

## Response to the Second Referee.

First, we would like to thank the Referee for her/his comments, all of which we have attempted to address. We think that the paper has been improved by them. Now we detail our response to each comment.

1. **Despite the intuitive, valid fractal analysis, I feel that the manuscript does not have many new elements to showcase. The reference to turbulence in efforts to physically connect solar, interplanetary, and magnetospheric timeseries is biased in its framework. The manuscript effectively shows the effect of intermittency in the fractal dimension of timeseries, regardless of turbulence. Intermittency is a term that is broader than turbulence: turbulent timeseries may be intermittent, but not all intermittent timeseries stem from turbulent systems.**

The scatter diagram of Figure 1 creates some “dust-like” fractals in case of intermittency (in this case, storm-time dips in Dst). Dust-like structures typically give rise to a fractal dimension smaller than  $D_{\max}=1$ , where  $D_{\max}$  is the embedding (i.e., Euclidean) dimension of the studied space. In Figure 1,  $D_{\max}=2$ , hence the dust-like structures in the lower-left part of the image show a fractal dimension  $D < 1$  (see, e.g. Schroeder, M.: *Fractal, Chaos, Power Laws. Minutes from an Infinite Paradise*, Freeman, New York, NY). If no significant intermittency is present, one is left with the upper right part of Figure 1 that typically gives  $1 < D < 2$ .

**Interpreting intermittency in general as turbulence and drawing physical conclusions from it is the main drawback of the manuscript.**

It was not our intention to force a connection between all three systems studied through the concept of turbulence. As stated in the manuscript, the GOY shell model has been shown to exhibit dissipative events whose distribution follows the same power-law statistics as observed in turbulent magnetized plasmas (Boffetta et al., 1999; Lepreti et al., 2004; Carbone et al., 2002), and our main goal was to test whether such

bursty behavior exhibited fractal features similar to those found in the *Dst* analysis.

Although various works suggest the presence of turbulence in the Earth's magnetosphere, the question of the validity of the GOY shell model to describe such phenomena is far beyond the scope of our paper. As the referee correctly points out, it is the intermittency of the time series which is the relevant feature, and in fact that is what justifies the use of certain values of  $\nu$  and  $\eta$  for the simulations, since, in general, intermittency levels similar to the *Dst* timeseries are not observed for arbitrary values of these parameters.

We have attempted to clarify this issue in various parts of the text. For instance, in the second paragraph of Sec. 4.

The new text reads:

*We first notice that, in general, setting parameters  $\nu$  and  $\eta$  with arbitrary values yields  $\epsilon_b(t)$  series which do not have the necessary intermittency level to resemble the *Dst* time series. Compare, for instance, the different panels in Fig. 16 in Domínguez et al. (2017), which shows that  $Pm = 0.2$  leads to a very noisy output, unlike simulations with  $Pm = 1.0$  or  $2.0$ , where individual, large peaks can be easily identified from the background. In fact, previous studies have shown that the statistics of bursts follows a power law for  $Pm = 1$  (Boffetta et al., 1999; Lepreti et al., 2004; Carbone et al., 2002), and for this reason we start by taking  $Pm = \nu/\eta = 1$ .*

Also, in the final paragraph of the same section.

The new text reads:

*Results suggest that the intermittency level of the output time series is relevant, which has led us to perform the analyses for the shell model within a certain range of values of the Prandtl number, as well as of the viscosity and resistivity.*

- 2. This leads to insufficiently justified conclusions such as the correlations between D from *Dst* timeseries and the solar flare**

/ coronal indices over tens of days (Figure 9). Indeed, there is connection if an eruptive flare (flare + coronal mass ejection) leads to a magnetospheric storm within 1–3 days. However, the correlation seen in Figure 9 is not due to physics but due to the fact that any two intermittent timeseries with intermittent excursions roughly matching in time will show similar correlations. I am afraid this is a common fallacy, appearing in several interdisciplinary studies of timeseries giving, not surprisingly, incidental correlations.

Thanks for pointing out this issue. The figure that the referee mentions (Fig. 9 in the previous version of the manuscript), was part of an exploration of possible correlations between fractal dimensions and various indices, using solar and geomagnetic timeseries, which was made in Domínguez et al. (2014). As the referee says, a better statistical and physical analysis is needed to state whether these correlations hold or not. Besides, in the context of the present manuscript, it is not a relevant discussion, since we focus on the series themselves, not on their correlations with others. We have thus dropped Fig. 9 in this version of the manuscript.

3. **Another unjustified conclusion is the one drawn from Figures 7, 8, namely that “results suggest that the box-counting dimension consistently decreases when the storm approaches” (p.8; top). However, the decrease is not due to the storm but due to the pre-storm disturbances (hours > 1400 and up to the storm’s onset). These disturbances are not necessarily related to the storm. Similar disturbances appear at times < 500 hours in the absence of a storm. Not surprisingly, D in this interval is very similar to the pre-storm D that is indeed decreasing. Again, it is the (most likely incidental, as it starts ~300 hours prior to the storm) minor intermittency in the timeseries that causes the decrease in both cases, regardless of the storm. Finding a unique pre-storm signature is the challenge here and the manuscript does not seem to contribute significantly to this cause.**

We agree that conclusions need to be toned down, and that the present analysis cannot suggest that the decrease observed before the storm is related to the storm itself. However, our aim in this manuscript is focused rather on the dissipative events themselves and the fractal dimension, not on the finding of precursors for geomagnetic activity, an issue which requires further, detailed analysis.

Thus, we have changed the wording in the sentence mentioned (now at the bottom of page 5).

The new text reads:

*As shown in Domínguez et al. (2014), the box-counting dimension of the Dst index decreases as the storm approaches for all cases studied. Moreover, this decrease occurs before the window includes the geomagnetic storm, as marked by the vertical lines in Fig. 5. Whether this is relevant for forecasting geomagnetic storm needs further study, as it may simply be due to an increase of the intermittency in the time series, unrelated to the upcoming dissipative event.*

- 4. The above issues render the penultimate conclusion of the manuscript (p.14) also biased. I see no point in re-doing the analysis unless more physical and statistical arguments for the apparent correlations are used alongside the analysis of the fractal dimension.**

We have indeed performed more systematic analysis than the ones mentioned in this manuscript, but were left in the cited references and not included in the current text.

Cross correlation analyses between the *Dst* timeseries and its fractal dimension were performed. This is not a direct calculation, as both time series have different resolutions, and thus interpolation of the fractal dimension time series is needed to match the resolution of the geomagnetic index. This analysis was made for individual storms and full year data, and is included in Domínguez et al. (2014). This is mentioned in the final paragraph of Sec. 3.

On the other hand, *p*-value analyses were systematically done for the

shell model simulations, for a wide range of values of  $\nu$  and  $\eta$ , considering  $Pm = 1$  and  $Pm \neq 1$ . This allowed us to find a range of values of the simulation parameters where the correlation between  $\epsilon_b(t)$  and its fractal dimension is statistically significant.

We have mentioned this issue in the first paragraph of page 8, relating it to the problem of intermittency.

The new text reads:

*We first notice that, in general, setting parameters  $\nu$  and  $\eta$  with arbitrary values yields  $\epsilon_b(t)$  series which do not have the necessary intermittency level to resemble the Dst time series. Compare, for instance, the different panels in Fig. 16 in Domínguez et al. (2017), which shows that  $Pm = 0.2$  leads to a very noisy output, unlike simulations with  $Pm = 1.0$  or  $2.0$ , where individual, large peaks can be easily identified from the background. In fact, previous studies have shown that the statistics of bursts follows a power law for  $Pm = 1$  (Boffetta et al., 1999; Lepreti et al., 2004; Carbone et al., 2002), and for this reason we start by taking  $Pm = \nu/\eta = 1$ .*

And again in the last paragraph of Sec. 4.

The new text reads:

*In Domínguez et al. (2017) a more detailed analysis is carried out on the shell model results, exploring other simulation parameters ( $\nu$ ,  $\eta$ , magnetic Prandtl number), other criterion for defining active states ( $n = 5$ ), and a systematic study of the correlations between the fractal dimension and the occurrence of dissipative events by means of the Student's  $t$ -test. Results suggest that the intermittency level of the output time series is relevant, which has led us to perform the analyses for the shell model within a certain range of values of the Prandtl number, as well as of the viscosity and resistivity.*