

Prof. Vicente Perez-Munuzuri

NPG Editor

Dear Prof. Perez-Munuzuri

You will find ci-joint the new version of our manuscript “Nonlinear analysis of the occurrence of hurricanes in the Gulf of Mexico and the Caribbean Sea” by Berenice Rojo-Garibaldi, David Alberto Salas-de-León, María Adela Monreal-Gómez, Norma Leticia Sánchez-Santillán, and David Salas-Monreal. We are sending two versions (tracking) in which you can see the changes made according to the comments of the reviewers and another (new) which is the version with the changes included

New comments for the reviewers:

The other methods used for the nonlinear analysis of the time series of hurricane events in the Gulf of Mexico and the Caribbean Sea, confirmed that this system has a chaotic behavior. When performing the corresponding spectral and finding a periodic behavior at the same time, it can be concluded that the system has a chaotic "edge" behavior.

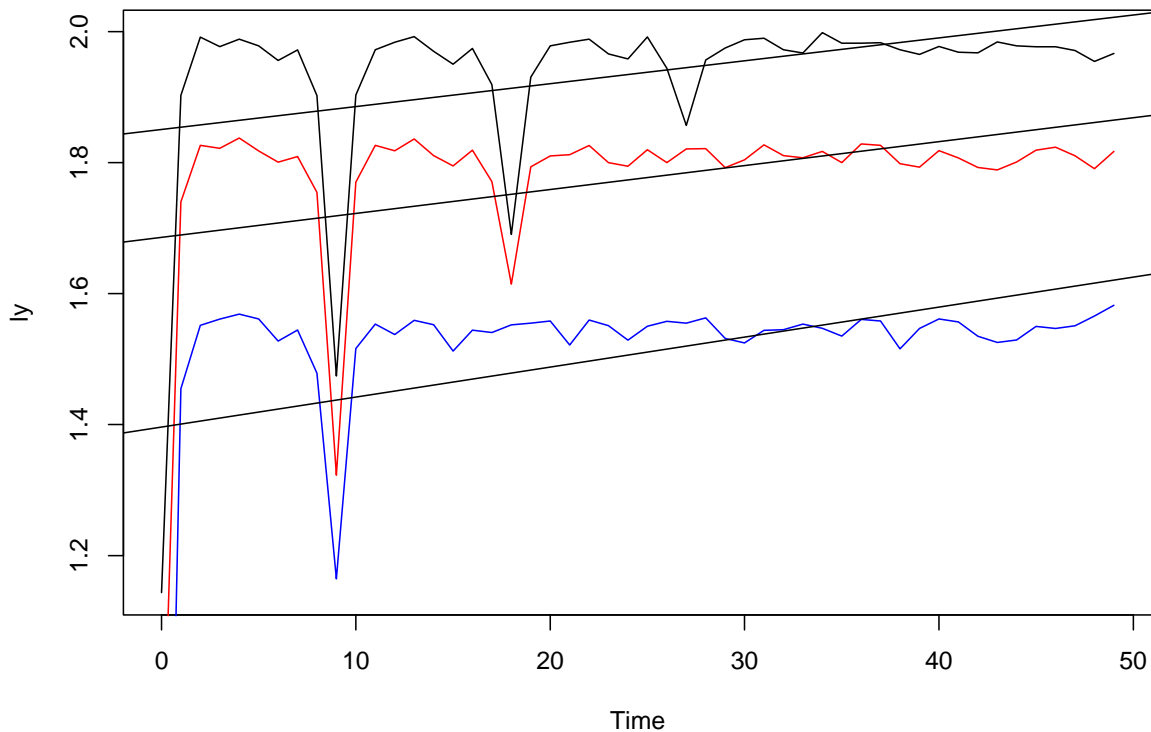
Different points must be taken into account to validate our results and these are the following:

1. To verify that our system has a chaotic behavior, the exponent of Lyapunov was obtained, but first we draw an embedding dimension and a delay time, this was done with the help of Theiler's window, which according to the chaos theory, it is necessary to not get the spurious dimensions. The exponent of Lyapunov was obtained by two different methods the Kantz (1994) and the Rosenstein (1993) method, in both cases the results were positive, indicating in this way that our system is chaotic.
2. The Poincaré diagram also gave us a chaotic behavior.

3. Since we were aware that our database was short, a bibliographic review was made to know if it was possible to work with this data number. We found that the number of data showed here was in agreement with the number of data expected by Eckmann and Ruelle (1992), therefore the number of data was sufficient to ensure obtaining a reliable Lyapunov exponent (this is discussed in the manuscript).
4. Another way to make sure that our system was chaotic was done through a visual inspection of the Lyapunov exponent graphs, once it approached the dimension of the embedment. According to Rosenstein (1993) the embedment dimension presents a flat behavior in the graph, in order to be not chaotic. This type of behavior was not observed here, on the contrary, the form is maintained (this was also mentioned in the manuscript).
5. For the spectra of the Lyapunov exponent, the following values were obtained: 0.09983, -0.07443, -0.23387 and -0.73958, whose sum has a value of  $\sum \lambda_i = -0.9480$ , according to the theory, it is enough to find at least one positive exponent in the spectrum to prove that our system has a chaotic behavior. Another observation made with respect to the spectrum of the exponents of Lyapunov, is the result obtained by the sum, in this case the value was negative, which indicates that there is a stable attractor.
6. In the case of the dimension of the attractor, we also were sure to get the requirement of Ruelle (1990), in order to have a reliable result of the dimension obtained for the attractor.
7. For the dimension of the attractor, the dimension of Hausdorff and Kaplan-Yorke were obtained, resulting in  $D_2 = 2.2$  and  $D_{k-y} = 2.26$ , this suggests that the result obtained for the dimension of the attractor is robust.
8. Finally, the IFS method was used, this suggests that when the points cover the whole space it may indicate two things: 1) The system is a white noise or 2) The system has a chaotic behavior of high dimensionality. As referred in the manuscript, the white noise

was discarded according to the Hurst exponent, therefore it was inferring that the system is chaotic and of high dimensionality.

9. All these analyzes helped to confirm that the system has a chaotic behavior, giving us at the same time more solid foundations for our result. However, and taking into account the suggestion of the reviewers, an additional analysis was carried out to check if our system is indeed chaotic, the exponent of Lyapunov was plotted and its value was obtained as the embedment dimension decreased. This was done at different dimensions ( $m = 4.3$  and  $2$ ) and with different radii ( $\epsilon = 16, 24$  and  $30$ ), with this analysis we can highlight two things: 1) The shape of the curve in the exponent of Lyapunov, is keeps invariant before transformations and 2) As suggested by the reviewer, the exponent of Lyapunov increases as the embedment dimension decreases (see figure below).



Where the exponents were:

$\lambda_4=0.0349$ , for  $m=4$

$\lambda_3=0.0365$ , for  $m=3$

$\lambda_2=0.0480$ , for  $m=2$

In this case, the numerical result differs from that obtained previously due to the different value of the radius and the number of iterations used here, however the important result is that the exponent remains invariant and increases its value by decreasing the embedment dimension. We must point out that, however, the values presented in the original document are reliable since the data were carefully chosen and analyzed, in this last analysis the calculations were made only in an exploratory way, in order to corroborate what was suggested by the reviewer.

# Nonlinear analysis of the occurrence of hurricanes in the Gulf of Mexico and the Caribbean Sea

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**Abstract.** ~~Abstract.~~ Hurricanes are complex systems that carry large amounts of energy. Their impact produces, ~~the majority,~~ most of the time, natural disasters involving the loss of human lives and ~~of materials~~ and such as infrastructure evaluated in billions of US dollars. However, not everything is negative, as hurricanes are the main source of rainwater for the regions where they develop. ~~In~~ This study, ~~we perform~~ shows a nonlinear analysis of the time series obtained from 1749 to 2012 of the occurrence of hurricanes in the Gulf of Mexico and the Caribbean Sea. The construction of the hurricane time series was carried out based on the hurricane database of The North Atlantic-basin Hurricane Database (HURDAT), and the published historical information. The Lyapunov exponent indicated that the system presented chaotic dynamics, and the time-series' spectral analysis along with the nonlinear analysis of the hurricanes time series showed chaotic edge behavior. One possible explanation for this edge is the individual

chaotic behavior of hurricanes, either by category or individually, regardless of their category, and their behavior on a regular basis.

## **Introduction**

Hurricanes have been studied since ancient times, their activity is related to disasters and loss of life. In recent years, there has been considerable progress in predicting their trajectory and intensity, once their tracking began, as well as their number and intensity from one year to the next. However, their long-term, and very short prediction remains a challenge (Halsey and Jensen, 2004), and damage to both materials and lives remains considerable. Therefore, it is important to make a greater effort in the study of hurricanes to reduce the damage they cause. The periodic behavior of hurricanes and their relationship with other natural phenomena have usually been performed with linear-type analyses, which have provided valuable information. However, we decided to make a different contribution by carrying out a non-linear analysis of time series of hurricanes that occurred in the Gulf of Mexico and the Caribbean Sea, since the dynamics of the system is controlled by a set of variables of low dimensionality (Gratrix and Elgin, 2004; Broomhead and King, 1986).

Theories of deterministic chaos and fractal structure have already been applied to atmospheric boundary data (Tsonis and Elsner, 1988; Zeng et al., 1992), to the pulse of severe rain time series (Sharifi et al., 1990; Zeng et al., 1992), and to the tropical cyclone trajectory (Fraedrich and Leslie, 1989; Fraedrich et al., 1990). Natural phenomena occur in nature within different contexts; however, they often exhibit common characteristics, or they may be understood using similar concepts. Deterministic chaos and fractal structure in dissipative dynamical systems are among the most important non-linear paradigms (Zeng et al., 1992). For a detailed analysis of deterministic chaos, the Lyapunov exponent is utilized as a key point and several methods have been developed to calculate it. It is possible to define different Lyapunov exponents for a dynamic system. The maximal Lyapunov exponent can

be determined without the explicit construction of a time-series model. A reliable characterization requires that the independence of the embedded parameters and the exponential law for the growth of distances can be explicitly tested (Rigney et al., 1993; Rosenstein et al., 1993). This exponent provides a qualitative characterization of the dynamic behavior and also provides the predictability measurement (Atari et al., 2003). The algorithms usually employed to obtain the Lyapunov exponent are those proposed by Wolf (1986), Eckmann and Ruelle (1992), Kantz (1994), and Rosenstein et al. (1993). The methods of Wolf (1986) and Eckmann and Ruelle (1992) assume that the data source is indeed a deterministic dynamic system and that irregular fluctuations in time-series data are due to deterministic chaos. Blind application of this algorithm to an arbitrary set of data will always produce numbers, i.e., these methods do not provide a strong test of whether the calculated number can actually be interpreted as Lyapunov exponents of a deterministic system (Kantz et al., 2013). The Rosenstein et al. (1993) method follows directly from the definition of the Lyapunov maximal exponent and is accurate because it takes advantage of all available data. The algorithm is fast, easy-to- implement, and robust for changes in the following quantities: embedded dimensions; data-set size; delay reconstruction, and noise level. The Kantz (1994) algorithm is similar to that of Rosenstein et al. (1993).

We constructed a database of occurrences of hurricanes in the Gulf of Mexico and the Caribbean Sea to make a nonlinear analysis of the time series, the results can help in the construction of models of occurrence of hurricanes and this in turn, will help to reinforce the measures of prevention before this type of hydrometeorological phenomena.

## **2 Materials and methods**

### **2.1 Data set description**

~~A historical database of hurricane occurrence in the Gulf of Mexico and the~~

Caribbean Sea was constructed. A detailed analysis of the recorded hurricanes' historical reports was provided by the ships sailing near the reported hurricanes; aerial reconnaissance data for recent hurricanes were obtained from 1749-2012. The aerial surveys began in 1944. Finally, the hurricanes reported by Fernández-Partagas and Díaz (1995a; 1995b; 1996a; 1996b; 1996c; 1997; 1999), were added, along with information obtained from the HURDAT re-analysis project.

A detailed analysis of the historical reports provided by the ships that were used was carried out in order to obtain the annual time series of the occurrence of hurricanes, from category one to five, on the Saffir-Simpson scale, in the study region from 1749-2012. The time series was composed by the historical ship track of all vessels sailing close to a registered hurricanes, the aerial reconnaissance data for hurricanes since 1944 and the hurricanes reported by Fernández-Partagas and Díaz (1995a, 1995b; 1996a; 1996b; 1996c; 1997; 1999). All this information together with the database of the HURDAT re-analysis project was used in a comparative way in order to build our time series, which is so far, the longest time series of hurricanes for the Gulf of Mexico and the Caribbean Sea. All that information was used to build the hurricane time series (Fig. 1).

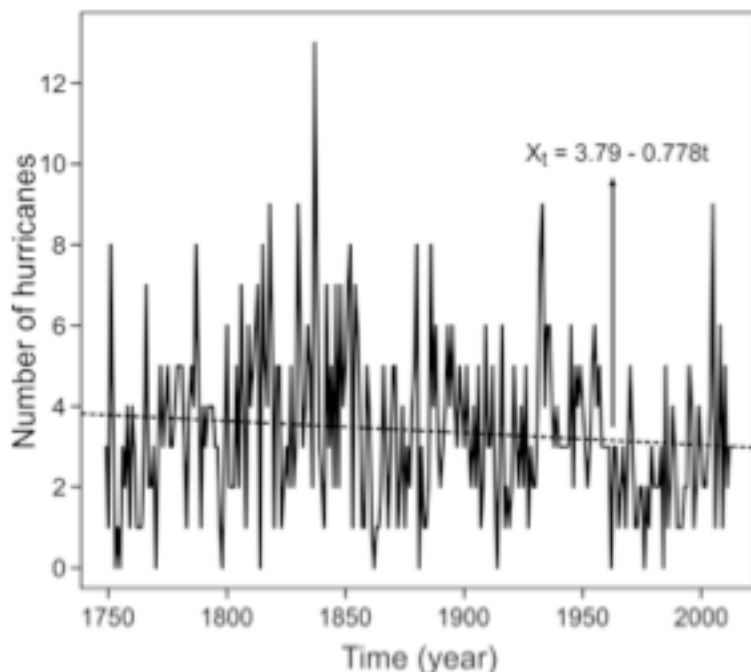




Figure 1. Hurricanes from the years 1749-2012. The line shows their linear trend (after Rojo-Garibaldi et al., 2016).

~~Only historical hurricanes that were reported in two or more databases were considered valid; on the other hand, the hurricane trajectory was followed to avoid counting more than once a specific hurricane that was reported in different places within a very short time. This was done in order to avoid overcounting the number of hurricanes; to do this, we follow the proposed method by Rojo-Garibaldi et al. (2016).~~

Only the historical hurricanes reported in two or more databases were considered as valid, considering both criteria, the reported hurricanes that touched land and those that remained in the ocean; on the other hand, the followed hurricanes, were studied considering their average duration and their maximum time (9 and 19 days, respectively). This was done in order to avoid counting more than one specific hurricane reported in different places within a short period of time; To do this, we follow the proposed method by Rojo-Garibaldi et al. (2016).

## **2.2 Data reduction and procedures**

Before performing the non-linear analysis to the time series, we removed the trend, thus, the series was prepared according to what is required for this type of analysis.

To know the properties of the system requires more than estimating the dimension of the attractor (Jensen et al., 1985); so three methods were applied in this study: 1) the Hurst exponent, which is a measure of the independence of the time series as an element to distinguish fractal series. It is basically a statistical method which provide the number of occurrence of rare events and is usually called re-scaling (RS) rank

analysis (Gutiérrez, 2008), therefore it could be applied to any time series. The RS helps to find the Hurst exponent, which provides the numerical value which makes possible to determine the autocorrelation in a data series. 2) The Lyapunov exponent is invariant under soft transformations, because it describes the long-term behavior, providing an objective characterization of the corresponding dynamics (Kantz and Schreiber, 2004). The presence of chaos in dynamic systems can be solved by this exponent, since it quantifies the exponential convergence or divergence of initially close trajectories in the state space and estimates the amount of chaos in a system (Rosenstein et al., 1993; Haken, 1981; Wolf, 1986). The Lyapunov exponent ( $\lambda$ ) can take one of the following four values:  $\lambda < 0$  corresponds to a stable fixed point,  $\lambda = 0$  which is for a stable limit cycle,  $0 < \lambda < \infty$  indicates chaos and  $\lambda = \infty$  for a Brownian process, which agrees with the fact that the entropy of a stochastic process is infinite (Kantz and Schreiber, 2004). 3). The Iterated Function Analysis (IFS) is an easier and simpler way to visualize the fine structure of the time series, it can reveal correlations in the data and help to characterize its color, referring to color to the type of noise (Miramontes et al., 2001). Together with the Lyapunov exponent, the phase diagrams, the False Close Neighbours method, the Space-Time Separation plot, the Correlation Integral plot, and the Correlation Dimension were taken into account, the latter two to identify whether the system attractor was fractal type. It is important to emphasize that in order to compute the Lyapunov exponent the algorithms proposed by Kantz (1994) and Rosenstein et al. (1993) were used in this study.

To detect some kind of chaotic behavior we plotted the Poincaré surface, known as a strobe map, which can be interpreted as a discrete dynamic system in a state space with a smaller dimension than the original continuous system, i.e. once a dimensional hyperplane ( $m - 1$ ) is taken in the dimensional space (embedded)  $m$ , a compressed time series of only the intersections of the continuous time path is created with this hyperplane in a predefined orientation (Kantz and Schreiber, 2004;

Hegger et al., 1999). Using these method three types of trajectories can be obtained: 1) A simple periodic orbit of the original dynamic system that becomes a single fixed point of the section of Poincaré, 2) A quasi-periodical trajectory that becomes the image of a closed curve, and 3) A chaotic movement that becomes a zone of erratically distributed points.

### 3 Results and discussions

Figure 1 shows the evolution of the number of hurricanes from 1749 to 2012 and their linear trend. In order to have a qualitative idea of the number of hurricanes that occurred in the Gulf of Mexico and the Caribbean Sea between 1749 to 2012 a phase diagram was performed using the "delay method" (Fig. 2). This was also used to elucidate the time lag for an optimal embedding in the dataset. The optimal time lag ( $\tau$ ) obtained from Fig. 2 was equal to 9. In our case, the hurricane dynamics were not distinguish through with the phase diagram; however, since any hurricane trajectory starts at a close point location on the attractor dataset that diverges exponentially, it is a primary evidence of a chaotic motion according to Thompson and Stewart (1986).

Another way to visualize the dynamics of the system is through the Poincaré Surface Section (Fig. 3), which helped us to observe the presence of chaos involved in our data. [It is worth to mention that the 2-D projection presented here, is valid for a first exploration of chaos](#)

. We observed that the points are scattered in the constructed plane, indicating that there is a chaotic behavior. However, the most robust method to identify chaos within the system is the Lyapunov exponent. Prior to obtaining the exponent, it was necessary to calculate the time lag using the Theiler window and the embedding dimension. The time lag was obtained with three different methods: 1) The method of constructing delays, 2) The method of mutual information, which yields a more reliable result since it works with the nonlinear part of the systems, and 3) The autocorrelation function methods (Fig. 4). There are two ways to obtain the time lag

from the autocorrelation function. We used the criterion of the first zero, because the exponent of Hurst ( $H = 0.032$ ) indicated that it was a short memory process; therefore, the criterion of the first zero is optimal method in this type of cases. The Hurst exponent aids us to identify the criteria to find a time lag, and it also describes

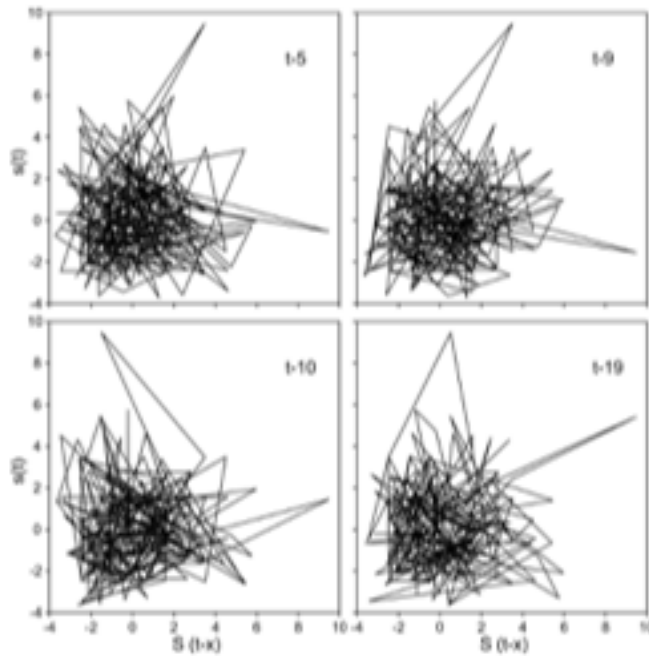


Figure 2. Phase diagrams corresponding to the time series of hurricanes that occurred between the years 1749 and 2012 in the Gulf of Mexico and the Caribbean Sea. The x-axis in the four plots indicates the time lag ( $\tau$ ).

system behavior (Quintero and Delgado, 2011), this could indicate that the system does not have chaotic behavior; however, the remaining methods have indicated the opposite and as mentioned previously, the Lyapunov exponent is considered the most appropriate method for this type of dataset. Therefore, different methods will provide different results, but the time series will indicate the best method and the result we should use.

It was possible to observe the difference in the time lag obtained by both methods; however, it is necessary to use only one result. Through the space-time separation

graphic and the False Close Neighbours method, we obtained embedding dimensions of  $m = 4$  for a  $\tau = 9$ , and  $m = 5$  for  $\tau = 10$  and the Theiler window (Fig. 5).

The idea of the False Close Neighbours algorithm is that at each point in the time series,  $S_t$  its neighbour  $S_j$  should be searched in a  $m$ -dimensional space. Thus, the distance  $\|S_t - S_j\|$  is calculated iterating both points, given by:

$$R_i = \frac{S_{i+1} - S_{j+1}}{\|S_i - S_j\|} \quad (1)$$

Therefore, it may have some False Close Neighbours even when working with the correct embedding dimension. The result of this analysis may depend on the time lag (Kantz and Schreiber, 2004).

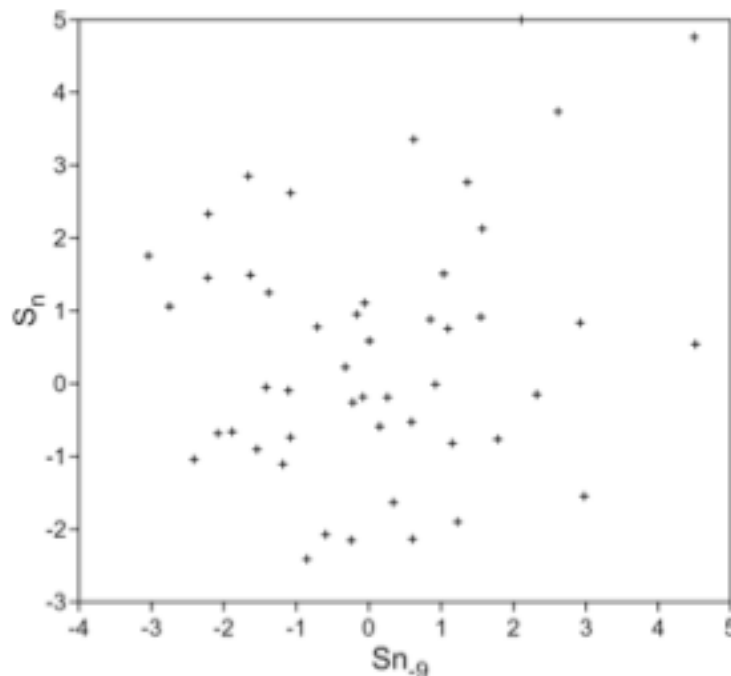


Figure 3. Poincaré surface section for the time series of hurricanes that occurred between the years 1749 and 2012 in the Gulf of Mexico and the Caribbean Sea.

The Lyapunov ( $\lambda$ ) exponents were obtained using the Kantz and Rosenstein methods taking the time lag and the embedding dimension. The Kantz (1994) method using a value of  $m = 4$  and  $\tau = 9$  give us an exponent of  $\lambda = 0.48392$  and for  $m = 5$  and  $\tau = 10$  the exponent was of  $\lambda = 0.48392$ . Since  $\lambda$  is a positive value, it was inferred that our system is chaotic. In addition, the value of  $\lambda$  obtained for both imbibing dimensions was the same, suggesting that our result is accurate. Using the Rosenstein et al. (1993) method, the value obtained for  $m=4$  and  $\tau=9$  was of  $\lambda=0.1056$  and for  $m=5$  and  $\tau=10$ , the exponent was  $\lambda=0.1123$  (Fig. 6).

There was a difference between placing the attractor in an embedding dimension of  $m = 4$  and one of  $m = 5$ , a better unfolding of the attractor in the embedding dimension was observed in  $m = 4$ . ~~Continuing with the analysis, the Integral Correlation was obtained for the occurrence of hurricanes in our study area (Fig. 7).~~

~~As observed in Fig. 7, as  $m$  increases, the structure eventually becomes invariant in high embedding dimensions; this is an indicator that no extra variables are needed to elucidate the dynamics of the system. The Integral of Correlation was obtained for a total of 10 different dimensions. There was a linear correspondence among the dimension; however, after the 4<sup>th</sup> dimension the behavior becomes invariable, which was something expected according to the method of False Neighbors and the results obtained from the Lyapunov exponent. Here it was also confirmed that the minimal dimension needed to study the system dynamics was fourth dimension. Fig. 8 shows the Correlation Dimension  $D_2$  for multiple length scales.~~

[It was possible to obtain the Correlation Dimension  \$D\_2\$  \(Figure 7\) and the Correlation Integral \(Figure 8\), using the embedding dimension, the delay time and the Theiler window, following the method of Grassberger and Procaccia \[1983a, 1983b \]. This was done in order to obtain the possible dimension of the attractor.](#)

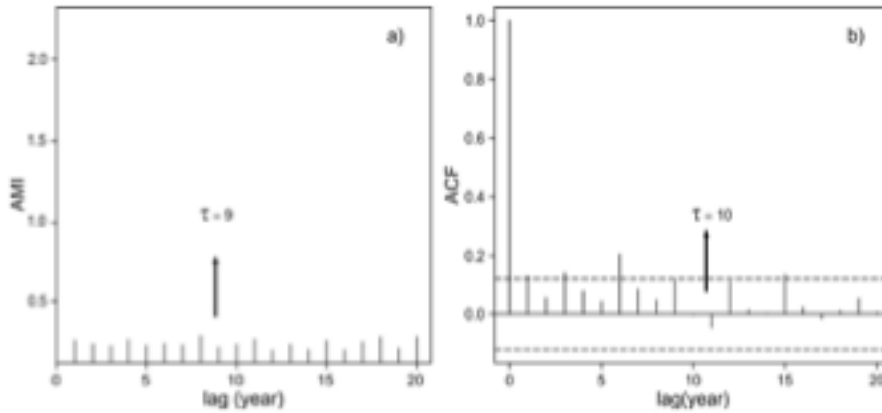


Figure 4. The left panel shows the Mutual Information Method, the x-axis indicates the time lag against the Mutual Information Index (AMI), the arrow indicates the first most pronounced minimal with a value of  $\tau = 9$ . The right panel shows the autocorrelation function, the x-axis indicates the time lag versus the value of the autocorrelation function, and the arrow denotes where the first zero of the function  $\tau = 10$  was obtained.

The right panel on Fig. 98 shows the slope trend of the majority of the slopes of the Correlation Integral ( $\epsilon$ ). In the range of  $1 < \epsilon < 10$  requires to have straight lines as an indicator of the self-similar geometry. The value obtained here correspond to  $D_2 = 2.20$  which is the aforementioned slope value. Another method to see the attractor dimension is the Kaplan-Yorke Dimension ( $D_{ky}$ ) which is given by:

$$D_{KY} = k + \sum_{i=1}^k \frac{\lambda_i}{|\lambda_{k+1}|} \quad (2)$$

Where  $k$  is the maximal integer, such that the sum of the  $k$  major exponents is not

negative. The fractal dimension with this method yielded a value of  $D_{ky} = 2.26$ , which is similar to the one obtained previously.

Even when all the requirements necessary to apply the non-linear analysis to our time series are present, one final requirement must be fulfilled to know whether we can obtain a dimension and whether the complete spectrum of Lyapunov exponents still need to be employed (another method to visualize chaos). Eckmann and Ruelle (1992) discuss the size of the dataset required

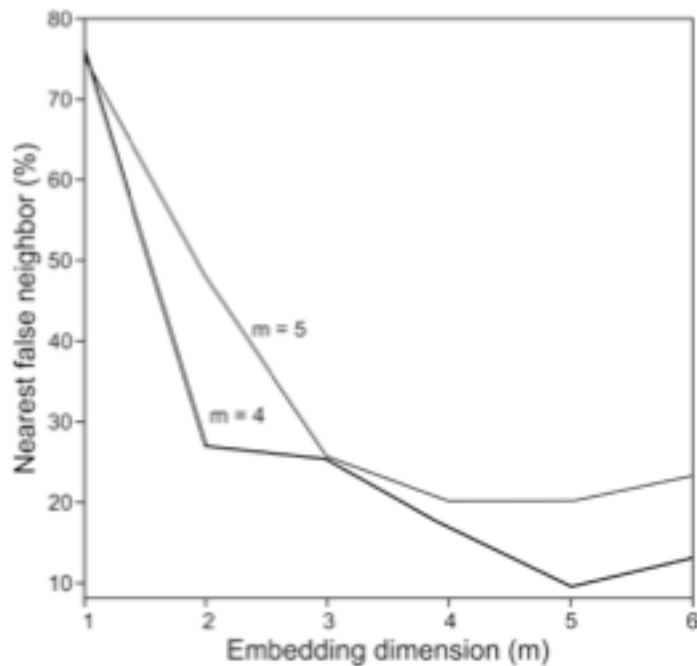


Figure 5. False Close Neighbours with a time lag of 10, where of the embedding dimension of 5 has a 9.4%, and the embedding dimension of 4 has a 16.66% False Close Neighbours (lower line). False Close Neighbours with a time lag of 9, where of the embedding dimension of 5 has a 20.15%, and the embedding dimension of 4 has a 20.12% False Close Neighbours (upper line). The values in each line indicate the optimal dimension for each lag.

to estimate Lyapunov dimensions and exponents. When these measure the divergence rate with near-initial conditions, they require a number of neighbors for a given reference point. These neighbours may be within a sphere of radius  $r$ , and of a given diameter ( $d$ ) the reconstructed attractor; ~~Therefore:~~



$$\frac{\tau}{d} = \rho \ll 1 \quad (3)$$

Eckmann and Ruelle (1992) suggested that  $\rho$  should be maximum around 0.1. In addition, the number of candidates for neighbors ( $\Gamma(r)$ ), must be much greater than one. Therefore:

$$\Gamma(r) \gg 1 \quad (4)$$

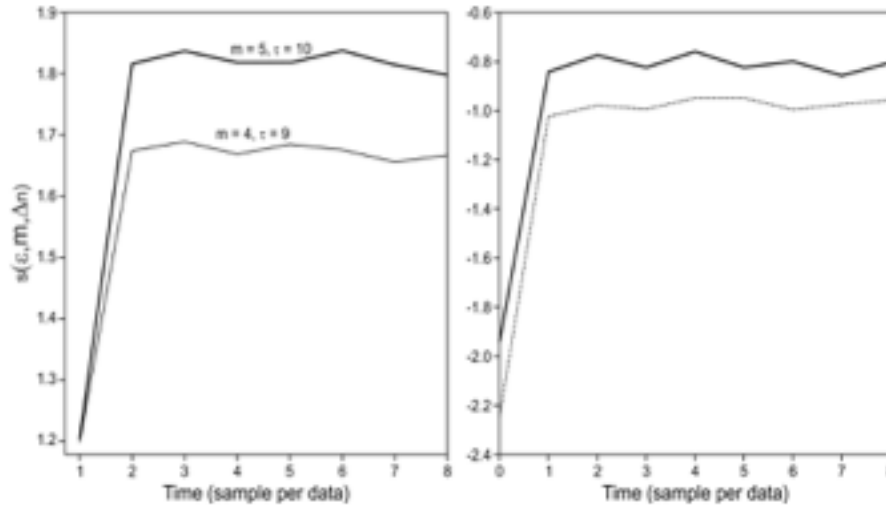


Figure 6. Left panel: Lyapunov exponent with  $m = 4, \tau = 9$  and  $m=5, \tau=10$ , with the Kantz method. Right panel: Lyapunov exponent with the same values with the Rosenstein method.

where:

$$\Gamma(r) \approx \text{constante} * r^{-D} \quad (5)$$

Then:

$$\Gamma(d) \approx N \quad (6)$$

Where  $D$  is the attractor dimension and  $N$  is the number of points or data. The following relation was then obtained from equations (4) and (6):

$$\Gamma(r) \approx N \left( \frac{r}{d} \right)^D \gg 1 \quad (7)$$

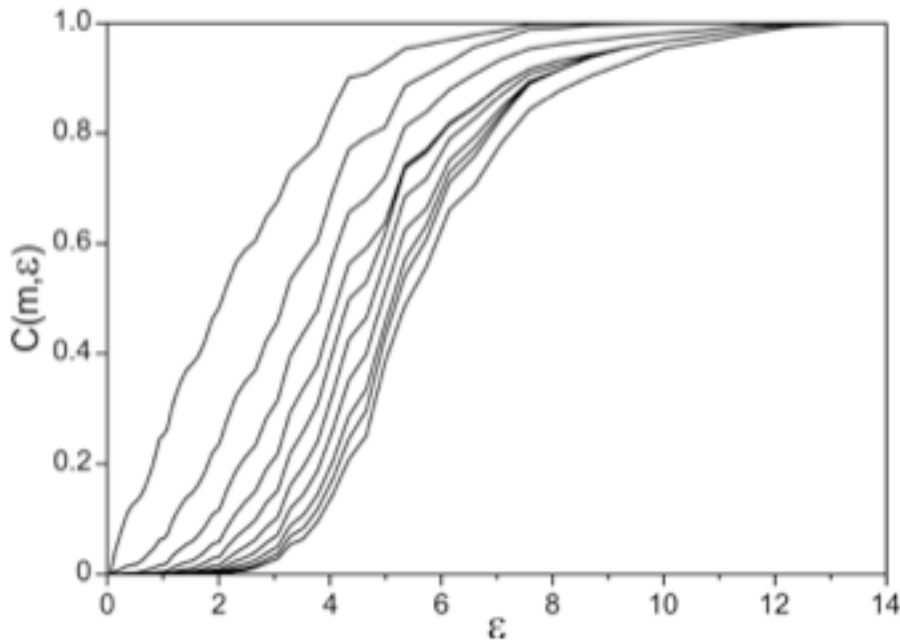


Figure 7. Integral Correlation for the occurrence of Hurricanes between the years 1749 and 2012 in the Gulf of Mexico and the Caribbean Sea. Each slope is a different  $m$  value;  $m$  takes values from 1 to 10.

Finally, on combining equations (3) and (7), this yields In this way we have the requirement ~~to for~~ the Eckmann and Ruelle (1992) condition ~~condition~~ to obtain the Lyapunov exponents:

$$\log N > D \log \left( \frac{1}{\rho} \right) \quad (38)$$

Where  $D$  is the dimension of the attractor,  $N$  is the numer of data points  $y \frac{r}{d} = \rho$  -For  $\rho = 0.1$  in equation (38),  $N$  may be chosen such that:

$$N > 10^D \quad (49)$$

Where  $N$  is the number of data and  $D$ , the dimension of the attractor. Our time series met this requirement; therefore, it support our previous results.

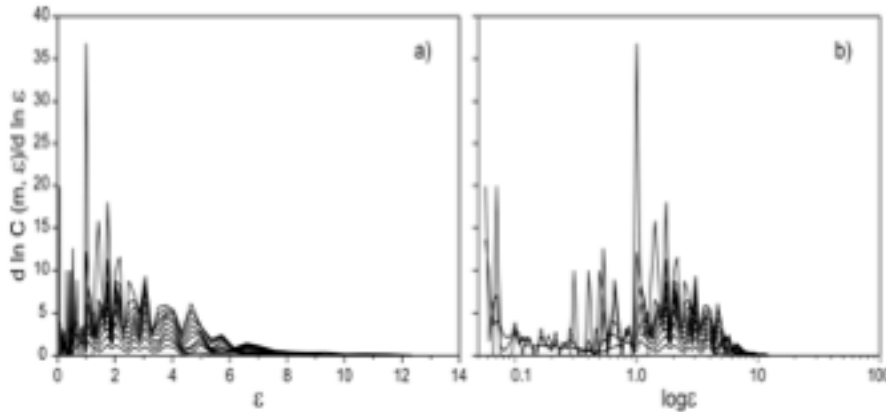


Figure 87. Correlation dimension  $D_2$  corresponding to the occurrence of hurricanes in the years 1749-2012 in the Gulf of Mexico and the Caribbean Sea. Left-panel curves for different dimensions of the attractor (y-axis). Right panel, same information for the  $D_2$  with a logarithmic scale on the x-axis.

The attractor dimension was mainly obtained because this value tells us the number of parameters or degrees of freedom necessary to control or understand the temporal evolution of our system in the phase space, and helps us to know how chaotic our system is. Using the previous methods, a final dimension of  $D_2 = 2.2$  was obtained for the fractal dimension. Following the embedding laws, it must be fulfilled that  $m > 2D_2$ , but, in the case of the correlation dimension, it is sufficient if it satisfies  $m > D_2$  (Sauer and Yorke, 1993; Kantz and Schreiber, 2004). [The criterion of Ruelle \[1990\] was used to corroborate that the obtained dimension of the attractor is reliable, where it must be fulfilled that  \$N = \lceil 10 \rceil^{\(D\_2 / 2\)}\$ , once the data fulfill this requirement we can say that the dimension of the attractor is reliable.](#)

Finally, the results indicated that at least three parameters are needed to characterize our system, since the 2.2 dimension indicates that the attractor dimension falls between 2 and 3.

The spectrum of the Lyapunov exponent gives: 0.09983, -0.07443, -0.23387 and -0.73958; therefore, the total sum was of  $\lambda_i = -0.9480$ , and, according to the previous theory, it is enough have at least one positive exponent in the spectrum of our system in order to have a chaotic behavior. Finally, the total sum of the spectrum of the Lyapunov exponents was negative, indicating that there is a stable attractor, as mentioned previously. However, since the stable attractor was not easily distinguished, we used a final method in order to confirm this result. This method comprised the Iterated Functions System test (IFS) (Fig. 109).

Using Fig. 109, it can be observed that the points representing our system occupy the entire space; according to this, the distribution could be described as a white noise signal, in systems without experimental noise the point distribution gives a single curve (Jensen et al., 1985), [or a high dimension chaotic behavior](#). However, the previous Hurst exponent obtained was not equal to zero; then, the white noise

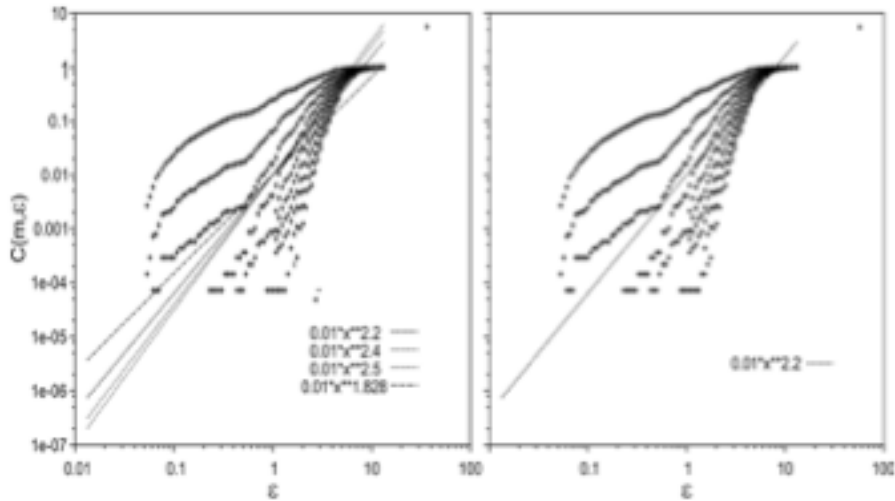


Figure 98. Integral Correlation for the occurrence of Hurricanes between the years 1749 and 2012 in the Gulf of Mexico and the Caribbean Sea. Each slope is a different m-value. Left panel; slopes that indicate possible fractal dimensions of the attractor. Right panel; the slope that covers the majority of the slopes of the Correlation Integral.

was also discarded with the autocorrelation function. On the other hand, our [Lyapunov exponent figure was not flat and it does not seem to flatten as the dimension of embedding increases](#)

~~IFS test was not flat and does not appear to be flattened by increasing the embedding dimension~~ which according to Rosenstein et al. (1993), our system would not be chaotic. It was then also possible to obtain a dimension of the attractor and a positive Lyapunov exponent. Fraedrich and Leslie (1989) analyses the trajectories of hurricanes in the region of Australia and calculate the dimensionality of this process, obtaining a result of between 6 and 8, i.e., a chaotic process of high dimensionality, which is similar to what we find with ~~this~~ the IFS method, [where the system was not considered as white noise either](#). On the other hand, Halsey and Jensen (2004) postulate that hurricanes contain a large number of dimensions in phase space.

Our results were not easy to interpret because the series presented certain periodic characteristics in an oscillatory fashion, and chaotic behavior at the same time.

According to Rojo-Garibaldi et al. [2016] the series of hurricanes with the spectral analyzes carried out, presented strong periodicities, corresponding to sunspots, which gives the system the periodic behavior mentioned above. An explanation was given with the IFS test. The IFS test showed that the occurrence of hurricanes in the Gulf of Mexico and the Caribbean Sea is chaotic with high dimensionality.

One possible explanation is localized within a boundary where chaos and order are separated; this boundary is commonly known as the "edge of chaos" (Langton, 1990; Miramontes et al., 2001). Miramontes et al. (2001) found this type of behavior in ants of the genus *Leptothorax*, when studying them individually and in groups. For the former case, the behavior was periodic, while for the second case, the behavior was chaotic. In our case, the chaotic behavior is due to the individual behavior or by hurricane category, while the periodic response is due to the behavior of hurricanes as a whole.

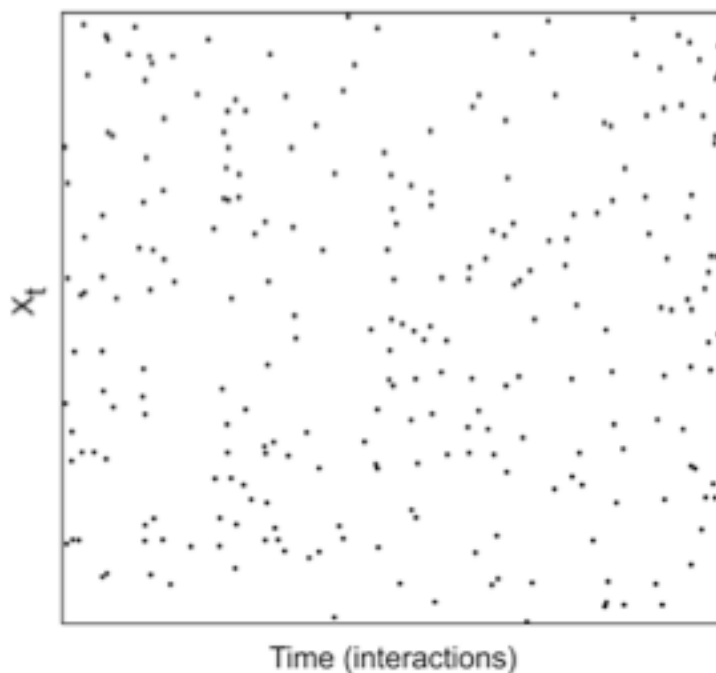


Figure 409. Iterated Functions System (IFS) test applied to the time series of the number of hurricanes that occurred in the Gulf of Mexico and the Caribbean Sea between the years

1749 and 2012.

#### **4 Conclusions**

The results obtained with the non-linear analysis suggested a chaotic behaviour in our system, mainly based on the Lyapunov exponents and correlation dimension, among others. However, the Hurst exponent indicated that our system did not follow a chaotic behaviour, and in order to be able to corroborate our results, we employed the IFS method, which led us to think that the hurricanes time series in the Gulf of Mexico and the Caribbean Sea from 1749 to 2012 has a chaotic edge.

#### New References

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