

Interactive comment on “Utsu aftershock productivity law explained from geometric operations on the permanent static stress field of mainshocks” by Arnaud Mignan

A. Mignan

arnaud.mignan@sed.ethz.ch

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Dear reviewer,

Thank you for your comments on the discussion paper by Mignan (2017). Below is my two-part answer to (1) show that the Solid Seismicity Postulate is supported by seismicity data and (2) discuss in more detail the mismatch between theoretical scaling break and lack of break in real data. A third section answers to your other comments.

1 Support of the Solid Seismicity Postulate (SSP) by aftershock data

The SSP should indeed be verified to be consistent with the spatial distribution of seis-

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micity data. I first clarify that the step-like function of event density in space is only expected for the case of an idealised smooth static stress field. I now compare this case (Fig. X1a-b) with the case of a stress field with uniform noise (Fig. X1c-d). While the ideal case is used to develop analytical solutions, a heterogeneous stress field described by additive uniform noise was already used in past studies to simulate non-stationary background events (King and Bowman, 2003; Mignan et al., 2007; Mignan, 2011). Addition of such noise blurs the “aftershock solid”, which reflects in the aftershock spatial density distribution, switching from a step function to a power-law of the form $\rho(r)$ proportional to r^{-q} , with ρ the linear spatial density and exponent $q = 1.7$. Figure X1 will be inserted in the revised manuscript as a new Figure 2 (with a new paragraph inserted line 111).

As shown in Figure X2 (new Figure 5 in the revised manuscript), the power law exponent obtained from the SSP with noisy static stress field matches the power law exponent found in Southern California. In the literature, $1.3 < q < 2.5$ centred around $q = 1.4-1.8$ was found for California (Felzer and Brodsky, 2006; Lipiello et al., 2009; Marsan and Lengliné, 2010; Richards-Dinger et al., 2010; Shearer, 2012; Gu et al., 2013; Moradpour et al., 2014; van der Elst and Shaw, 2015). This demonstrates that the SSP is not “too simple” or “unrealistic”. Comparison of Figure X1d with Figure X2a shows that “the spatial patterns of real earthquakes are reproduced” by the SSP (i.e., the power-law behaviour) with a realistic q -value (without any tuning required). A short review of past studies on the spatial distribution of aftershocks and a discussion of Figure X2 will be added line 184 in the revised manuscript (end of section 3 on “Observations & Model Fitting”).

This work goes beyond the results of Hainzl et al. (2010) since an analytical formulation is explicit while the physical driver of a simulation output is implicit and potentially ambiguous. In the King and Bowman (2003) study for example, a power-law behaviour of precursory seismicity emerged from their static stress simulations. However the result was ambiguous. It was not clear if the behaviour emerged from the stress field

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geometry, implemented Gutenberg-Richter power-law, or else. It led to the first study on Solid Seismicity, which demonstrated that the power-law time-to-failure equation derived from the geometry of the stress field (Mignan et al., 2007). While such ambiguity may not be present in the simulations of Hainzl et al. (2010), we are still left wondering which parameters are critical to the emergence of the Utsu productivity law, i.e., “it remains unclear how K_0 and α relate to the underlying physical parameters” (line 50).

Here are two “new and meaningful perspectives as a result of the introduction” of the SSP: (i) It is first of importance to demonstrate that the Solid Seismicity theory can explain the aftershock productivity law, since it already explains both tectonic foreshocks (Mignan, 2012) and induced seismicity (Mignan, 2016). If such physical framework can explain the main seismicity patterns observed in Nature, it becomes a potential candidate for a unified theory of seismicity. (ii) Figure X2 goes farer into the Solid Seismicity analysis, showing how to estimate its main parameters (intermediary parameter r_{-}^* , main parameters δ_{+} and $\Delta\sigma_{-}^*$). We first note that the $q = 1.7$ theoretical estimate (SSP + uniform noise) is compatible with observations (Fig. X2a). I here focus on the largest mainshocks to avoid the scattering and scaling break issues at small M . On the same plot, we can roughly estimate $r_{-}^* = 1$ km (maximum r at which the ρ plateau breaks – in analogy with Fig. X1d). It is constant for any large M ($> M_{\text{break}}$) since the stress drop is a constant, $c = w_0$ is a constant, and $\Delta\sigma_{-}^*$ is also a priori a constant (one of the 2 main parameters of the Solid Seismicity approach; Eq. 7). Now let us calculate δ_{+} from the commonly used parameter K_0 . We first note from Eq. (11) that the second term is negligible for large M , yielding

$$K(M > M_{\text{break}}) \approx 2\delta_{+}(m_0) r_{-}^* \exp[\ln(10)(M-4)] \quad (\text{X1})$$

Rearranging m_0 and $M-4$ and comparing to the original Utsu Eq. (1), we get

$$\delta_{+}(m_0) = K_0 \exp[\ln(10)(4-m_0)] / (2r_{-}^*) \quad (\text{X2})$$

With $\alpha = \ln(10)$ fixed and K_0 estimated from the MLE for $M > 6$, we get $K_0 = 0.027$ and thus $\delta_{+}(m_0 = 2) = 1.35$ events/km³ (fit represented in Fig. X2b). If correct, the

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linear density below r_* (plateau) for any given large M should be

$$\rho(r < r_*, M) = \delta_{+} \exp[\ln(10)(M-4)] \quad (X3)$$

which is represented on Fig. X2a and matches the data (Eq. (X3) simply calculates the linear density of events ρ from the volumetric density of events δ_{+}). This suggests that δ_{+} is also constant, at least for the four largest strike-slip mainshocks in Southern California. One could have also estimated δ_{+} directly from $\rho(r)$ (as done for r_*) to directly derive the aftershock productivity law of Southern California with Eq. (X1). This shows the direct link between aftershock productivity and aftershock spatial distribution (or geometry). As for the parameter $\Delta\sigma_*$, its estimation remains ambiguous as it depends on the seismogenic width w_0 . We get the ratio $\Delta\sigma_*/\Delta\sigma_0 = \{-0.5, -1.0, -1.4\}$ for $w_0 = \{5, 10, 15\}$ km, respectively (Eq. 7). This analysis as well as Figure X2 (new Figure 5) will be inserted at the end of section 3 in the revised manuscript. This of course remains a preliminary analysis. However I hope that additional analyses of aftershocks, foreshocks as well as induced seismicity in different regions will provide useful information as to the distribution of the $\Delta\sigma_*$ and δ_{+} parameters. Are they universal? Is a same regional value applicable to all types of seismicity? Are there any correlations? Those are important questions I wish to answer in the near future. To do so, the theoretical framework must first be conveyed for each class of seismicity pattern.

2 Theoretical scaling break & mismatch with seismicity data

The discussion paper already indicates that: “Possible biases of aftershock selection may explain the lack of break” (lines 18-20, abstract) and “while such a bias is possible, it yet does not prove that the break in scaling exists” (line 208) – This clearly suggests that it is only one possible option. It is indeed a weak argument (since based on a negative result) but it is so far the best one available (all existing declustering techniques assuming no break in magnitude). “It is also possible that Eq. (16) is incorrect”, true, but so would the clock-advance model in such premise, which the reviewer describes

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as “a simple but realistic physical assumption”. No explanation for the lack of break in real data was given in Hainzl et al. (2010). The present paper provides one possible explanation. Any criticism on the scaling break mismatch shall apply the same way to the present study and the published one of Hainzl et al. (2010). An alternative view is that both studies found the same scaling break, hence supporting this result as characteristic of the static stress process.

Following on the new results presented in Figure X1d, the explanation of lack of break due to aftershock selection bias becomes a more realistic one. It is NOT “a vague consequence” since any study of the aftershock productivity law is based on the use of such a declustering method. The ETAS simulation does NOT “violate the self-consistency of this MS” either since the power-law spatial distribution is now shown to be verified by the SSP. The theoretical value $q = 1.7$ is very close to the value I already used in the ETAS simulations ($q = 1.47$) and observed here for the largest strike-slip mainshocks (Fig. X2a). Since the aftershock selection bias is only one option, another alternative will be discussed: The proposed productivity equation assumes moment magnitude while the earthquake catalogue is in local magnitude. Deichmann (2017) recently demonstrated that while M_L is proportional to M_w at large M , M_L is proportional to $1.5M_w$ at small M . This would cancel the kink observed in the real data. However the scaling break predicted by Deichmann (2017) occurs at several magnitude units below the geometric one expected by static stress.

3 Other aspects

On the introduction of the Zero-Inflated Poisson (ZIP) distribution: Explaining the distribution of earthquakes, from the static stress process to their occurrence on a fractal network of faults remains out of the scope of the present study. Since the ZIP does not lead to significant changes in the α -value and since section 3 will be completed with an analysis of the spatial distribution of aftershocks (Fig. X2), the ZIP part will be deleted from the revised manuscript.

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On $\alpha = 2.04$ (line 197): This is the maximum likelihood estimate of α obtained for Southern California in the present study (see line 164). α is thus constrained from large magnitude data (Fig. 4a) and the simulated break at lower magnitudes is estimated from the theoretical value $3/2 \alpha$.

Figures

Figure X1. Spatial distribution of aftershocks following the SSP. (a) Smooth static stress field as a function of distance r from the mainshock, with $\Delta\sigma_0 = -10$ bar and $c = 10$ km (Eq. 6); (b) Step-like aftershock spatial linear density ρ with $\delta_{+} = 1000$ events per km, $\delta_0 = 1$ event per km and $\Delta\sigma_* = -0.3\Delta\sigma_0$ (ad-hoc ratio yielding $r_* = 3.5$ km; Eq. 7 – event distances sampled from the $\delta(r)$ distribution, repeated 100 times). Such distribution is not observed in Nature; (c) Same as (a) but with random uniform noise representative of spatial heterogeneities added to the regional stress field; (d) Power-law-like aftershock spatial density ρ with power exponent maximum likelihood estimate $q = 1.7$, representative of real aftershock observations (see Fig. X2a), due to the addition of uniform noise to the static stress field.

Figure X2. Estimating the Solid Seismicity parameters from the aftershock spatial distribution: (a) Linear spatial distribution $\rho(r)$ of the four largest strike-slip mainshocks in Southern California (first aftershock generation as defined from the nearest-neighbour method). $r_* = 1$ km and $\delta_{+}(m_0 = 2) = 1.35$ events/km³ can be retrieved from the observed plateau at low r , in agreement with the SSP. Note that the spatial power-law decay at high r is similar to the one expected by the SSP in the case of a static stress field with additive uniform noise (see Fig. X1; $q = 1.7$ represented by the dashed black lines); (b) Aftershock productivity K for $M > 6$. The black line represents the Utsu law as defined by Solid Seismicity (Eq. X1, simplified case). We see that taking into account the second term of the productivity law (full second line of Eq. 12 with r_* known) has no significant impact on the result (dashed and dotted curves).

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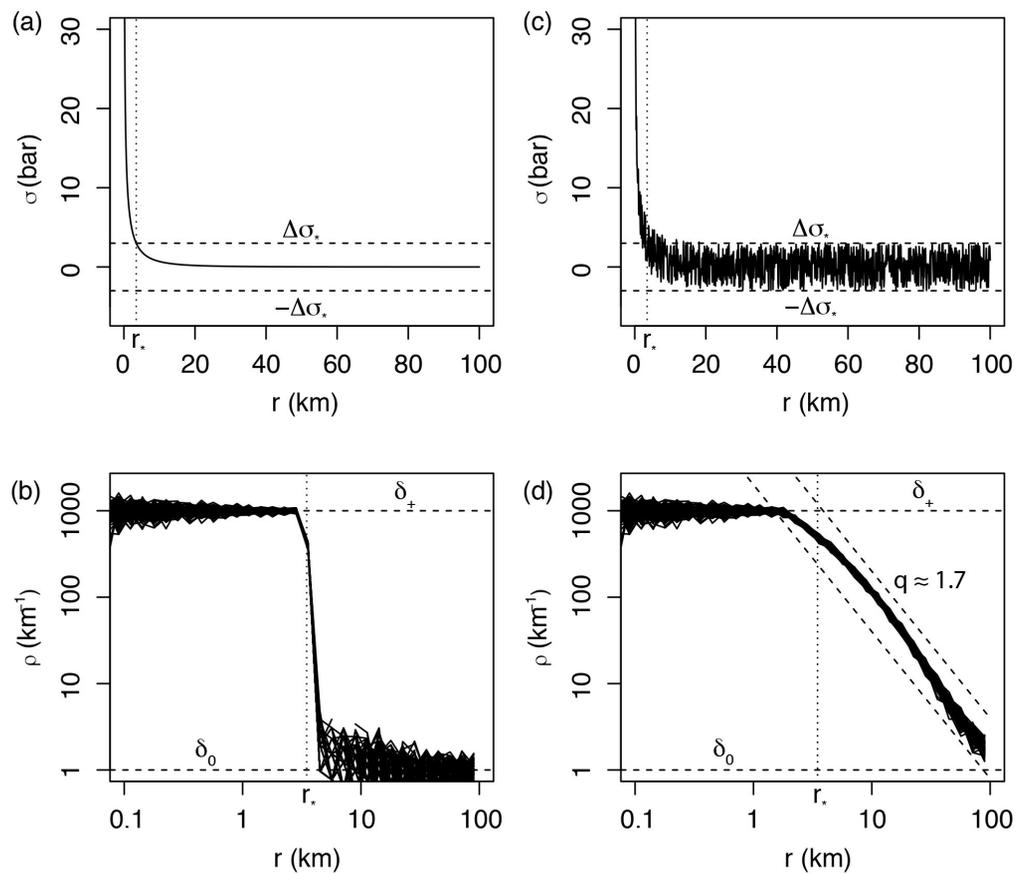


Fig. 1.

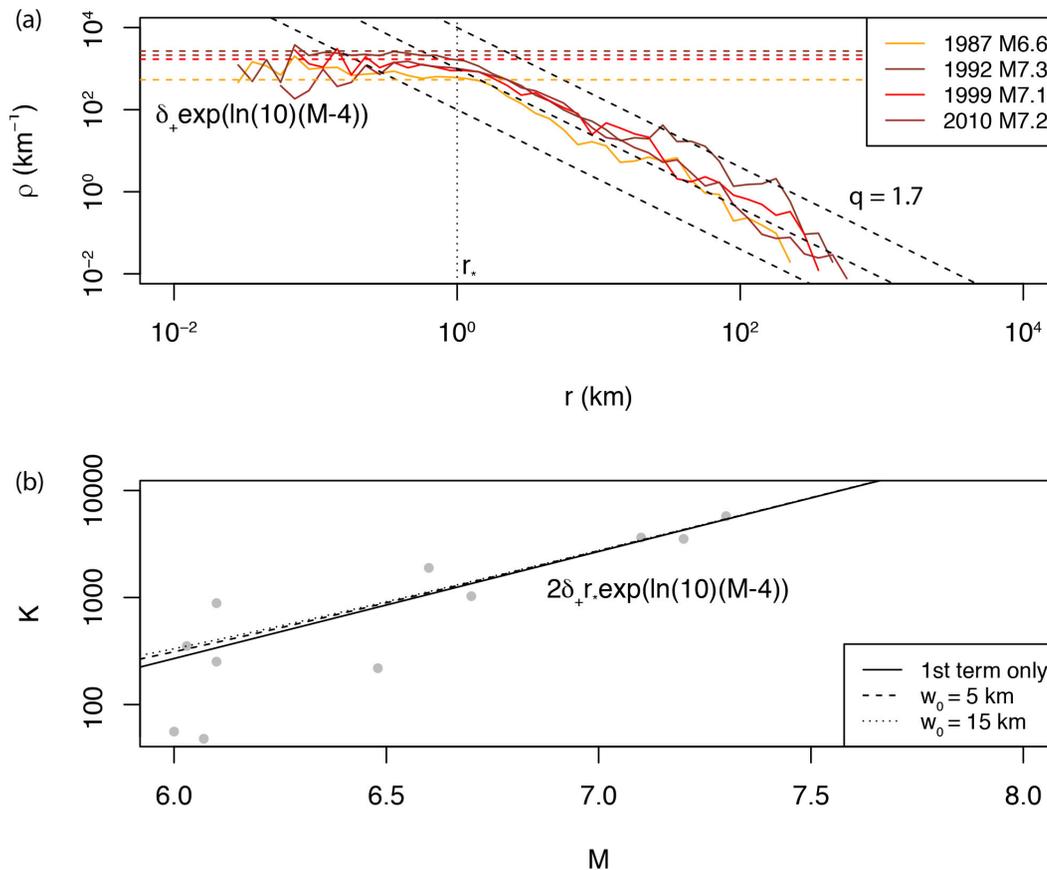


Fig. 2.