Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg-2017-55
Manuscript under review for journal Nonlin. Processes Geophys.
Discussion started: 6 October 2017

Nonlinear Processes in Geophysics
Discussions
Open Access

Nonlinear analysis of the occurrence of hurricanes in the Gulf of Mexico and the Caribbean Sea

Berenice Rojo-Garibaldi [1], David Alberto Salas-de-León [2], María Adela Monreal-Gómez [2], Norma Leticia Sánchez-Santillán [3], and David Salas-Monreal [4]

[1]Posgrado en Ciencias del Mar y Limnología, Universidad Nacional Autónoma de México, Av. Universidad 3000, Col. Copilco, Del. Coyoacán, Cd.Mx. 04510, México
[2]Instituto de Ciencias del Mar y Limnología, Universidad Nacional Autónoma de México, Av. Universidad 3000, Col. Copilco, Del. Coyoacán, Cd.Mx. 04510, México
[3]Dep. El Hombre y su Ambiente, Universidad Autónoma Metropolitana, Calz. del Hueso 1100, Del. Coyoacán, Villa Quietud, Cd.Mx. 04960, México
[4]Instituto de Ciencias Marinas y Pesquerías, Universidad Veracruzana, Hidalgo No. 617 Col. Río Jamapa, C.P. 94290 Boca del Río, Veracruz, México

Correspondence to: D. A. Salas-de-León (dsalas@unam.mx)

Abstract. Abstract. Hurricanes are complex systems that carry large amounts of energy. Their impact produces, the majority of the time, natural disasters involving the loss of human lives and of materials and infrastructure in billions of US dollars. However, not everything is negative, as hurricanes are the main source of rainwater for the regions where they develop. In this study, we perform a nonlinear analysis of the time series obtained from 1749 to 2012 of the occurrence of hurricanes in the Gulf of Mexico and the Caribbean Sea. The construction of the hurricane time series was carried out based on the hurricane database of The North Atlantic-basin Hurricane Database (HURDAT), and the published historical information. The Lyapunov exponent indicated that the system presented chaotic dynamics, and the time-series' spectral analysis along with the nonlinear analysis of the hurricanes time series showed chaotic edge behavior. One possible explanation for this edge is the individual chaotic behavior of hurricanes, either by category or individually, regardless of their category, and their behavior on a regular basis.

1 Introduction

Hurricanes have been studied since ancient times, their activity is related to disasters and loss of life. In recent years, there has been considerable progress in predicting their trajectory and intensity, once their tracking began, as well as their number and intensity from one year to the next. However, their long-term, and very short prediction remains a challenge (Halsey and Jensen, 2004), and damage to both materials and lives remains considerable. Therefore, it is important to make a greater effort in the study of hurricanes to reduce the damage they cause. The periodic behavior of hurricanes and their relationship with other natural phenomena have usually been performed with linear-type analyses, which have provided valuable information. However, we decided to make a different contribution by carrying out a non-linear analysis of time series of hurricanes that occurred in the Gulf of Mexico and the Caribbean Sea, since the dynamics of the system is controlled by a set of variables of low dimensionality (Gratrix and Elgin, 2004; Broomhead and King, 1986). Theories of deterministic chaos and fractal structure have already been applied to atmospheric boundary data (Tsonis and Elsner, 1988; Zeng et al., 1992), to the pulse of severe rain time series (Sharifi et al., 1990; Zeng et al., 1992), and to the tropical cyclone trajectory (Fraedrich and Leslie, 1989; Fraedrich et al., 1990). Natural phenomena occur in nature within different contexts; however, they often exhibit common characteristics, or they may be understood using similar concepts. Deterministic chaos and fractal structure in dissipative dynamical systems are among the most important non-linear paradigms (Zeng et al., 1992). For a detailed analysis of deterministic chaos, the Lyapunov exponent is utilized as a key point and several methods have been developed to calculate it. It is possible to define different Lyapunov exponents for a dynamic system. The maximal Lyapunov exponent can be determined without the explicit construction of a time-series model. A reliable characterization requires that the independence of the embedded parameters and the exponential law for the growth of distances can be explicitly tested (Rigney et al., 1993; Rosenstein et al., 1993). This exponent provides a qualitative characterization of the dynamic behavior and also provides the predictability measurement (Atari et al., 2003). The algorithms usually employed to obtain the Lyapunov exponent are those proposed by Wolf (1986), Eckmann and Ruelle (1992), Kantz (1994), and Rosenstein et al. (1993). The methods of Wolf (1986) and Eckmann and Ruelle (1992) assume that the data source is indeed a deterministic dynamic system and that irregular fluctuations in time-series data are due to deterministic chaos. Blind application of this algorithm to an arbitrary set of data will always produce numbers, i.e., these methods do not provide a strong test of whether the calculated number can actually be interpreted as Lyapunov exponents of a deterministic system (Kantz et al., 2013). The Rosenstein et al. (1993) method follows directly from the definition of the Lyapunov maximal exponent and is accurate because it takes advantage of all available data. The algorithm is fast, easy-to-implement, and robust for changes in the following quantities: embedded dimensions; data-set size; delay reconstruction, and noise level. The Kantz (1994) algorithm is similar to that of Rosenstein et al. (1993).

We constructed a database of occurrences of hurricanes in the Gulf of Mexico and the Caribbean Sea to make a nonlinear analysis of the time series, the results can help in the construction of models of occurrence of hurricanes and this in turn, will help to reinforce the measures of prevention before this type of hydrometeorological phenomena.

2 Materials and methods

2.1 Data set description

A historical database of hurricane occurrence in the Gulf of Mexico and the Caribbean Sea was constructed. A detailed analysis of the recorded hurricanes' historical reports was provided by the ships sailing near the reported hurricanes; aerial reconnaissance data for recent hurricanes were obtained from 1749-2012. The aerial surveys began in 1944. Finally, the hurricanes reported by Fernández-Partagas and Díaz (1995a; 1995b; 1996a; 1996b; 1996c; 1997; 1999), were added, along with information obtained from the HURDAT re-analysis project. All that information was used to build the hurricane time series (Fig. 1).

Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg-2017-55
Manuscript under review for journal Nonlin. Processes Geophys.
Discussion started: 6 October 2017

Nonlinear Processes in Geophysics
Discussions
Open Access

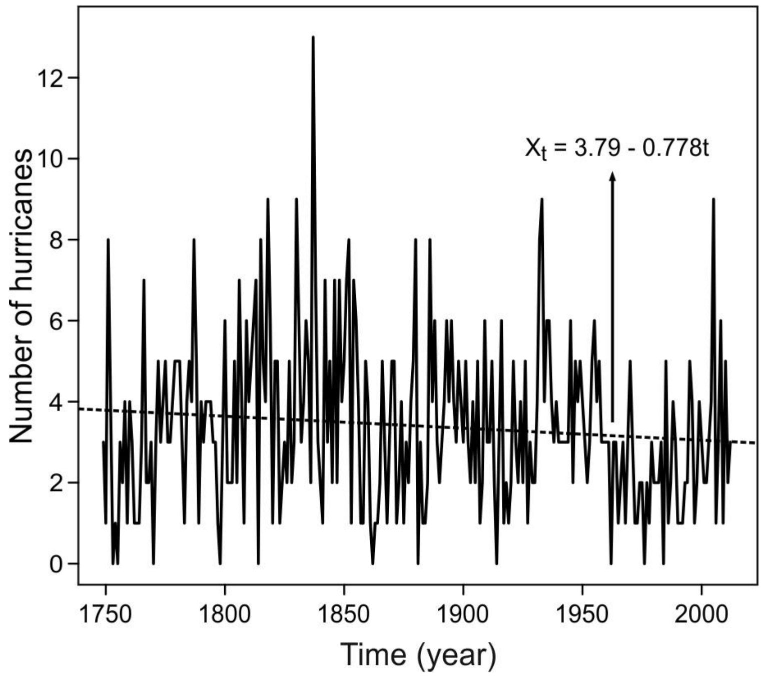

Figure 1. Hurricanes from the years 1749-2012. The line shows their linear trend (afther Rojo-Garibaldi et al., 2016).

Only historical hurricanes that were reported in two or more databases were considered valid; on the other hand, the hurricane trajectory was followed to avoid counting, more than, once a specific hurricane that was reported in different places within a very short time. This was done in order to avoid overcounting the number of hurricanes; to do this, we follow the proposed method by Rojo-Garibaldi et al. (2016).

2.2 Data reduction and procedures

To know the properties of the system requires more than estimating the dimension of the attractor (Jensen et al., 1985); so three methods were applied in this study: 1) the Hurst exponent, which is a measure of the independence of the time series as an element to distinguish fractal series. It is basically a statistical method which provide the number of occurrence of rare events and is usually called re-scaling (RS) rank analysis (Gutiérrez, 2008), therefore it could be applied to any time series. The RS helps to find the Hurst exponent, which provides the numerical value which makes possible to determine the autocorrelation in a data series. 2) The Lyapunov exponent is invariant under soft transformations, because it describes the long-term behavior, providing an objective characterization of the corresponding dynamics (Kantz and Schreiber, 2004). The presence of chaos in dynamic systems can be solved by this exponent, since it quantifies the exponential convergence or divergence of initially close trajectories in the state space and estimates the amount of chaos in a system (Rosenstein et al., 1993; Haken, 1981; Wolf, 1986). The Lyapunov exponent (A) can take one of the following four values: $A < 0$ corresponds to a stable fixed point, $A = 0$

Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg-2017-55
Manuscript under review for journal Nonlin. Processes Geophys.
Discussion started: 6 October 2017

which is for a stable limit cycle, $0 < A < \infty$ indicates chaos and $A = \infty$ for a Brownian process, which agrees with the fact that the entropy of a stochastic process is infinite (Kantz and Schreiber, 2004). 3). The Iterated Function Analysis (IFS) is an easier and simpler way to visualize the fine structure of the time series, it can reveal correlations in the data and help to characterize its color, referring to color to the type of noise (Miramontes et al., 2001). Together with the Lyapunov exponent, the phase diagrams, the False Close Neighbors method, the Space-Time Separation plot, the Correlation Integral plot, and the Correlation Dimension were taken into account, the latter two to identify whether the system attractor was fractal type. It is important to emphasize that in order to compute the Lyapunov exponent the algorithms proposed by Kantz (1994) and Rosenstein et al. (1993) were used in this study.

To detect some kind of chaotic behavior we plotted the Poincar surface, known as a strobe map, which can be interpreted as a discrete dynamic system in a state space with a smaller dimension than the original continuous system, i.e. once a dimensional hyperplane $(m-1)$ is taken in the dimensional space (embedded) m, a compressed time series of only the intersections of the continuous time path is created with this hyperplane in a predefined orientation (Kantz and Schreiber, 2004; Hegger et al., 1999). Using these method three types of trajectories can be obtained: 1) A simple periodic orbit of the original dynamic system that becomes a single fixed point of the section of Poincar, 2) a quasi-periodical trajectory that becomes the image of a closed curve, and 3) a chaotic movement that becomes a zone of erratically distributed points.

3 Results and discussions

Figure 1 shows the evolution of the number of hurricanes from 1749 to 2012 and their linear trend. In order to have a qualitative idea of the number of hurricanes that occurred in the Gulf of Mexico and the Caribbean Sea between 1749 to 2012 a phase diagram was performed using the "delay method" (Fig. 2). This was also used to elucidate the time lag for an optimal embedding in the dataset.

The optimal time lag (τ) obtained from Fig. 2 was equal to 9. In our case, the hurricane dynamics were not distinguish through with the phase diagram; however, since any hurricane trajectory starts at a close point location on the attractor dataset that diverges exponentially, it is a primary evidence of a chaotic motion according to Thompson and Stewart (1986).

Another way to visualize the dynamics of the system is through the Poincare Surface Section (Fig. 3), which helped us to observe the presence of chaos involved in our data. We observed that the points are scattered in the constructed plane, indicating that there is a chaotic behavior. However, the most robust method to identify chaos within the system is the Lyapunov exponent. Prior to obtaining the exponent, it was necessary to calculate the time lag using the Theiler window and the embedding dimension. The time lag was obtained with three different methods: 1) The method of constructing delays, 2) The method of mutual information, which yields a more reliable result since it works with the nonlinear part of the systems, and 3) The autocorrelation function methods (Fig. 4).

There are two ways to obtain the time lag from the autocorrelation function. We used the criterion of the first zero, because the exponent of Hurst ($H = 0.032$) indicated that it was a short memory process; therefore, the criterion of the first zero is optimal method in this type of cases. The Hurst exponent aids us to identify the criteria to find a time lag, and it also describes

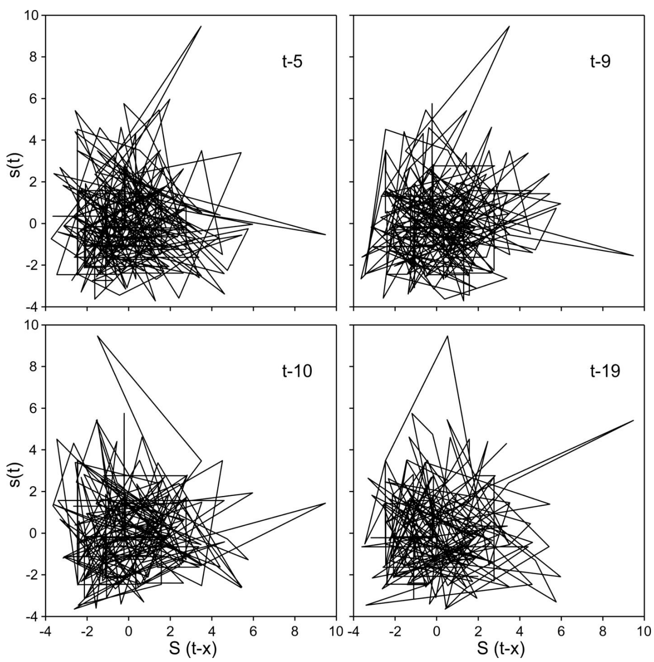

Figure 2. Phase diagrams corresponding to the time series of hurricanes that occurred between the years 1749 and 2012 in the Gulf of Mexico and the Caribbean Sea. The x-axis in the four plots indicates the time lag (τ).

system behavior (Quintero and Delgado, 2011), this could indicate that the system does not have chaotic behavior; however, the remaining methods have indicated the opposite and as mentioned previously, the Lyapunov exponent is considered the most appropriate method for this type of dataset. Therefore, different methods will provide different results, but the time series will indicate the best method and the result we should use.

It was possible to observe the difference in the time lag obtained by both methods; however, it is necessary to use only one result. Through the space-time separation graphic and the False Close Neighbors method, we obtained embedding dimensions of $m = 4$ for a $r = 9$, and $m = 5$ for $r = 10$ and the Theiler window (Fig. 5).

The idea of the False Close Neighbors algorithm is that at each point in the time series, $\bar{S}_t$ its neighbour $\bar{S}_j$ should be searched in a m-dimensional space. Thus, the distance $\|S_t - S_j\|$ is calculated iterating both points, given by:

$$R_i = \frac{S_{i+1} - S_{j+1}}{\|\bar{S}_i - \bar{S}_j\|} \quad (1)$$

Therefore, it may have some False Close Neighbours even when working with the correct embedding dimension. The result of this analysis may depend on the time lag (Kantz and Schreiber, 2004).

Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg-2017-55
Manuscript under review for journal Nonlin. Processes Geophys.
Discussion started: 6 October 2017

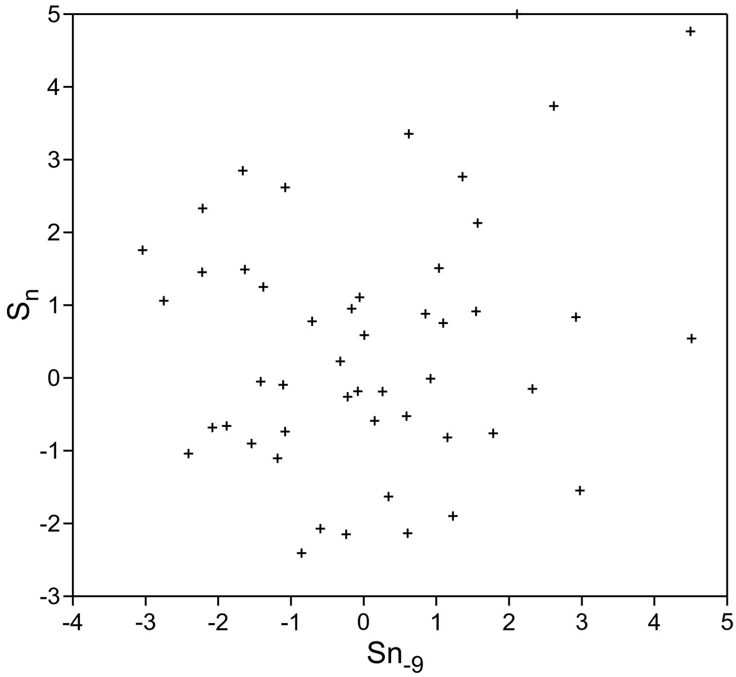

Figure 3. Poincare surface section for the time series of hurricanes that occurred between the years 1749 and 2012 in the Gulf of Mexico and the Caribbean Sea.

The Lyapunov (A) exponents were obtained using the Kantz and Rosenstein methods taking the time lag and the embedding dimension. The Kantz (1994) method using a value of $m = 4$ and $r = 9$ give us an exponent of $A = 0.48392$ and for $m = 5$ and $r = 10$ the exponent was of $A = 0.48392$. Since A is a positive value, it was inferred that our system is chaotic. In addition, the value of A obtained for both imbibing dimensions was the same, suggesting that our result is accurate. Using the Rosenstein et al. (1993) method, the value obtained for $m = 4$ and $r = 9$ was of $A = 0.1056$ and for $m = 5$ and $r = 10$, the exponent was $A = 0.1123$ (Fig. 6).

There was a difference between placing the attractor in an embedding dimension of $m = 4$ and one of $m = 5$, a better unfolding of the attractor in the embedding dimension was observed in $m = 4$. Continuing with the analysis, the Integral Correlation was obtained for the occurrence of hurricanes in our study area (Fig. 7).

As observed in Fig. 7, as m increases, the structure eventually becomes invariant in high embedding dimensions; this is an indicator that no extra variables are needed to elucidate the dynamics of the system. The Integral of Correlation was obtained for a total of 10 different dimensions. There was a linear correspondence among the dimension; however, after the 4^{th} dimension the behavior becomes invariable, which was something expected according to the method of False Neighbors and the results obtained from the Lyapunov exponent. Here it was also confirmed that the minimal dimension needed to study the system dynamics was fourth dimension. Fig. 8 shows the Correlation Dimension D_2 for multiple length scales.

Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg-2017-55
Manuscript under review for journal Nonlin. Processes Geophys.
Discussion started: 6 October 2017

Nonlinear Processes in Geophysics
Discussions
Open Access
EGU

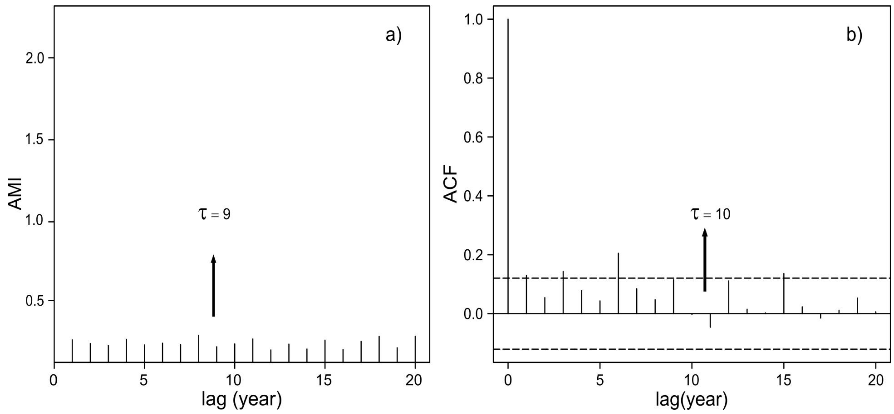

Figure 4. The left panel shows the Mutual Information Method, the x-axis indicates the time lag against the Mutual Information Index (MII), the arrow indicates the first most pronounced minimal with a value of r = 9. The right panel shows the autocorrelation function, the x-axis indicates the time lag versus the value of the autocorrelation function, and the arrow denotes where the first zero of the function r = 10 was obtained.

The right panel on Fig. 9 shows the slope trend of the majority of the slopes of the Correlation Integral (ϵ). In the range of $1 < \epsilon < 10$ requires to have straight lines as an indicator of the self-similar geometry. The value obtained here correspond to $D_2 = 2.20$ which is the aforementioned slope value. Another method to see the attractor dimension is the Kaplan-Yorke Dimension (D_{ky}) which is given by:

$$D_{ky} = k + \sum_{i=1}^{k} \frac{\lambda_i}{|\lambda_{k-1}|} \tag{2}$$

Where k is the maximal integer, such that the sum of the k major exponents is not negative. The fractal dimension with this method yielded a value of $D_{ky} = 2.26$, which is similar to the one obtained previously.

Even when all the requirements necessary to apply the non-linear analysis to our time series are present, one final requirement must be fulfilled to know whether we can obtain a dimension and whether the complete spectrum of Lyapunov exponents still need to be employed (another method to visualize chaos). Eckmann and Ruelle (1992) discuss the size of the dataset required

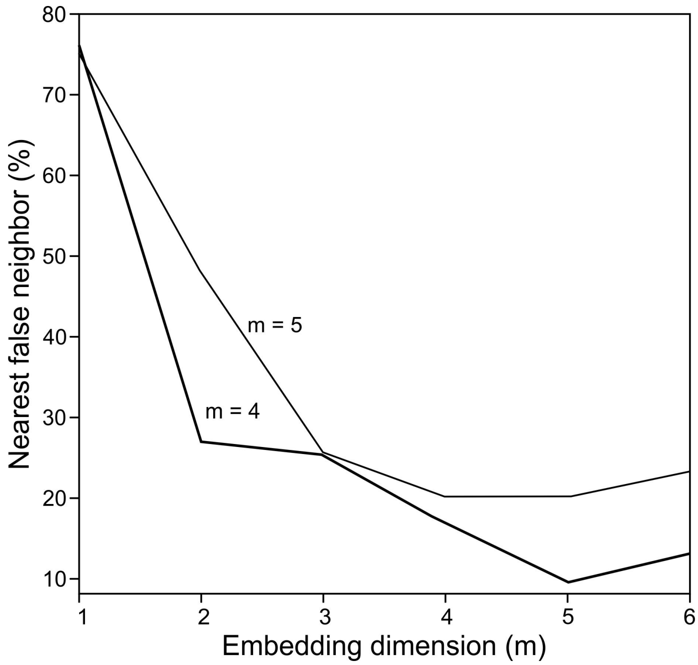

Figure 5. False Close Neighbors with a time lag of 10, where of the embedding dimension of 5 has a 9.4%, and the embedding dimension of 4 has a 16.66% False Close Neighbors (lower line). False Close Neighbors with a time lag of 9, where of the embedding dimension of 5 has a 20.15%, and the embedding dimension of 4 has a 20.12% False Close Neighbors (upper line). The values in each line indicate the optimal dimension for each lag.

to estimate Lyapunov dimensions and exponents. When these measure the divergence rate with near-initial conditions, they require a number of neighbors for a given reference point. These neighbors may be within a sphere of radius r, and of a given diameter (d) the reconstructed attractor; Therefore:

$$\frac{r}{d} = \rho << 1 \quad (3)$$

Eckmann and Ruelle (1992) suggested that ρ should be maximum around 0.1. In addition, the number of candidates for neighbors $(\Gamma(r))$, must be much greater than one. Therefore:

$$\Gamma(r) >> 1 \quad (4)$$

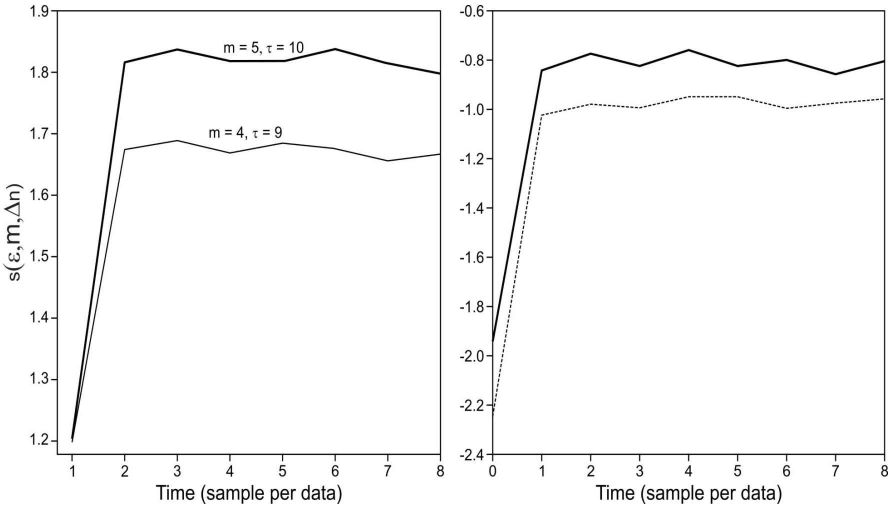

Figure 6. Left panel: Lyapunov exponent with m = 4, r = 9 and m=5, r=10, with the Kantz method. Right panel: Lyapunov exponent with the same values with the Rosenstein method.

where:

$$\Gamma(r) \approx constant \times r^D \tag{5}$$

Then:

$$\Gamma(d) \approx N \tag{6}$$

Where D is the attractor dimension and N is the number of points or data. The following relation was then obtained from equations (4) and (6):

$$\Gamma(r) \approx N(\frac{r}{d})^D >> 1 \tag{7}$$

Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg-2017-55
Manuscript under review for journal Nonlin. Processes Geophys.
Discussion started: 6 October 2017

Nonlinear Processes in Geophysics
Discussions
Open Access

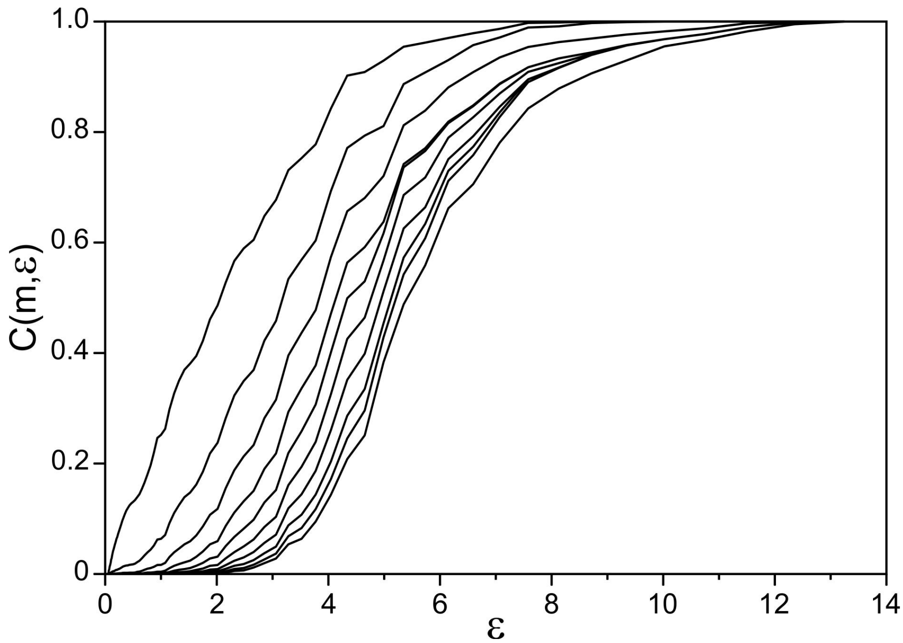

Figure 7. Integral Correlation for the occurrence of Hurricanes between the years 1749 and 2012 in the Gulf of Mexico and the Caribbean Sea. Each slope is a different m-value; m takes values from 1 to 10.

Finally, on combining equations (3) and (7), this yields to the Eckmann and ruelle (1992) condition to obtain the Lyapunov exponents:

$$\log(N) > D\log(\frac{1}{\rho}) \tag{8}$$

For $\rho = 0.1$ in equation (8), N may be chosen such that:

$$N > 10^D \tag{9}$$

Where N is the number of data and D, the dimension of the attractor. Our time series met this requirement; therefore, it support our previous results.

Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg-2017-55
Manuscript under review for journal Nonlin. Processes Geophys.
Discussion started: 6 October 2017

Nonlinear Processes in Geophysics
Discussions
Open Access

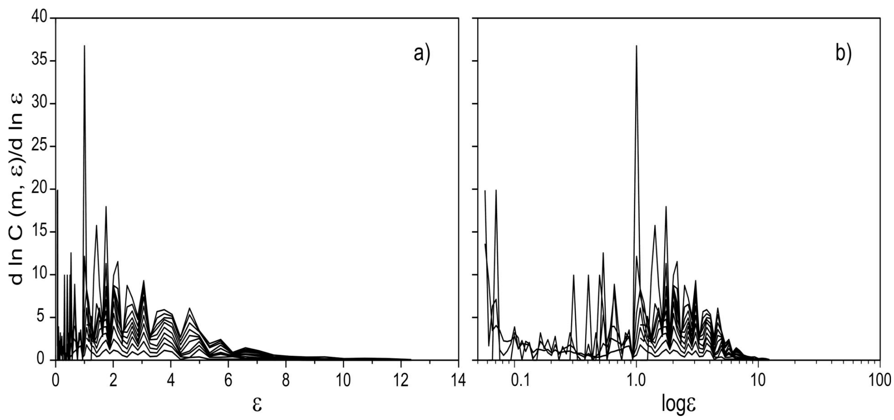

Figure 8. Correlation dimension D_2 corresponding to the occurrence of hurricanes in the years 1749-2012 in the Gulf of Mexico and the Caribbean Sea. Left-panel curves for different dimensions of the attractor (y-axis). Right panel, same information for the D_2 with a logarthmic scale on the x-axis

The attractor dimension was mainly obtained because this value tells us the number of parameters or degrees of freedom necessary to control or understand the temporal evolution of our system in the phase space, and helps us to know how chaotic our system is. Using the previous methods, a final dimension of $D_f = 2.2$ was obtained for the fractal dimension. Following the embedding laws, it must be fulfilled that $m > 2D_f$, but, in the case of the correlation dimension, it is sufficient if it satisfies $m > D_f$ (Sauer and Yorke, 1993; Kantz and Schreiber, 2004). Finally, the results indicated that at least three parameters are needed to characterize our system, since the 2.2 dimension indicates that the attractor dimension falls between 2 and 3.

The spectrum of the Lyapunov exponent gives: 0.09983, -0.07443, -0.23387 and -0.73958; therefore, the total sum was of $\sum \lambda_i = -0.9480$, and, according to the previous theory, it is enough have at least one positive exponent in the spectrum of our system in order to have a chaotic behavior. Finally, the total sum of the spectrum of the Lyapunov exponents was negative, indicating that there is a stable attractor, as mentioned previously. However, since the stable attractor was not easily distinguished, we used a final method in order to confirm this result. This method comprised the Iterated Functions System test (IFS) (Fig. 10).

Using Fig. 10, it can be observed that the points representing our system occupy the entire space; according to this, the distribution could be described as a white noise signal, in systems without experimental noise the point distribution gives a single curve (Jensen et al., 1985). However, the previous Hurst exponent obtained was not equal to zero; then, the white noise

Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg-2017-55
Manuscript under review for journal Nonlin. Processes Geophys.
Discussion started: 6 October 2017

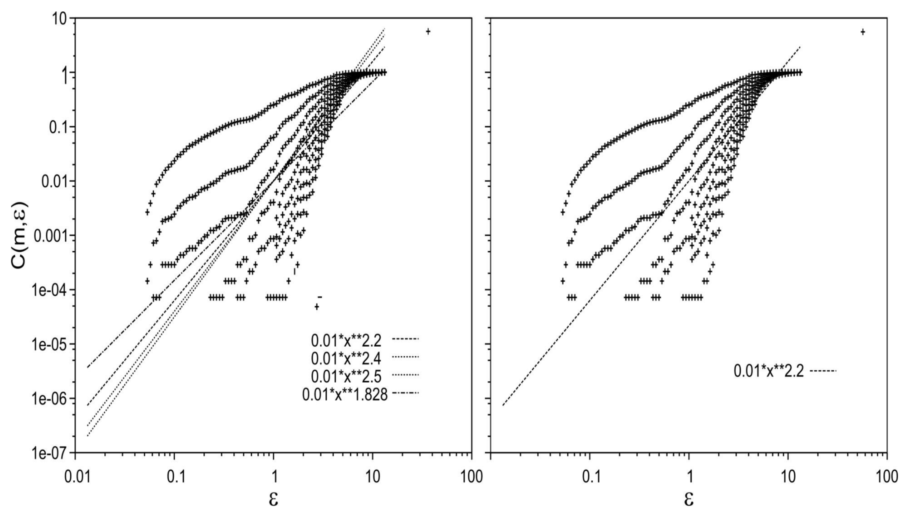

Figure 9. Left panel; slopes that indicate possible fractal dimensions of the attractor. Right panel; the slope that covers the majority of the slopes of the Correlation Integral.

was also discarded with the autocorrelation function. On the other hand, our IFS test was not flat and does not appear to be flattened by increasing the embedding dimension, which according to Rosenstein et al. (1993), our system would not be chaotic. It was then also possible to obtain a dimension of the attractor and a positive Lyapunov exponent. Fraedrich and Leslie (1989) analyses the trajectories of hurricanes in the region of Australia and calculate the dimensionality of this process, obtaining a result of between 6 and 8, i.e., a chaotic process of high dimensionality, which is similar to what we find with this method. On the other hand, Halsey and Jensen (2004) postulate that hurricanes contain a large number of dimensions in phase space.

Our results were not easy to interpret because the series presented certain periodic characteristics in oscillatory fashion, and chaotic behavior at the same time. An explanation is that the occurrence of hurricanes in the Gulf of Mexico and the Caribbean Sea is high- dimensionality -chaotic, and the system is localized within a boundary where chaos and order are separated; this boundary is commonly known as the "edge of chaos" (Langton, 1990; Miramontes et al., 2001). Miramontes et al. (2001) found this type of behavior in ants of the genus Leptohorax, when studying them individually and in groups. For the former case, the behavior was periodic, while for the second case, the behavior was chaotic. In our case, the chaotic behavior is due to the individual behavior or by hurricane category, while the periodic response is due to the behavior of hurricanes as a whole.

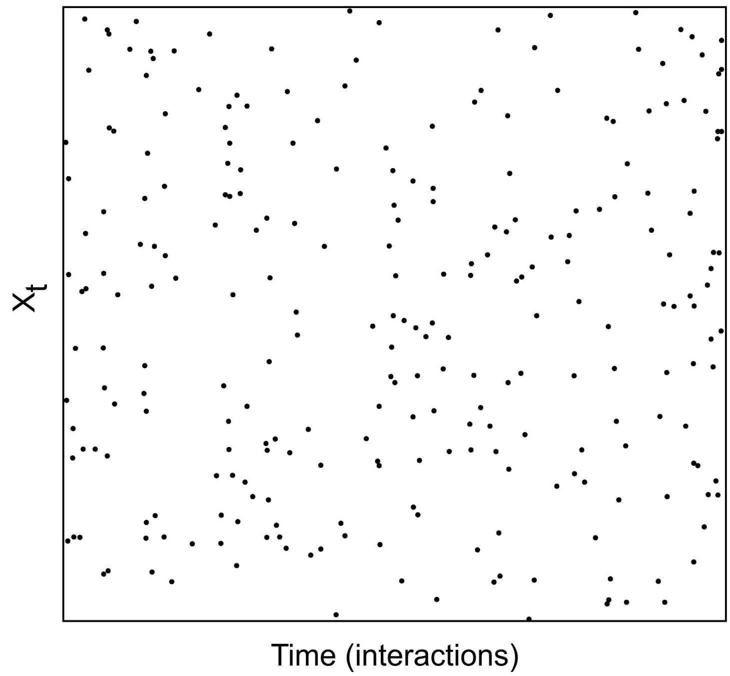

Figure 10. Iterated Functions System (IFS) test applied to the time series of the number of hurricanes that occurred in the Gulf of Mexico and the Caribbean Sea between the years 1749 and 2012.

4 Conclusions

The results obtained with the non-linear analysis suggested a chaotic behaviour in our system, mainly based on the Lyapunov exponents and correlation dimension, among others. However, the Hurst exponent indicated that our system did not follow a chaotic behaviour, and in order to be able to corroborate our results, we employed the IFS method, which led us to think that the hurricanes time series in the Gulf of Mexico and the Caribbean Sea from 1749 to 2012 has a chaotic edge.

Author contributions. All the authors contributed equally to this work

Competing interests. The authors declare that they have no conflict of interest

Acknowledgements. This work was financially supported by the Instituto de Ciencias del Mar y Limnología de la Universidad Nacional Autónoma de México, projects 144 and 145. BR-G is grateful for the CONACYT Scholarship that supported her study at the Posgrado en Ciencias del Mar y Limnología, Universidad Nacional Autónoma de México

References

Ataei, M., Khaki-Sedigh, A., Lohmann, B., and Lucas, C.: Determining embedding dimension from output time series of dynamical systems-scalar and multiple output cases, in: Proceedings of the 2^{nd} International Conference on System Identification and Control Problems, Moscow, Russia, 2003.

Broomhead, D. S., and King, G. P.: Extracting qualitative dynamics from experimental data. Physica D: Nonlinear Phenomena, 20(2-3), 217-236, 1986.

Quintero, O. Y. and Delgado, J. R.: Estimación del exponente de Hurst y la dimensión fractal de una superficie topográfica a través de la extracción de perfiles, Geomática UD.GEO, 5, 84-91, 2011 (In Spanish).

Eckmann, J. P. and Ruelle, D.: Fundamental limitations for estimating dimensions and Lyapunov exponents in dynamical systems, Physica D, 56, 185-187, 1992.

Fernández-Partagás, J. J. and Diáz, H. F.: A reconstruction of historical tropical cyclone frequency in the Atlantic from documentary and other historical sources. Part I, 1851-1870, Climate Diagnostics Center, Environmental Research Laboratories, NOAA, Boulder, CO., 1995a.

Fernández-Partagás, J. J. and Diáz, H. F.: A reconstruction of historical tropical cyclone frequency in the Atlantic from documentary and other historical sources. Part II, 1871-1880, Climate Diagnostics Center, Environmental Research Laboratories, NOAA, Boulder, CO., 1995b.

Fernández-Partagás, J. J. and Diáz, H. F.: Atlantic hurricanes in the second half of the nineteenth century, B. Am. Meteorol. Soc., 77, 2899-2906, 1996a.

Fernández-Partagás, J. J. and Diáz, H. F.: A reconstruction of historical tropical cyclone frequency in the Atlantic from documentary and other historical sources. Part III, 1881-1890, Climate Diagnostics Center, Environmental Research Laboratories, NOAA, Boulder, CO., 1996b.

Fernández-Partagás, J. J. and Diáz, H. F.: A reconstruction of historical tropical cyclone frequency in the Atlantic from documentary and other historical sources. Part IV, 1891-1900, Climate Diagnostics Center, Environmental Research Laboratories, NOAA, Boulder, CO., 1996c.

Fernández-Partagás, J. J. and Diáz, H. F.: A reconstruction of historical tropical cyclone frequency in the Atlantic from documentary and other historical sources. Part V, 1901-1908, Climate Diagnostics Center, Environmental 320 Research Laboratories, NOAA, Boulder, CO., 1997.

Fernández-Partagás, J. J. and Diáz, H. F.: A reconstruction of historical tropical cyclone frequency in the Atlantic from documentary and other historical sources. Part VI, 1909-1910, Climate Diagnostics Center, Environmental Research Laboratories, NOAA, Boulder, CO., 1999.

Fraedrich, K. and Leslie, L.: Estimates of cyclone track predictability. I: Tropical cyclones in the Australian region, ?Q.J.R. Meteorol. Soc., 115, 79-92, 1989.

Fraedrich, K., Grotjahn, R., and Leslie, L. M.: Estimates of cyclone track predictability. II: Fractal analysis of mid-latitude cyclones, Q. J. Roy. Meteor. Soc., 116, 317-335, 1990.

Gratrix, S., and Elgin, J. N.: Pointwise dimensions of the Lorenz attractor. Physical review letters, 92(1), 014101, 2004

Gutiérrez, H.: Estudio de geometría fractal en roca fracturada y series de tiempo, M. Sc. Thesis, Universidad de Chile, Facultad de Ciencias Físicas y Matematicas, Departamento de Ingeniería Civil, Santiago de Chile, Chile, 2008 (In Spanish).

Halsey, T. C., and Jensen, M. H.: Hurricanes and butterflies. Nature, 428(6979), 127-128, 2004

Haken H.: Chaos and Order in Nature. In: Haken H. (eds) Chaos and Order in Nature. Springer Series in Synergetics, vol 11. Springer, Berlin, Heidelberg, 1981.

Hegger, R., Kantz H., and Schreiber, T.: Practical implementation of nonlinear time series methods: The Tisean package, Chaos, 9, 413-435, 1999.

Jensen, M. H., Kadanoff, L. P., Libchaber, A., Procaccia, I., and Stavans, J.: Global universality at the onset of chaos: results of a forced Rayleigh-Bénard experiment. Physical review letters, 55(25), 2798, 1985.

Kantz, H.: A robust method to estimate the maximal Lyapunov exponent of a time series, Phys. Lett. A, 185, 77-87, 1994.

Kantz, H. and Schreiber, T.: Nonlinear time series analysis, Vol. 7, Cambridge University Press, Cambridge, 2004.

Kantz, H., Radons G., and Yang H.: The problem of spurious Lyapunov exponents in time series analysis and its solution by covariant Lyapunov vectors. J. Phys. A: Math. Theor., 46, 254009.

Langton, C. G.: Computation at the edge of chaos: phase transitions and emergent computation, Physica D, 42, 12-37, 1990.

Miramontes, O., Sole, R. V., and Goodwin, B. C.: Neural networks as sources of chaotic motor activity in ants and how complexity develops at the social scale, Int. J. Bifurcat. Chaos, 11, 1655-1664, 2001.

Rigney, D. R., Goldberger, A. L., Ocasio, W., Ichimaru, Y., Moody, G. B., Mark, R.: Multi-Channel Physiological Data: Description and Analysis, in Weigend, A. S., Gershenfeld, N. A., Eds: Predicting the Future and Understanding the Past: A Comparison of Approaches. Santa Fe Institute Studies in the Science of Complexity, Proc. Vol. XV, Addison-Wesley, Reading, MA., 1993.

Rojo-Garibaldi, B., Salas-de-León, D. A., Sánchez, N. L., and Monreal-Gómez, M. A.: Hurricanes in the Gulf of Mexico and the Caribbean Sea and their relationship with sunspots, J. Atmos. Sol.-Terr. Phy., 148, 48-52, 2016.

Rosenstein, M. T., Collins, J. J., and De Luca, C. J.: A practical method for calculating largest lyapunov exponents from small data sets, Physica D, 65, 117-134, 1993.

Sauer, T. and Yorke J. A.: How many delay coordinates do you need? Int. J. Bifurcat. Chaos, 3, 737-744, 1993.

Sharifi, M., Georgakakos, K., and Rodriguez-Iturbe, I.: Evidence of deterministic chaos in the pulse of storm rainfall, J. Atmos. Sci., 47, 888-893, 1990.

Thompson, J. and Stewart, H.: Nonlinear dynamics and chaos: geometrical methods for engineers and scientists, John Wiley & Sons Ltd, New York, 1986.

Tsonis, A. and Elsner, J.: The weather attractor over very short timescales, Nature 333, 545-547, 1988.

Wolf, A.: Quantifying chaos with Lyapunov exponents, Princeton University Press, Princeton, NJ, 1986.

Zeng, X., Pielke, R., and Eykholt, R.: Estimating the fractal dimension and the predictability of the atmosphere, J. Atmos. Sci., 49, 649-659, 1992.