

our knowledge this has not been discussed in the literature before. Also note that in previous work we have not discussed traveling waves for $F < 0$.

The example of the double-Hopf bifurcation for $n = 12$ also appears in arXiv:1704.05442 (v3), but in the current manuscript we show that the number double-Hopf bifurcations grows quadratically with $n$. In addition, we show that for many dimensions of the 2-parameter model there is a pair of double-Hopf bifurcations close to the $F$-axis which causes the co-existence of *three* stable waves. This phenomenon has not been discussed before.

We feel that our results are of interest to NPG readers. Among these readers there are researchers who use the Lorenz-96 model for testing purposes. From Table 1 it becomes clear that many researchers stick with the canonical choices $n = 36$ and $n = 40$. As argued above, such choices can influence the results of numerical experiments. We hope that by giving an coherent overview of the dynamics for various values of $n$ and both $F > 0$ and $F < 0$ users of the Lorenz-96 model will be able to make appropriate parameter choices that suit their purposes.

**Comment 3**

After re-reading this paragraph we agree with the Referee that our explanation is not sufficiently detailed. After a Hopf bifurcation there is a periodic orbit. We can then take a Poincaré section to obtain a discrete-time dynamical system with a stable fixed point. The latter bifurcates through a Neimark-Sacker bifurcation which gives rise to a closed invariant curve, which corresponds to a 2-dimensional torus in the continuous-time system.

In the revised manuscript we will provide a more elaborate explanation of this bifurcation scenario and give additional references. We will also provide a clearer definition of the "multi-stability lobe". The fact that this exists near a Hopf-Hopf bifurcation follows from the normal form theorem of the Hopf-Hopf bifurcation. In general this region does not need to be bounded (but whether it is bounded or not does not affect our results).

We will provide a more detailed explanation in the revised manuscript.

**Technical comments**

Finally, we thank the Referee for suggesting the technical corrections. We will incorporate these in the revision of our manuscript.

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