Dear Prof. Vicente Perez-Munuzuri
Editor
Nonlinear Processes in Geophysics

You will find ci-joint the new version of our manuscript “Nonlinear analysis of the occurrence of hurricanes in the Gulf of Mexico and the Caribbean Sea”. We provide a point-by-point list of each comments by the reviewers. English was significant improved by the Elsevier Language Editing Services, so we cannot indicate where in the manuscript the English revision appears. What we do is indicate the changes of the point-by-point list of each comments by the reviewers (page #, line #).

David Salas

Answers to the Second Reviewer

1. ***For example, it is not clear for me that the time series is stationary. May you calculate the probability density function for the half and full data lenght and compare the results. Of course after the detrend has been done.***

Reply: Two tests were done to see the stationarity of the series, one of which was the Dickey-Fuller test (D-F), which is the standard test to prove the stationarity of a series. They consider three different regression equations that can be used to prove the presence of a unit root, the parameter of interest of these equations is **r**, if **r = 1** the series has a unit root. In this test the null hypothesis where Ho: **r = 1** shows that the series has unit root and is not stationary, and if Ha: **r** < 1 then the series is stationary. Using this test, a value of D-F = -5.7753 was obtained with a p-value = 0.01, which is statistically significant and therefore we can say that our series is stationary.

Now, following the test suggested by the reviewer. Since a series is stationary over time when its mean and variance are constant over time, the respective values were obtained for the middle of the series and for the complete series. The values of the mean and variance of the complete series were: mean = 0.138 and variance = 0.020. The values for the middle of the series were: mean = 0.123 and variance = 0.0199, so this requirement is also met. Finally, the probability density function was plotted for 1749-1881 and 1749-2012, as well as their respective histograms, with this we can see how the form of the function and the histogram are conserved over time.

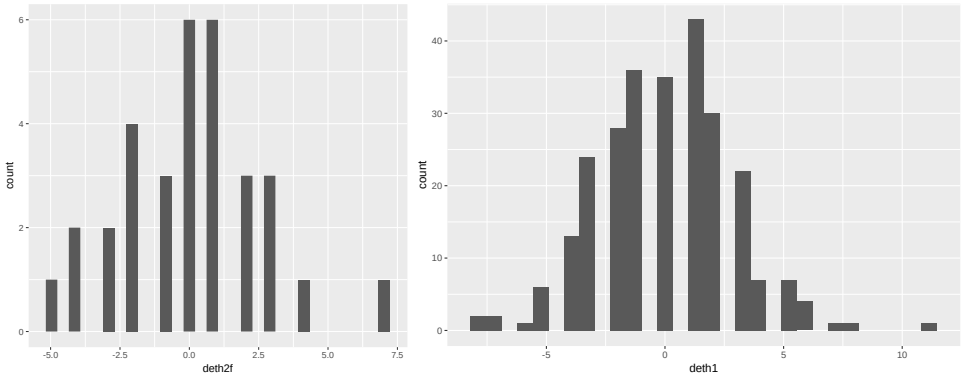

Figure 1. (Left) Histogram from the Hurricane series 1749-2012. (Right) Histogram for the Hurricane series 1749-1881. It is possible to observe how the distribution of the histogram is preserved for the middle of the series and for the complete series.

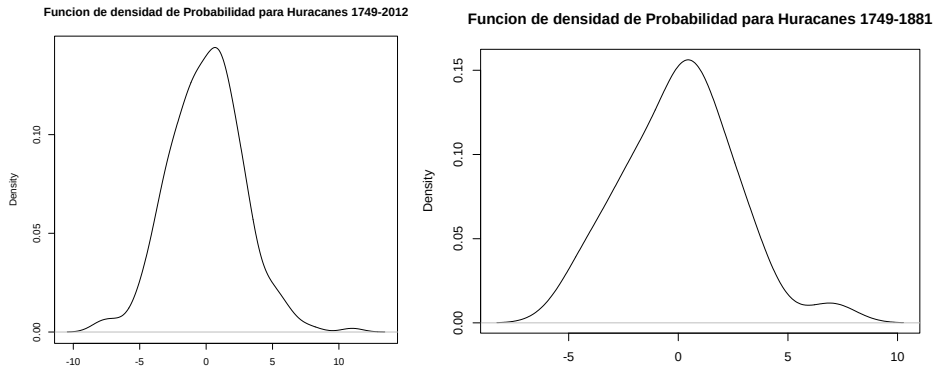

Figure 2. (Left) Probability density function of the Hurricane series 1749-2012. (Right) Probability density function for the Hurricane series of 1749-1881. It is possible to observe how the distribution of the function is preserved for the middle of the series and for the complete series.

2. ***Concerning the Poincaré map I continue to have some doubts as may not be useful. I can think of a 5-dimensional quasiperiodic signal that shows an irregular two-dimensional Poincaré map, and however the original time series is not chaotic I recommend deleting that figure and the corresponding comments.***

Reply: The section corresponding to this part has already been deleted.

3. ***The new results of the Lyapunov coefficient as a function of the embedding dimension and the time delay which are shown in the answer do not appear in the manuscript. I think they should.***

Reply: The new results were already put in the text. See page 14, lines 313-315

4. ***One of the main objectives of using these nonlinear methods not only lie in obtaining an embedding dimension and showing the chaotic nature of the time series, but to perform some forecasting. What will happen if you try to reconstruct the time series for example, using only half of its length. This was mentioned in my previous review.***

Reply: = By means of non-linear methods, the entropy test was performed, which showed a predictability value of 2.78 years, and means of the locally linear prediction (making the prediction at one step), the same value was obtained. The procedure for this method is as follows: The last known state of the system, represented by a vector $x = [x(n), x(n + \tau), ..., x(n + (m-1)\ \tau)]$, is determined, where m is the embedment dimension and τ is the delay time. Then we have found p nearby states (usually close neighbors of x) of the system that has happened in the past, from calculating their distances of x. The idea is then to adjust a map that extrapolates x and its neighboring p to determine the following values" (Dasan et al., 2002). Based on the above, the value of the embedding dimension and the delay

time were changed, in order to see in which values better results were obtained; this was possible with a dimension of $m = 4$ and $\tau = 9$, which are the values with which the attractor of the system was obtained. Therefore, a good prediction is possible until $t = t_0 + 3$. It is not possible to reproduce half of the series since the system tells us that we can only do it for two years.

Dasan, J., Ramamohan, R. T., Singh, A., y Prabhu, R. N. (2002) Stress fluctuations in sheared Stokesian suspensions, Phys. Rev., E, 66, pp.021409-1-021409-14.

Answers to the Third Reviewer

1. ***The manuscript presents nonlinear analysis of the occurrence of hurricanes in the Gulf of Mexico and the Caribbean Sea, which is interesting. The subject addressed is within the scope of the journal.***

2. ***However, the manuscript, in its present form, contains several weaknesses. Appropriate revisions to the following points should be undertaken in order to justify recommendation for publication.***

3. ***Full names should be shown for all abbreviations in their first occurrence in texts. For example, 2-D in p.6, etc.***

   Reply: done

4. ***For readers to quickly catch your contribution, it would be better to highlight major difficulties and challenges, and your original achievements to overcome them, in a clearer way in abstract and introduction.***

   Reply: Already added to the text. See page 1 y 2, lines 21-22, 40-44

5. ***It is shown in the reference list that the authors have a pertinent publication in this field. This raises some concerns regarding the potential overlap with their previous works. The authors should explicitly state the novel contribution of this work, the similarities and the differences of this work with their previous publications.***

   Reply: Indeed, we have a publication about the hurricanes that occurred in the same study area for the same time interval (1749-2012). However, in this case, our analysis focused on a different approach, for this article we apply different methods of spectral analysis (Wavelets, Fast Fourier Transform, Multi-taper and Maximum Entropy), in order to see if there was a relationship between the occurrence of hurricanes and the periodicity of sunspots.

   An exhaustive analysis was carried out on the type of correlation between the two systems, as well as the lag between both events. Finding a considerable relationship between the two systems, not only with the periodicities, but also with the type of correlation.

   The substantial difference between the previous article and this one, is that this time we are focusing on the behavior of only the hurricanes, performing a non-linear analysis, the main objective was to find out if the hurricanes belong to the chaotic dynamic systems. When comparing both studies (spectral analysis and non-linear analysis) we realized that hurricanes have the so-called "chaotic edge". This is the most important contribution of our work, since it tells us that hurricanes can behave periodically and

follow a pattern, but this does not hold all the time, their behavior goes to a threshold at which it is chaotic.

6. It is mentioned in p.1 that historical records of 1749 to 2012 are taken. Why are more recent data not included in the study?

Reply: This was a study that was carried out during the year of 2012, until then, recent information was taken.

6.1. Is there any difficulty in obtaining more recent data?

Reply: Of course not, the HURDAT page updates its data every year after the end of hurricane season in the Atlantic Ocean, Caribbean Sea and Gulf of Mexico. Well it's the official hurricanes record of USA for this area.

6.2. Are there any changes to situation in recent years? What are its effects on the result?

Reply: We must not forget that this type of studies provides us with information about the dynamics of an underlying system, so that, once we have found that our system is chaotic, it will not let being chaotic even if we obtain the record of the missing years (i.e., 2013-2017). Therefore, the absence of this data does not affect our result.

7. It is mentioned in p.1 that the Gulf of Mexico and the Caribbean Sea are adopted as the case study. What are other feasible alternatives?

Reply: We can study the hurricanes of the Pacific Ocean, which is another area impacted by the occurrence of hurricanes, for this part we can analyze those occurred in the Mexican territory, and those observed in China, Japan and the Philippines. In the same way, they are observed in Australia and India.

7.1. What are the advantages of adopting this particular case study over others in this case?

Reply: The importance of this project and the reason why this region was studied, is because there was a fairly substantial record of the hurricanes that occurred in the Gulf of Mexico and the Caribbean Sea since 1749. The other regions impacted by hurricanes do not have such a long historical record. Having a record of this magnitude was important for the spectral analysis that was developed, on the other hand the length of the time series, also allowed the development of non-linear analysis, since with less available data it would then be impossible to think to do this type of analysis.

7.2. How will this affect the results? The authors should provide more details on this.

Reply: As mentioned above, the study area was chosen taking into account the long historical record that was available. Even at present, the data in the other regions impacted by hurricanes, would not be enough to perform a non-linear analysis, at least for the case in which we want to study the occurrence of these phenomenon.

8. It is mentioned in p.1 that HURDAT is adopted as the database. What are other feasible alternatives?

Reply: There is the National Hurricane Center (NHC) -NOAA (National Oceanic and Atmospheric Administration), which is a regional specialized meteorological center for the North Atlantic and the Eastern Pacific, created with the purpose of creating a hurricane warning network. HURDAT considered the annual registration of this center.

8.1. What are the advantages of adopting this particular database over others in this case? How will this affect the results? The authors should provide more details on this.

Reply: Both HURDAT and NHC-NOAA are official records of the United States and HURDAT bases its database on the NHC report, apart from its own re-analysis project, therefore, we consider that our results could not be affected. We have not only reliable historical data, but also data obtained from the official records of the United States.

9. It is mentioned in p.1 that spectral analysis is adopted for the nonlinear analysis of the hurricanes time series. What are other feasible alternatives?

Reply: In fact, the first hint of chaotic behavior can be seen from a spectral analysis. "Spectral power analysis is often used to distinguish chaotic or quasi-periodic behavior from periodic structures and to identify different periods embedded in a chaotic signal" (Zeng et al., 1990). According to Schuster (1988) and Tsonis (1992) the power spectrum is not only characteristic of a process of deterministic chaos, but also of a linear stochastic process. That is why more studies should be done and for this reason. Other than this first approach, we have the Hurst exponent and the phase space graph, these results are presented in our result section.

9.1. What are the advantages of adopting this particular approach over others in this case? How will this affect the results? The authors should provide more details on this.

Reply: As we mentioned before, spectral analysis is a tool that is used to see possible indications of a chaotic behavior, however, it cannot always be appreciated and that is why the corresponding methods of non-linear analysis are used.

10. ***It is mentioned in p.5 that three methods are adopted to know the properties of the system. What are other feasible alternatives?***

Reply: There are the bifurcation diagrams which are abrupt changes of the geometry of the attractor or of the topology in a critical value of the control parameter. In this type of diagrams, it is possible to see periodic and chaotic regimes, there are five different types of bifurcation diagrams, which provide a route to chaos. In fact, these types of diagrams can be compared with the graph of the Lyapunov exponent. For example, for a logistic map, it can be seen how the Lyapunov exponent goes from negative values in the regular regions of the bifurcation diagram, to positive values in the chaotic regions, becoming zero at the bifurcation points. In the chaotic region we can see regular behavior windows, in which the exponent becomes negative again. On the other hand, we have the Horizontal Visibility graph method, which "offers a promising new method for the development of time series analysis, mainly because it has been corroborated that the fundamental nature of quite different complex dynamic processes is inherited for the associated visibility charts" (Núñez et al., 2013). The Horizontal Visibility graphic allow us to describe chaotic, fractal-stochastic and dissipative processes.

10.1. What are the advantages of adopting these particular methods over others in this case? How will this affect the results? The authors should provide more details on this.

Reply: The methods used are as good as those mentioned above. If the Visibility graph had been used, the advantage would have been not having to calculate the rest of the parameters such as: delay time, Theiler window, embedding dimension, etc.; however, we do not use this method because it requires thousands of data.

11. ***It is mentioned in p.6 that the algorithms proposed by Kantz (1994) and Rosenstein et al. (1993) are adopted to compute the Lyapunov exponent. What are other feasible alternatives? What are the advantages of adopting these particular algorithms over others in this case? How will this affect the results? The authors should provide more details on this.***

Reply: There is the algorithm of Wolf et al. (1985), in which the maximum exponent of Lyapunov can also be calculated from a data set, following the long-term evolution of one of the main axes. However, it is a highly sensitive method and can easily lead to an erroneous result. Rosenstein and Kantz, more than suggesting a trajectory, used the complete data set and essentially calculated a trajectory for each pair of nearby neighbors. The two algorithms are substantially similar and calculate the maximum exponent of Lyapunov by looking for all the neighbors within a neighborhood of the reference trajectory and calculating the average distance between the neighbors and that trajectory as a function of time or relative time scale for the data sampling rate. Having used the algorithm of Wolf et al. (1985) a positive Lyapunov exponent would have been obtained without guaranteeing that the system has a chaotic dynamic, since this algorithm always gives a positive exponent.

12. ***It is mentioned in p.6 that the Poincaré surface is adopted to detect some kind of chaotic behavior. What are other feasible alternatives? What are the advantages of adopting this particular approach over others in this case? How will this affect the results? The authors should provide more details on this.***

Reply: The other alternatives are those presented at the present work; In fact, in our study we did not rely on the results obtained by a single method, but we used several methods, in order to corroborate each result.

13. ***It is mentioned in p.6 that the "delay method" is adopted to have a qualitative idea of the number of hurricanes that occurred. What are other feasible alternatives? What are the advantages of adopting this particular method over others in this case? How will this affect the results? The authors should provide more details on this.***

Reply: The "delay method" was used to construct the phase space of the system, once the delay time and the embedment dimension were obtained. It was a method created precisely for the case where we have a discrete system, that is, a set of data; so far it is the only method.

14. ***It is mentioned in p.7 that three different methods are adopted to calculate the time lag. What are other feasible alternatives? What are the advantages of adopting these particular methods over others in this case? How will this affect the results? The authors should provide more details on this.***

Reply: The delay time can also be obtained by constructing the phase space from an arbitrary time delay, later, by trial and error, testing with other values until the trajectories are more visible; however, this form does not give a very reliable value of the delay time. That is why we used the mentioned methods, which are, until now, the most reliable.

15. ***It is mentioned in p.10 that the Kaplan-Yorke Dimension is adopted to see the attractor dimension. What are other feasible alternatives? What are the advantages of adopting this particular method over others in this case? How will this affect the results? The authors should provide more details on this.***

Reply: There is a whole family of fractal dimensions D_q, which are called Renyi dimensions, the way you can see them is through a partition of the phase space: For the number of boxes N_ε of size ε, you need to cover a fractal set with scales of dimension D_0. In D_0 we have another type of dimension, which is the so-called capacity dimension, which is closely related to the dimension of Hausdorff, which is from the mathematical point of view, the most natural concept to characterize fractal sets. On the other hand, there is also the Information dimension, which takes into account the relative frequency of visits of the trajectory, making this type of dimension more attractive for physical systems. However, the Integral and the Correlation Dimension were made to characterize measured data, as well as being more robust and efficient estimators. Successive elements of a time series are not usually independent, but the

last two mentioned methods involve phase space vectors such as the location of points in an attractor. This is why they are the most used and most reliable.

16. ***It is mentioned in p.13 that the criterion of Ruelle (1990) is adopted to corroborate that the obtained dimension of the attractor is reliable. What are other feasible alternatives? What are the advantages of adopting this particular criterion over others in this case? How will this affect the results? The authors should provide more details on this.***

    Reply: We also have the criterion suggested by Tsonis et al. (1993), assuming that $M \sim 10^{2+0.4\nu_2}$ data points are needed for a reliable estimate of the fractal dimension ν_2. If this criterion is used, our data does not meet the requirement; however, they agree with the requirements of Ruelle (1990) and there is no stipulation that requires that the systems must have both requirements, so it is sufficient for the system to agree with one of them in order for the dimension to be reliable.

17. ***It is mentioned in p.14 that the Iterated Functions System test is adopted to confirm that there is a stable attractor. What are other feasible alternatives? What are the advantages of adopting this particular test over others in this case? How will this affect the results? The authors should provide more details on this.***

    Reply: The Iterated Functions system is used to make an adequate visualization of fine details that are present in the time series, including the self-similarity, therefore it can reveal the correlations in the data and help characterize its "color" (referring to the type of noise). As for the techniques that are used for the characteristic of the data, we also have the Hurst exponent, which also characterizes the color of the noise. Both methods are used in our study. The results obtained from both methods help to complement our discussion.

18. ***It is mentioned in p.15 that "…test showed that the occurrence of hurricanes in the Gulf of Mexico and the Caribbean Sea is chaotic with high dimensionality. One possible explanation is…." More justification should be furnished on this issue.***

    Reply: It has already been added to the text, see page 14 y 15, line 316-343.

19. ***Some key parameters are not mentioned. The rationale on the choice of the particular set of parameters should be explained with more details. Have the authors experimented with other sets of values? What are the sensitivities of these parameters on the results?***

    Reply: We do not understand to what key parameters it refers. The parameters that were used were: 1) The Theiler window, which was obtained from the space-time separation graph. The value of this window is very important because it prevents spurious dimensions from being obtained in the attractor. In fact, if a good Theiler window is not chosen, it is not possible to extract the embedding dimension. 2) On the other hand we have the delay time and the dimension of embebimiento, with these values the tests were also made, as it is explained in the article; the purpose of

changing these values was to corroborate the existence of the chaotic behavior in a quantitative way, through the change in the values of the exponent of Lyapunov with the decrease of the dimension of embebimiento, and qualitatively, when observing the invariant behavior in the graphs of the exponent. 3) Change in the radius of the neighborhood in which the reference point was chosen; this radius should be as small as possible but large enough so that on average each reference point has at least some neighbors. 4) Reference points, initially 500 points are an appropriate choice but should be changed if the data are intermittent or the computation time is fast and 5) Neighbors close to these points.

20. ***Some assumptions are stated in various sections. Justifications should be provided on these assumptions. Evaluation on how they will affect the results should be made.***

Reply: Added to the text, see page 3, 5, 6, 7-10, 14-15, line 88-94, 114-116, 149-164, 178-184, 191-205, 214-216, 220-225, 238, 305-311, 320-334, 338-343

21. ***The discussion section in the present form is relatively weak and should be strengthened with more details and justifications.***

Reply: Added to the text, see discussion section

22. ***Moreover, the manuscript could be substantially improved by relying and citing more on recent literatures about contemporary real-life case studies of modelling techniques in hydrologic engineering such as the followings:***

Reply: The authors are grateful for the comment, the following references were taken into account

Taormina, R., et al., "Neural network river forecasting through base flow separation and binary-coded swarm optimization", Journal of Hydrology 529 (3): 1788-1797 2015.

Gholami, V., et al., "Modeling of groundwater level fluctuations using dendrochronology in alluvial aquifers", Journal of Hydrology 529 (3): 1060-1069 2015.

Chen, X.Y., et al., "A comparative study of population-based optimization algorithms for downstream river flow forecasting by a hybrid neural network model," Engineering Applications of Artificial Intelligence 46 (A): 258-268 2015.

Wang, W.C., et al., "Improved annual rainfall-runoff forecasting using PSO-SVM model based on EEMD," Journal of Hydroinformatics 15 (4): 1377-1390 2013.

Wu, C.L., et al., "A flood forecasting neural network model with genetic algorithm," International Journal of Environment and Pollution 28 (3-4): 261-273 2006.

Chau, K.W., et al., “A split-step particle swarm optimization algorithm in river stage forecasting,” Journal of Hydrology 346 (3-4): 131-135 2007.

23. In the conclusion section, the limitations of this study, suggested improvements of this work and future directions should be highlighted.

Reply: Already added to the text, see page 16-17, line 355-361.