Reviewer#1.

The authors are most grateful for your comments. We have followed your suggestions and revised the manuscript accordingly in many places. Please, find our responses below.

GENERAL

This paper uses primarily 2-D simulations to study the collision of internal solitary waves with trapped cores of different amplitudes. The motivation is observed collisions of Morning Glory clouds in Australia. Results focus on the phase shift, amplitude change and kinematic mechanisms underlying the actual collision. I find this paper to be an interesting read which, nevertheless, leaves several questions. Numerous questions exist about how the simulations sweep parameter space, how the initial trapped core waves are set up and the physical mechanisms behind the actual collision. In terms of the latter, I am greatly concerned about the adequacy of the 2-D and 3-D resolution of the simulations, particularly in light of the use of a Schmidt number of $O(10^3)$?!? How well do these simulations resolve the finer features one expects, even in 2-D, due to the wind-up of the isopycnals by the K-H billows and how can we truly speak of turbulence and mixing at the resolutions used ? How much are the computed fields smeared at the finest-resolved scale by numerical diffusion ? Finally, there are a few points where the English needs polishing. One general grammatical comment: When describing the results, the authors often shift between past and present tense. Please keep the verb tenses consistent throughout the text. I list my specific comments below. If the authors address them I will gladly consider re-reading the paper to recommend it for publication.

Answer. See answers to specific comments.

SPECIFIC

Abstract

Line 12: Change "monotonous" to monotonic.

Answer. Done.

Introduction

Page 2, Line 2: The English feels awkward here. I would change to "… experiments and numerical solutions of both the DJL equation and the actual Navier-Stokes equations.

Answer. Done.

Section 2

1. *Use of a Schmidt number of Sc = ν/≈1,000 is highly perplexing. Such a value of Sc should allow the formation of very fine scale patterns in the density field: 2-D runs can support very sharp gradients, either due to the straining of the pycnocline during collision or due to the roll-up of isopycnal lines by K-H instabilities, which are most likely below grid resolution. In 3-D, one would expect a Batchelor scale (presuming the K-H billows can attain some level of turbulence) which is equal to 10001/2 times smaller than the Kolmogorov scale. Are the simulations resolving this scale ?*

The authors need to clarify the following points:

a. Have they conducted grid independence studies at least for their 2-D higher-amplitude ISW collision runs, where we expect the finest-scale patterns to form in the density field ?

Answer. We carried out doubling-grid tests to verify that chosen grid adequately described flow fields. The comparison for wave A13 is shown in Figs. A1 and A2 (see answers to Comments 1b-1c). The text was added accordingly.

p. 4 l. 25 “Most of the runs were performed in a two-dimensional setting with a grid resolution of 3000×400 (length and height, respectively), whereas several runs for waves A9-A13 were also carried out with a grid resolution of 6000×800 (length and height, respectively) to verify effect of grid resolution on the wave interaction and to make the fine structure clearer. Comparison of the baseline and doubled grid resolution showed the equivalence of the calculated fields, with the exception of wave A13 for which 6000×800 resolution was used.”

b. How many grid points span the actual pycnocline ? My back-of-the-envelope calculations show that the pycnocline is very coarsely resolved. Upon wave collision, it’ll even be further strained and less resolved. Numerical diffusion of the low-order method underlying the authors’ model can artificially smooth out things.

Answer. For the series A number of grid points span the pycnocline was 17 for grid 3000x400 and 35 for grid resolution 6000x800, for the series B the number of grid points span the pycnocline was 34 for grid 3000x400,whereas for the series C the number of grid points span the pycnocline was 68 for grid 3000x400.

c. In a 2-D run, how many grid points does one have across a K-H billow associated with instabilities along the wave ? One would need at least 30 grid points to guarantee that the resultant transverse instabilities are properly resolved in 3-D.

Answer. In our simulations about 45 grid points were placed across KH billow in the case (A13;A13) and Sc=1000 for grid resolution 3000x400 and more than 90 grid points covered KH billow for grid resolution 6000x800 as shown in Fig. A1-A2. For the rest of series of experiments this coverage was greater.

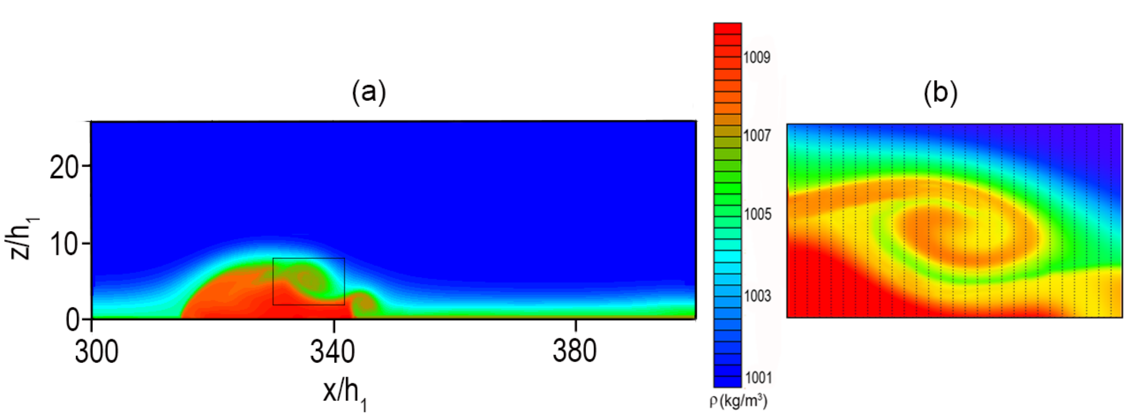

Fig. A1 Snapshot of the density field for case (A13;A13) at $\tau=175$ and Sc=1000 for grid resolution 3000x400 (a) and extended snapshot of KH billow with grid points (b).

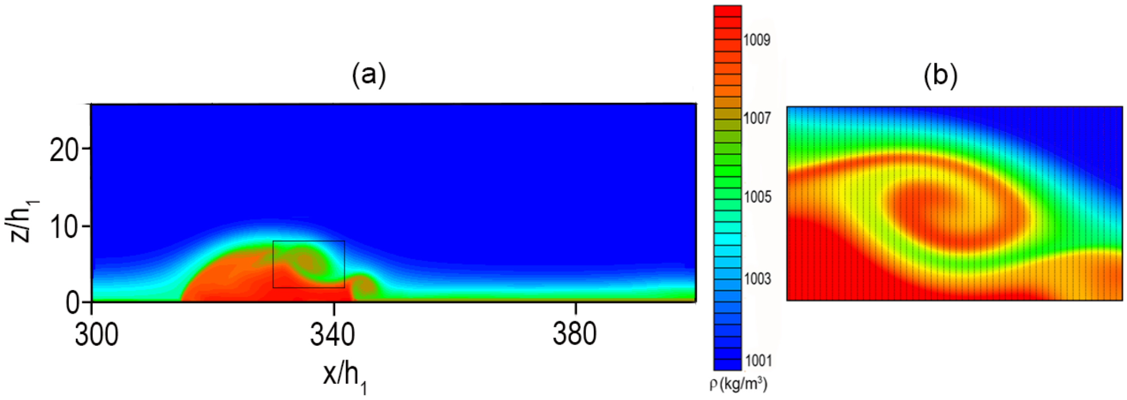

Fig. A2 Snapshot of the density field for case (A13;A13) at $\tau = 175$ and Sc=1000 for grid resolution 6000x800 (a) and extended snapshot of KH billow with grid points (b).

d. When 3-D runs are conducted, what is the local Reynolds number (based on local value of shear and B-V frequency along the wave-strained pycnocline) in the regions where K-H billows are observed, prior to K-H billow formation ? Is this Reynolds number high enough for actual turbulence to form within these billows or do they simply form, possibly pair and support some weak transverse instability ? How do we know that there are not scales smaller than the transverse instability that form ? Again, numerical diffusion can drive some very spurious results here.

Answer. We excluded results of 3D simulation from this paper.

e. MOST IMPORTANTLY: In 2-D, the authors should conduct a comparison of one simulation of high amplitude ISW collision at Sc = 1 and 1000, where I would hope/assume Sc = 1 is well-resolved by the authors' choice of grid. How do the results compare ? The Sc=1 case is presumably more relevant to the atmospheric Morning Glory case which motivates this study.

Answer. Text and figure were added to consider the impact of small diffusivity on the collision processes.

p. 7 l.8 "In the ocean and in the most of the laboratory experiments the Schmidt number is about 700-800. The used grid does not allow the whole range of inhomogeneities in salinity (density) to be resolved. Therefore, it is important to evaluate the effect of molecular diffusion of salinity on the dynamics of waves and to verify the possibility that diffusion can be neglected in the wave collision for large Sc. Two cases for large amplitude waves were considered (A9;A9) and (A13;A13). We performed runs for Sc=1; 10 and 1000. In the collision case (A9;A9) the behaviour of colliding waves are the same, whereas the difference between runs for Sc=1 and Sc=1000 was less than 1% of $\Delta\alpha/\alpha$ and $\Delta\theta$ values. The comparison of the density snapshots during collision in case (A13;A13) for different Schmidt numbers is shown in Fig. 9. Figure clearly depicts difference between structure of interacting waves for cases Sc=1 and Sc=10. The corresponding values of $\Delta\alpha/\alpha$ and $\Delta\theta$ differ by 5% and 0.6%, respectively. This was in agreement with the results by Deepwell and Stastna (2016), where it was shown essential effect of molecular diffusivity on the mass transport by mode-2 ISW in the range $1 \leq \mathrm{Sc} < 20$. At the same time, the results of calculations at Sc=10 and Sc=1000 in Fig.9b and 9c practically coincide, which indicates that molecular diffusion may not be taken into account when studying the global properties of colliding waves. This conclusion agrees with (Terez and Knio, 1998) as they estimate that the value of Sc=100 was "sufficiently high for density diffusion to be ignored during simulation period" and the results of the Deepwell and Stastna (2016) simulation, according to which the mass transfer is virtually independent of Sc already at Sc>20. However, diffusion can be important for small scale mixing processes in tiny density structures (see e.g. Galaktionov et al., 2001) forming in result of instability and turbulent cascade processes (Deepwell and Stastna, 2015) and persisting over time in a wake behind moving bulge of trapped fluid (Terez and Knio, 1998). These subgrid scale structures in our simulations were smashed by numerical diffusion which did not affect larger scale due to use of second order total variation diminishing (TVD) scheme for advective terms in transport equation. "

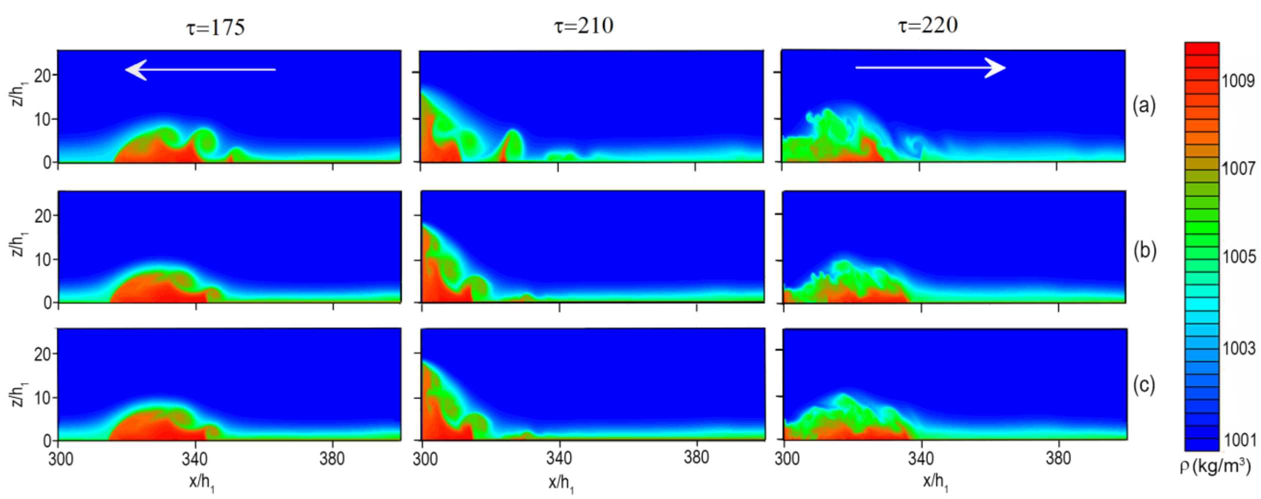

Figure 9. Comparison of the density snapshots during collision of ISWs in case (A13;A13) for different Schmidt numbers. (a) Sc=1. (b) Sc=10. (c) Sc=1000. The right half of the numerical flume is shown due to the symmetry of the interaction process.

The authors need to answer all the above questions. If they cannot they should at least be honest that their results are highly contingent on the degree of pycnocline resolution and the degree of numerical diffusion in their low-order numerical method.

***2.** Page 3, Line 10: The authors discuss at this point the various scaling parameters they use. Later on in the paper, in page 7, there's a discussion as to how such a scaling does not work for the Euler equations. To this end, it would help greatly if the scaled Navier-Stokes eqns. were written out explicitly hereand a warning was given to the reader about potential inapplicability of this finding to the Euler eqns.*

Answer. We included dimensionless NS equations and clarified discussion on complete and incomplete similarity on non-dimensional parameters as you suggested.

p. 3 l.27 "Generally, however, the flow dependence on the viscosity, diffusivity and depth can retain at $\mathrm{Re} \to \infty$, $\mathrm{Sc} \to \infty$ and $\varepsilon \to \infty$ and scaling on them is called incomplete (Barenblatt,1996). In most cases it is impossible to determine the kind of self-similarity *a priori*, until the solution of the full problem. Like Maderich et al. (2015), we follow suggestion by Barenblatt (1996) "assuming in succession complete similarity, incomplete similarity, lack of similarity - and then comparing the relations obtained under each assumption with data from numerical calculations, experiments, or the results of analytic investigations". The simulation results (Maderich et al., 2015), show that the flume depth in the range $23 \le \varepsilon \le 92$ does not affect the characteristics of the ISWs with trapped cores. The sensitivity of the wave dynamics to the values of ε was found by Carr et al., (2008) in the range $4 \le \varepsilon \le 11$. From these studies we conclude that results our simulations in the range $23 \le \varepsilon \le 92$ (Table 1) does not depend on ε. The possible effects of Schmidt and Reynolds numbers will be discussed in sections Sect. 3.4 and Sect.3.6."

p. 8 l. 28 "From dimensional arguments $\Delta E_{loss} = \Phi(\alpha, \mathrm{Re}_m, \mathrm{Sc})$, where Φ is function of three arguments. Assuming complete similarity on the $\mathrm{Re}_m \to \infty$ and $\mathrm{Sc} \to \infty$ consider dependence ΔE_{loss} on α. As seen in Fig. 12, this dependence given for symmetric collisions ($\alpha = \alpha_L = \alpha_R$) is not monotonic and is not universal, changing depending on the series of calculations. "

p. 9 l. 8 "The absence of complete self-similarity on the Reynolds and Schmidt numbers also means that the Euler equations do not describe the wave interaction processes in deep water even for the range of stable waves. As shown in Table 1, the parameter Re_m varies in Series A-C several times for waves of the same dimensionless amplitude α. The incomplete similarity scaling following Barenblatt (1996) results in relation: $\Delta E_{loss} = \Psi(\alpha)\mathrm{Re}_m^m \mathrm{Sc}^n$, where Ψ is function of α, *m* and *n* are exponents. However, this rescaling also did not result in universal dependence. We conclude that it is due to the different mechanisms governing collision process in ranges I-III: nonlinear wave interaction, collapse of collided trapped masses and instability. Another factor influencing the interaction may be the diffusivity effect (Deepwell and Stastna, 2016), which is described by the Schmidt number. However, in these experiments, the Schmidt number was large and constant. "

***3.** Same page, line 19: Correct to "The simulations of interacting ISWs". Now, when one turns to table 1, there is an exhaustive list of simulations, organized in 4 groups, A through D. This is not an easy table to read. Please separate groups A, B, C and D by a space. Also, both in the text of page 3 but also in the figure caption, help the reader out by clearly stating what A, B, C and D represent. Finally, in the caption define what the first 5 parameters are so that the reader doesn't have to flip back and forth to the actual text.*

Answer. We added text to explain difference between groups A-D, and added text to caption and also separated groups A-D in the Table 1.

p. 4 l. 14 "The waves are divided into four groups: (A,B,C) depending on the thickness of the stratified layer and D for simulation of ISW reflection from a vertical wall in the laboratory experiment (Stamp and Jacka, 1995)".
p. 14 Table 1. "Summary of parameters of interacting ISWs: pycnocline thickness parameter *h*, wave amplitude *a*, wavelength $\lambda_{0.5}$, ratio ε, dimensionless ISW amplitude α, Froude number Fr_{max}, minimum Richardson number Ri_{min}, Reynolds number Re_m and class of ISW."

***4.** Same page, line 23: Apparently, the authors are using these runs to double up for both simulations of mode-1 waves with trapped cores, for a near-surface stratification, and mode-2 waves in a two layer* *stratification. The latter assumes perfectly symmetry of the solution around the middle of the pycnocline. Is this a realistic assumption and could it lead to misrepresentation of the actual physics ? How do the authors contrast this approach to that used by Stastna and Deepwell who examine the full domain.*

Answer. We carried out simulations in deep flume ($\varepsilon \gg 1$) when depth of flume effects were weak that allowed using the results of simulation near the bottom as for surface layer as for mode-2 waves assuming symmetry. The simulations by Stastna and Deepwell (2016) for mode-2 waves corresponded value of $\varepsilon = 10$ whereas we carried out simulations in range $23 \le \varepsilon \le 92$. The text was reworked accordingly.

p.4 l. 21 "For large ε, these allow for the simulation of the interaction of mode-1 ISWs with trapped core, propagating in stratified layers near the surface, and the ISWs interaction near the bottom, as considered here, and the interaction of mode-2 ISWs, assuming symmetry in the Boussinesq approximation around the horizontal midplane (Maderich et al., 2015)."

***5.** Same page, line 26: Is the no-flux condition applied to salinity or density ? The authors should clarify what active scalar they actually examine and what type of equation of state they use, if it is salinity they are actually working with.*

Answer. We used salinity stratified water. The text was added accordingly.

p. 2 l. 27 "A free-surface non-hydrostatic numerical model for variable-density flows using the Navier-Stokes equations in the Boussinesq approximation (Kanarska, Maderich, 2003; Maderich et al., 2012) was applied in the simulations of a numerical basin emulating a laboratory flume filled with salinity-stratified water.
p. 3 l. 3 "An equation of state $\rho = \rho(T, S)$ (Mellor, 1991) was used for constant temperature $T = 15^{o}C$."

***6.** How are the initial actual waves generated ? Are they produced by solving the DJL equation and then inserted into the Navier-Stokes solver to allow for the trapped core to actually evolve dynamically ? Alternatively, is some higher-density fluid released at the pycnocline as done by Stastna and Deepwell ?*

Answer. The ISWs were generated at both ends of the flume by the collapse of the mixed regions (see p. 3 bl.5).

***7.** See Comment 1 above: How do we know that the resolution used by the authors is sufficient ? Have grid-independence tests been conducted ? What is the resolution of various critical lengthscales of the problem ? I seriously question the utility of the 3-D runs, at least until the authors are honest about their limitations.*

Answer. See answer to comment 1.

Results

***8.** Page 4, Line 16: The reference to fluid having escaped both trapped cores and then subject to a buoyancy-driven collapse, countered by viscosity and diffusion of mass, raises the question: Are the trapped cores of the original waves subject to any leakage of mass in the first place ?*

Answer. We refined description of the experiment, accordingly.
p. 5 l. 17 "The trapped fluid slowly leaks from rear of trapped bulge similarly to the laboratory experiments (e. g. Maderich et al., 2001; Brandt and Shipley, 2014). However, after collision, the waves lost all fluid trapped by the wave cores."

9. *Page 4, line 29: What is a "small offset pycnocline" ?*

Answer. The text was rewritten accordingly.
p. 6 l.1 "Some mass exchange that occurred in the mode-2 experiment (Stamp and Jacka, 1995) was, perhaps, the result of a slight displacement of the pycnocline in the vertical direction, which is often observed in laboratory experiments (Carr et al., 2015).

10. *Page 5, Line 14 and onward: We suddenly are told that the numerical simulations include runs with internal waves with trapped cores reflecting off a side boundary. See my comment (3) above. Nowhere in section 2 are we told that reflecting internal waves are studied. Pre-dispose the reader please !*

Answer. The run with reflection from side boundary was separated in text and table, accordingly.

p. 4 l. 14 "The parameters of interacting ISWs are given in Table 1. The waves are divided into four groups: (A,B,C) depending on the thickness of the stratified layer and D for simulation of ISW reflection from a vertical wall in the laboratory experiment (Stamp and Jacka, 1995)."

11. *Same page, line 30: Beyond K-H instabilities, are the other mechanisms through which fluid can escape the trapped core ? Consulting Kevin Lamb's two JFM papers (2002 and 2003) might provide some useful insights in this regard.*

Answer. The text was added accordingly.
p. 7 l. 1 "The waves carry out trapped fluid, but the cores gradually lose trapped fluid to the wake through KH billows shifting to the wave rear and through recirculation in trapped core (Terez and Knio, 1998; Maderich et al., 2001; Lamb, 2002). "

12. *Same page, line 33: Can one truly speak of mixing in a 2-D context, when the actual process is turbulent but not resolved in 3-D ? At least qualify the statement by saying that "mixing, as represent in a 2-D context".*

Answer. We eliminated the "mixing" in this sentence. The text was added also to another sentence.
p. 6 l. 10. "Then, the fluid in the cores is entrained by the outgoing waves with some mixing, as represent in a 2-D context, arising due to instability."

13. *Figure 9 and relevant discussion in text: The top four panels need to be magnified by at least a factor of two. Any smaller-scale feature is barely visible and any transverse structure cannot be seen at all. This begs the question once again, how well-resolved are these transverse instabilities ? The authors use 45 spanwise grid-points and it seems that the domain is wide enough to capture about 4 (??) wavelengths thereof. Again, taking into account the numerical diffusion of their method, can we really* *speak of resolving anything below the scale of the transverse instability ? Please see my comment (1). As such, any mention of turbulence and mixing in this section should be made with great caution.*

Answer. We excluded results of 3D simulation from paper. Therefore, Figure 9 was substituted by figure with Sc impact analysis.

14. *Page 7, line 11: More detail is needed as to how ΔE dis is defined. Does one conduct a run of a single wave and measure the energy at the beginning at end of the run, with any losses driven by viscous decay (and apparently numerical diffusion) and shear instability ?*

Answer. We provided more detail on calculation of the energy loss due to the wave collision:

p.8 l.14 We defined the energy loss due to the wave collision (ΔE_{loss}) as the difference between the total loss of energy ΔE_{tot} due to the collision and the loss of energy by two single waves due to the viscous decay or instability ΔE_{dis}

$$\Delta E_{loss} = \Delta E_{tot} - \Delta E_{diss} \qquad (9)$$

The relative loss of energy due to the collision of ISWs can be calculated as the normalized difference in energy of waves before and after collision

$$\Delta E_{tot} = \frac{PSE_L^{(bf)} + PSE_R^{(bf)} - PSE_L^{(af)} - PSE_R^{(af)}}{PSE_L^{(bf)} + PSE_R^{(bf)}} \qquad (10)$$

$$\Delta E_{vis} = \frac{PSE_L^{(bf)} + PSE_R^{(bf)} - \overline{PSE}_L^{(af)} - \overline{PSE}_R^{(af)}}{PSE_L^{(bf)} + PSE_R^{(bf)}} \qquad (11)$$

where $PSE_L^{(bf)}$ and $PSE_R^{(bf)}$ are the pseudo-energies of the waves before collision at the cross-sections x_L and x_R,respectively, and $PSE_L^{(af)}$ and $PSE_R^{(af)}$ are the pseudo-energies of the waves after collision at the cross-sections x_L and x_R, respectively, whereas $\overline{PSE}_L^{(af)}$ and $\overline{PSE}_L^{(af)}$ are the energies of the transmitted waves without interaction at cross sections x_L and x_R, respectively. The pseudo-energy is the sum of the kinetic and available potential energies (Shepherd-1993) of waves before and after collision. The method for estimation of the available potential energy and energy fluxes was given in (Scotti et al., 2006; Lamb, 2007). A detailed description of the procedure of the pseudo-energy calculation was presented by Maderich et al., (2010).

15. *Same page, line 22 and onward: This is a very interesting discussion. However, please see my comment (2) above. Including the actual scaled Navier-Stokes in the text would help the reader understand why this scaling won't apply to the Euler equations. Moreover, the remaining discussion is confusing. Please clarify what is meant by "complete" and "incomplete" similarity. As always, my concern of use of a Schmidt number close to 1,000 arises.*

Answer. See answers to comment 2.

Conclusions

16. *Page 8, line 10: This study also examines mode-1 waves, simply with a near-surface stratification. Clarify that this contrast is made to mode-1 waves in a "two-layer stratification".*

Answer. Done.
p. 9 l.25 "The dependence is similar to the interaction of the mode-1 waves in a two-layer stratification (Terletska et al., 2017), with the difference being that the phase shift continues to grow for the collision of interfacial waves of mode-1.".

17. *Same page, line 15: Again, I doubt that this study resolves any turbulence. What we're seeing is the product of numerical diffusion. Also, correct "monotonous" to "monotonic".*

Answer. The text was changed accordingly.
p.9 l. 29 "The collision of locally shear unstable waves of class (iii) was accompanied by the development of instability."

18. *Trapped cores in internal solitary waves are efficient mechanisms for transporting particulate matter, not just mass (see the work of Lamb). Can the authors at least offer some comment here as to how much collision impacts the capacity for an ISW to transport mass ?*

Answer. We added text accordingly.
p. 9 l. 30 "We conclude that this kind of interaction reduces the capacity for an ISW to transport mass."

19. *It is clear to me that this study examines trapped core waves where the core forms due to near-surface stratification, i.e. one is looking at surface cores. However, the work of Lien et al. clearly observed subsurface cores in the South China Sea ; the localization of the cores in the subsurface orginates from the presence of a background current and the specifics of its vertical structure. Although I see an investigation of ISWs with subsurface cores to be outside of the scope of the particular study, it would help if the authors referenced such phenomena as a topic of future investigation.*

Answer. We mentioned work by Lien et al. (2012) in Introduction. The text was added accordingly.
p.10 l. 9 "The obtained results can be applied to the interaction dynamics of subsurface trapped core formed within a shoaling large amplitude internal waves (Lien et al., 2012)."

Reviewer#2.

The authors are most grateful for your comments. We have followed your suggestions and revised the manuscript accordingly in many places. Please, find our responses below.

I enjoyed reading this paper because the numerical simulations are of high quality, the experimental design is well devised, and the results yield interesting insight into the behavior of colliding nonlinear internal solitary waves with trapped cores. I have one suggestion for major revision although this won't require too much work, and some suggestions for clarification:

1) *My only suggestion for revision is that the authors remove the three-dimensional results and discussion of the mixing, dissipation, and energetics. I would only trust discussion of these if the authors demonstrated that they are truly resolved through discussion of grid resolution requirements for DNS, i.e. grid resolution via Kolmogorov scale. It is hard to imagine that the mixing is resolved given that the molecular diffusion is so small. In fact, unresolved two-dimensional simulations can lead to more mixing because the inverse energy cascade in two dimensions stretches density filaments and leads to more numerical mixing, even if the dissipation is actually lower in two dimensions (see Fringer and Street 2003; doi:10.1017/S0022112003006189, Arthur and Fringer, 2014; doi:10.1017/jfm.2014.641). An additional problem with discussion of the energetics in the paper is that the Reynolds number varies significantly for different runs. Arthur and Fringer (2014) showed that not accounting for Reynolds number effects can give a very different picture of the dynamics of breaking internal solitary waves on slopes. Such may be the case for the results in Figure 12, for which it is difficult to determine whether the behavior of the energy loss is due to alpha effects or Reynolds number effects. It may be that the two-dimensional simulations represent the energetics to a reasonable degree, as in many studies of internal wave energetics, although I would not necessarily trust the arguments concerning the mixing. Either way, I suggest that the authors discuss the three-dimensional effects and associated energetics in a different paper.*

Answer. We have revised the text accordingly:
(i) Results of 3D simulation were excluded from this manuscript;
(ii) The effect of Re_m was discussed in text;

p. 9 l. 8 "The absence of complete self-similarity on the Reynolds and Schmidt numbers also means that the Euler equations do not describe the wave interaction processes in deep water even for the range of stable waves. As shown in Table 1, the parameter Re_m varies in Series A-C several times for waves of the same dimensionless amplitude α. The incomplete similarity scaling following Barenblatt (1996) results in relation: $\Delta E_{loss} = \Psi(\alpha)\mathrm{Re}_m^m \mathrm{Sc}^n$, where Ψ is function of α, *m* and *n* are exponents. However, this rescaling also did not result in universal dependence. We conclude that it is due to the different mechanisms governing collision process in ranges I-III: nonlinear wave interaction, collapse of collided trapped masses and instability. Another factor influencing the interaction may be the diffusivity effect (Deepwell and Stastna, 2016), which is described by the Schmidt number. However, in these experiments, the Schmidt number was large and constant. "
(iii) Text was added on limitations using 2D setting:

p. 10 l.8 “Notice, however, that the destruction of the KH billows is essentially three-dimensional process, therefore, 3D high-resolution simulation is necessary to predict turbulence development (Arthur and Fringer,2014, Deepwell and Stasna, 2016). This is the subject of a separate study, whereas the interaction of the colliding waves as a whole can be described in 2D setting.”

2) *Please discuss how you chose the grid resolution for the two-dimensional simulations.*

Answer. We carried out doubling-grid tests to verify that chosen grid adequately described flow fields. The comparison for wave A13 is show in Figs. 1 and 2. The text was added accordingly.

p. 4 l. 25 “Most of the runs were performed in a two-dimensional setting with a grid resolution of 3000×400 (length and height, respectively), whereas several runs for waves A9-A13 were also carried out with a grid resolution of 6000×800 (length and height, respectively) to verify effect of grid resolution on the wave interaction and to make the fine structure clearer. Comparison of the baseline and doubled grid resolution showed the equivalence of the calculated fields, with the exception of wave A13 for which 6000×800 resolution was used.”

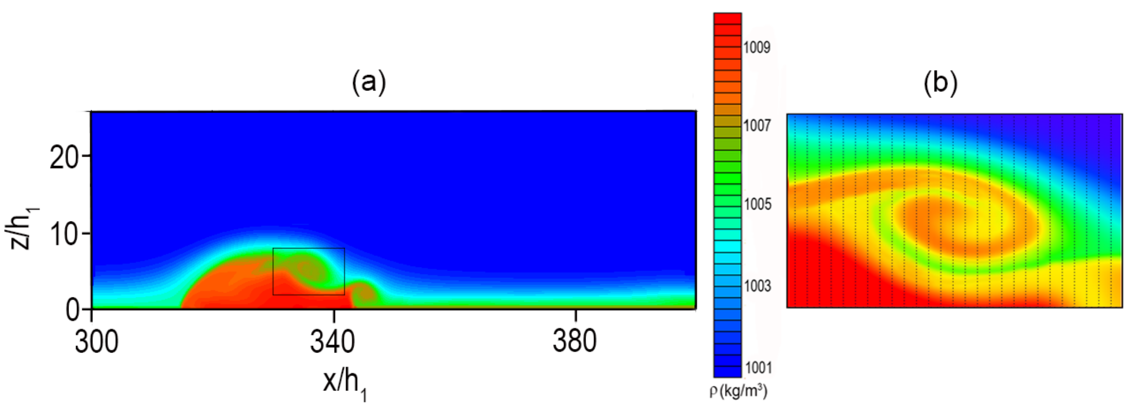

Fig. A1 Snapshot of the density field for case (A13;A13) at $\tau = 175$ and Sc=1000 for grid resolution 3000x400 (a) and extended snapshot of KH billow with grid points (b).

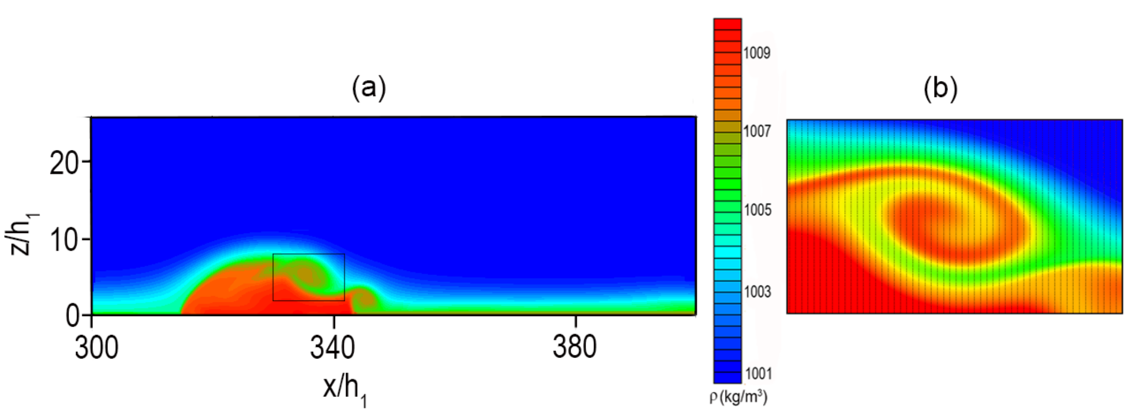

Fig. A2 Snapshot of the density field for case (A13;A13) at $\tau = 175$ and Sc=1000 for grid resolution 6000x800 (a) and extended snapshot of KH billow with grid points (b).

3) The Richardson number should be defined as $Ri_m = g'h/(U_m 2)$ so that it is consistent with the way the other nondimensional parameters are defined, i.e. in terms of the independent parameters following the Buckingham Pi theorem.

Answer. While there are various estimates of Froude, Richardson and Reynolds numbers, we found (Maderich etal., 2015) that definitions (5)-(7) allow one to adequately classify the state of colliding waves of large amplitude using local characteristics such as minimum Richardson number or the ratio of maximal local velocity to the ISW phase velocity.

p.4 l. 5 "The important features of the ISWs can be described by dimensionless amplitude is $\alpha = a/h$, the Froude, Richardson and Reynolds numbers based on local characteristics of waves (Maderich et al.,2015)."

4) *It would be helpful if, on page 3, you discussed the general features of Series A-D, and included a brief description in another column in Table 1, i.e. a column indicated by "Comments" which, for series A would state, "No trapped cores". Also please indicate whether the waves were in regimes (i), (ii), or (iii) in Table 1.*

Answer. We added column with class of colliding waves. In accordance with the definition of the class of the ISW, waves with trapped cores belong to the classes (ii) and (iii).

5) *What is the justification for choosing such a small molecular diffusion?*

Answer. Text and figure were added to consider the impact of small diffusivity on the collision processes.

p.7 l. 8 "In the ocean and in the most of the laboratory experiments the Schmidt number is about 700-800. The used grid does not allow the whole range of inhomogeneities in salinity (density) to be resolved. Therefore, it is important to evaluate the effect of molecular diffusion of salinity on the dynamics of waves and to verify the possibility that diffusion can be neglected in the wave collision for large Sc. Two cases for large amplitude waves were considered (A9;A9) and (A13;A13). We performed runs for Sc=1; 10 and 1000. In the collision case (A9;A9) the behaviour of colliding waves are the same, whereas the difference between runs for Sc=1 and Sc=1000 was less than 1% of $\Delta\alpha/\alpha$ and $\Delta\theta$ values. The comparison of the density snapshots during collision in case (A13;A13) for different Schmidt numbers is shown in Fig. 9. Figure clearly depicts difference between structure of interacting waves for cases Sc=1 and Sc=10. The corresponding values of $\Delta\alpha/\alpha$ and $\Delta\theta$ differ by 5% and 0.6%, respectively. This was in agreement with the results by Deepwell and Stastna (2016), where it was shown essential effect of molecular diffusivity on the mass transport by mode-2 ISW in range $1 \le Sc < 20$. At the same time, the results of calculations at Sc=10 and Sc=1000 in Fig.9b and 9c practically coincide, which indicates that molecular diffusion may not be taken into account when studying the global properties of colliding waves. This conclusion agrees with (Terez and Knio, 1998) as they estimate that the value of Sc=100 was "sufficiently high for density diffusion to be ignored during simulation period" and the results of the Deepwell and Stastna (2016) simulation, according to which the mass transfer is virtually independent of Sc already at Sc>20. However, diffusion can be important for small scale mixing processes in tiny density structures (see e.g. Galaktionov et al., 2001) forming in result of instability and turbulent cascade processes (Deepwell and Stastna, 2015) and persisting over time in a wake behind moving bulge of trapped fluid (Terez and Knio, 1998). These subgrid scale structures in our simulations were smashed by numerical diffusion which did not affect larger scale due to use of second order total variation diminishing (TVD) scheme for advective terms in transport equation.“

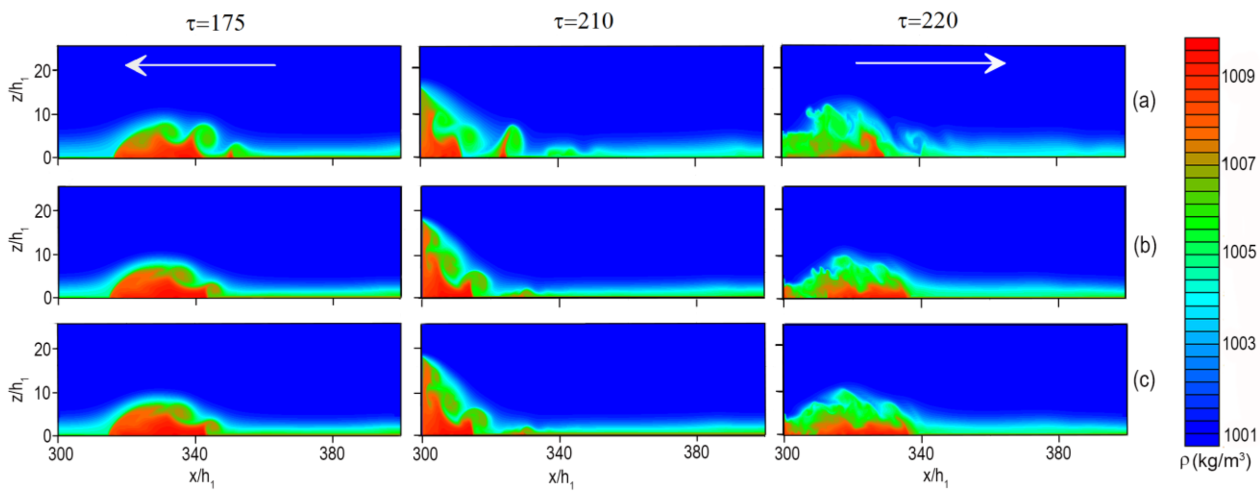

Figure 9. Comparison of the density snapshots during collision of ISWs in case (A13;A13) for different Schmidt numbers. (a) Sc=1. (b) Sc=10. (c) Sc=1000. The right half of the numerical flume is shown due to the symmetry of the interaction process.

6) *Page 4, Line 9: Please explain the meaning of and how you computed the phase shift* $\Delta\theta$ *, and how it is normalized by* τ_0 *.*

Answer. We estimated temporal phase shift by comparing trajectories of the wave crests with and without collision. This temporal phase shift was normalized to characteristic time

p.5 l. 9 "…whereas normalized to characteristic time τ_0 temporal phase shift $\Delta\theta$ is $\Delta\theta \sim \alpha$."
p.5 l. 10 "We estimated temporal phase shift by comparing trajectories of the wave crests with and without collision."

7) *Page 4, Line 10: Please explain how you expect Da/a and Dq to behave for limiting cases (* $\alpha \to 0$ *and* $\alpha \to \infty$ *). Why does* $\Delta\theta \to 4$ *as* $\alpha \to \infty$ *?*

Answer. We added text with explanations.

p. 6 l. 12 "As shown in Fig. 4a, for stable waves of class (ii), the runup excess $\Delta\alpha/\alpha$ still almost linearly increases in the range $2.3 < \alpha < 4.6$, whereas the increase in the phase shift $\Delta\theta$ is substantially slowed down when $\alpha > 1$, and then $\Delta\theta$ tends towards a constant value $\alpha > 4$. The distributions of $\Delta\alpha/\alpha$ and $\Delta\theta$ in Fig. 4 for stable waves were approximated by linear and exponential dependencies, respectively, which were based on the weakly-nonlinear asymptotics $\Delta\alpha/\alpha \sim \alpha$ and $\Delta\theta \sim \alpha$ (Matsuno,1998) for small α and obtained in numerical experiments almost constant distribution of $\Delta\theta$ at large α. "

8) *On Page 5, Line 5, you state that the colliding waves pass through each other. Theory suggests that nonlinear waves exchange momentum by bouncing off each other, just like billiard balls (e.g. Fringer and Holm 2001; doi:10.1016/S0167-2789(00)00215-3).*

Answer. The collision of large amplitude ISWs with trapped cores is complicated process, which in theory has not yet been described in detail, in particular for waves of different amplitude. To avoid misinterpretation of results we changed “transmitted” waves to “outgoing” waves.

9) *Please do not include the regressions on Page 5, line 13, unless you can justify the functional relationships through scaling or other means.*

Answer. See answer to comment 7)

10) *Minor: a. I don’t understand the meaning of the sentence starting with “The waves of class (ii) …” on line 8 of page 1. b. Throughout: monotonous à monotonic.*

Answer. The text was changed accordingly.

p. 1 l. 9 “The colliding waves of class (ii) lose fluid trapped by the wave cores when a normalized on thickness of pycnocline amplitudes are in the range of approximately between 1 and 1.75.”

Head-on collision of internal waves with trapped cores

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Abstract. The dynamics and energetics of a head-on collision of internal solitary waves (ISWs) with trapped cores propagating in thin pycnocline were studied numerically within the framework of the Navier-Stokes equations for a stratified fluid. The peculiarity of this collision is that it involves the trapped masses of a fluid. The interaction of ISWs differs for three classes of ISWs: (i) weakly nonlinear waves without trapped cores, (ii) stable strongly nonlinear waves with trapped cores, and (iii) shear unstable strongly nonlinear waves. The wave phase shift of the equal colliding waves grows as the amplitudes ~~of the interacting waves~~ increase for colliding waves of classes (i) and (ii) and remains almost constant for those of class (iii). The normalized to amplitude of waves excess of the maximum runup amplitude over the sum of the amplitudes of the equal colliding waves almost linearly increases with increasing amplitude of the interacting waves belonging to classes (i) and (ii); however, it decreases somewhat for those of class (iii). The colliding waves of class (ii) ~~with a normalized on thickness of pycnocline amplitude~~ lose fluid trapped by the wave cores when a normalized on thickness of pycnocline amplitudes are in the range of approximately between 1 and 1.75. The interacting stable waves of higher amplitude capture cores and carry trapped fluid in opposite directions with little mass loss. The collision of locally shear unstable waves of class (iii) is accompanied by the development of ~~three-dimensional instabilityand turbulence~~instability. The dependence of loss of energy on the wave amplitude is not ~~monotonous~~monotonic. Initially, the energy loss due to the interaction increases as the wave amplitude increases. Then, the energy losses reach a maximum due to the loss of potential energy of the cores upon collision and then start to decrease. With further amplitude growth, collision is accompanied by the development of instability and an increase in the loss of energy. The collision process is modified for waves of different amplitudes because of the exchange of trapped fluid between colliding waves due to the conservation of momentum.

1 Introduction

Internal solitary waves (ISWs) are widespread in stratified oceans and lakes (Helfrich and Melville, 2006). The observed ISWs are mostly waves of mode-1 propagated as waves of depression or as waves of elevation. When near-surface or near-bottom layers are stratified, then mode-1 ISWs of large amplitude can trap and transport fluid in their cores, as observed in the ocean (Moum et al., 1990; Lien et al., 2012; Klymak et al., 2003; Scotti and Pineda, 2004) and in the atmospheric boundary layer (the phenomenon known as “the morning glory” (Christie, 1992; Reeder et al., 1995)). These waves have been studied in laboratory experiments (Grue et al., 2000; Carr et al., 2008; Luzzatto-Fegiz and Helfrich, 2014). The fluid can also be trapped by ISWs of mode-2 (Yang et al., 2010; Shroyer et al., 2010; Ramp et al., 2015) propagating in the thin interface layer between two uniform density layers, as has been shown in laboratory experiments (e.g., Davis and Acrivos (1967); Maxworthy (1980); Kao and Pao (1980); Honji et al. (1995); Stamp and Jacka (1995); Maderich et al. (2001); Brandt and Shipley (2014); Carr et al. (2015)). The weakly nonlinear solution for the corresponding ISW (i.e., the Benjamin-Ono (BO) soliton (Benjamin, 1967; Ono, 1974)) agrees well with experimental data for a small amplitude ISW without mass transport. However, experiments and ~~solutions both of~~ numerical solutions of both the Dubreil-Jacotin-Long (DJL) equation and ~~within the framework of the~~ the actual Navier-Stokes equations (Lamb, 2002; Helfrich and White, 2010; Lamb and Farmer, 2011; Salloum et al., 2012; Maderich et al., 2015; Luzzatto-Fegiz and Helfrich, 2014; Deepwell and Stastna, 2016) show that BO solitons cannot even qualitatively describe the dynamics and transport features of large amplitude waves. A detailed review of laboratory and numerical experiments is given by ~~(Maderich et al., 2015).~~ Maderich et al. (2015).

Little is known regarding the interaction of waves with trapped cores. This kind of interaction is of special interest, as the masses of fluid trapped by waves are also involved in the interaction. The oblique interaction of "morning glories" over northern Australia was described by Reeder et al. (1995). Head-on collision as a special case of oblique interactions was considered by Matsuno (1998) using a second-order analysis of small-amplitude interfacial waves in deep fluid. A mostly qualitative description of the head-on collision of mode-2 waves with trapped cores, obtained through conducting several laboratory experiments (Kao and Pao, 1980; Honji et al., 1995; Stamp and Jacka, 1995) and via a numerical simulation (Terez and Knio, 1998), has been given. These experiments showed that (i) waves experience a phase shift during collision, (ii) large amplitude waves ~~in the interaction process exchange~~ transport trapped masses of ~~fluids between incident waves~~ fluid after collision, (iii) waves of unequal amplitude exchange masses of trapped fluid in the interaction process, and (~~iii~~iv) some trapped fluid is ejected upon collision. According to Gear and Grimshaw (1984), interaction processes can be distinguished as strong interactions when waves propagate in almost the same direction and the time of interaction is relatively long and as weak interaction when waves propagate in almost opposite directions and the time of interaction is relatively short. However, a numerical study of both overtaking and head-on collisions of large amplitude mode-1 and weakly nonlinear mode-2 ISWs Stastna et al. (2015) showed that these interactions are strong interactions resulting in the degeneration of mode-2 ISWs. In this paper, the dynamics and energetics of a head-on collision of ISWs with trapped cores for a wide range of amplitudes and stratifications are studied numerically within the framework of the Navier-Stokes equations. The paper is organized as follows: the numerical flume setup is described in section 2, the results of experiments on the collision of waves of equal and non-equal amplitudes are discussed in section 3, and the main results are summarized in section 4.

2 The numerical model setup

A free-surface non-hydrostatic numerical model for variable-density flows using the Navier-Stokes equations in the Boussinesq approximation (Kanarska and Maderich, 2003; Maderich et al., 2012) was applied in the simulations of a numerical flume emulating a laboratory ~~flume~~ basin filled with salinity-stratified water. The numerical flume and experimental configurations are shown in Fig. 1. ~~Here, (x,y,z)~~ The simulations were performed in a two-dimensional setting where (x,y) are the Cartesian coordinates in the longitudinal ~~, transverse~~ and vertical directions, respectively. The vertical coordinate z is directed upward.

The flume has a length $L_x = 3$ m ~~,~~ and a depth H=0.46 m~~and a width $L_y = 0.5$ m~~. It was filled with salinity S stratified water, in which the density of the upper layer is ρ_1, and with a thin pycnocline near the bottom, expressed ~~a~~ as

$$\rho(z) = \rho_0 \left(1 - \frac{\Delta\rho}{\rho_0} \tanh\left(\frac{z}{h}\right)\right), \tag{1}$$

where ρ_0 is density at the bottom, $\Delta\rho = \rho_0 - \rho_1$ is the density difference between the bottom and the surface, and h is ~~the pycnocline layer thickness.~~ a parameter of the pycnocline. An equation of state $\rho = \rho(S,T)$ (Mellor, 1991) was used for constant temperature $T = 15°C$.

The ISWs were generated at both ends of the flume by the collapse of the mixed regions with the density ρ_0. Following Maderich et al. (2015), the shape of the mixed region was selected to be half of a BO soliton to reduce the mixing due to the collapse. The wave amplitude varied according to the initial volume of the mixed fluid and thickness of the pycnocline h. The kinematic viscosity ν was 1.14×10^{-6} m^{2}s^{-1}, and the molecular diffusion χ was 10^{-9} m^{2}s^{-1}.

An ISW is characterized by an amplitude a, which represents the maximum displacement of the isopycnal (Fig. 1), a wave speed U_c, calculated as the velocity of the wave crest, and a wavelength $\lambda_{0.5}$, estimated to be a half-width with which the amplitude of the wave is reduced by half. The maximal speed of the wave is U_m. The wave parameters were evaluated in the sections x_L=x_R=0.5 m away from the centre of the laboratory tank x_C. For example, the amplitudes of waves propagating from the left to the right in the cross-sections x_L and x_R are defined as $a_L(x_L)$ and $a_L(x_R)$, while those propagating from the right to the left are defined as $a_R(x_L)$ and $a_R(x_R)$, respectively.

The simulation results are presented in dimensionless form. The coordinates $x_i = (x,z)$ are normalized to h, and the time $\tau = t/\tau_0$ is dimensionless, where t is time, $\tau_0 = \sqrt{2\rho_0 h/\Delta\rho g}$, and g is gravity; the velocity ~~$U = (U,V,W)$~~ $u_i = (u,w)$ is normalized to the long linear wave phase velocity ~~$c_0 = \sqrt{gh\Delta\rho/2\rho_0}$ (Benjamin, 1967). The important dimensionless parameters characterizing the waves are the ratio $\varepsilon = H/h$ and dimensionless amplitude~~ $C_0 = \sqrt{gh\Delta\rho/2\rho_0}$ (Benjamin, 1967); p is pressure deviation in the Boussinesq approximation normalized to the $\rho_0 gh/2$; $\rho' = (\rho - \rho_0)/\rho_0$ is non-dimensional density deviation. The governing Navier-Stokes equations for stratified fluid in non-dimensional form are written in the Boussinesq approximation as

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{2}$$

$$\frac{\partial u_i}{\partial \tau} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} - \frac{1}{2}\rho' \delta_{i,3} + \frac{1}{\mathrm{Re}} \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{3}$$

$$\frac{\partial \rho'}{\partial \tau} + u_j \frac{\partial \rho'}{\partial x_j} = \frac{1}{\mathrm{ReSc}} \frac{\partial^2 \rho'}{\partial x_j \partial x_j}, \tag{4}$$

where δ_{ij} is the Kronecker delta, Re=$C_0 h/\nu$ is the Reynolds number based on linear theory, $\mathrm{Sc} = \nu/\chi$ is the Schmidt number. The effect of the height of the computational tank on the ISW propagation is described by parameter $\varepsilon = H/h$. It can be assumed that the results of experiments and simulation for small viscosity (Re$\to \infty$), small diffusivity for water (Sc$\to \infty$) and for deep water ($\varepsilon \to \infty$) will not depend on the viscosity, diffusivity and the depth. That case is referred by Barenblatt (1996) as a complete similarity with the parameters Re, Sc and ε. Generally, however, the flow dependence on the viscosity, diffusivity and depth can retain at Re$\rightarrow\infty$, Sc$\rightarrow\infty$ and $\epsilon\rightarrow\infty$ and scaling on them is called incomplete (Barenblatt, 1996). In most cases it is impossible to determine the kind of self-similarity *a priori*, until the solution of the full problem. Like (Maderich et al., 2015), we follow suggestion by Barenblatt (1996) "assuming in succession complete similarity, incomplete similarity, lack of similarity - and then comparing the relations obtained under each assumption with data from numerical calculations, experiments, or the results of analytic investigations". The simulation results (Maderich et al., 2015) show that the flume depth in the range $23 \lesssim \varepsilon \lesssim 92$ does not affect the characteristics of the ISWs with trapped cores. The sensitivity of the wave dynamics to the values of ε was found by Carr et al. (2008) in the range $4 \lesssim \varepsilon \lesssim 11$. From these studies we conclude that results of our simulations in the range $23 \lesssim \varepsilon \lesssim 92$ (Table 1) do not depend on ε. The possible effects of Schmidt and Reynolds numbers will be discussed in sections 3.4 and 3.6.

The important features of the ISWs can be described by dimensionless amplitude is $\alpha = a/h$, the Froude, Richardson and Reynolds numbers based on local characteristics of waves (Maderich et al., 2015). The Froude number $\mathrm{Fr}_{\max}$ is defined as the ratio of the maximum local velocity U_m to the phase velocity U_c:

$$\mathrm{Fr}_{\max} = \frac{U_m}{U_c}. \tag{5}$$

The shear stability of an ISW can be described by the minimum Richardson number $\mathrm{Ri}_{\min}$ calculated for a wave crest:

$$\mathrm{Ri}_{\min} = \frac{g}{\rho_0}\frac{\partial\rho}{\partial z} \Bigg/ \left(\frac{\partial U}{\partial z}\right)^2, \tag{6}$$

where $\rho(x,y,z,t)$ is the density~~. The wave~~ , U is longitudional component of velocity. The ISW Reynolds number is defined as

$$\mathrm{Re_m} = \frac{U_m(a)}{\nu}. \tag{7}$$

The parameters of interacting ISWs are given in Table 1. ~~They~~ The waves are divided into four groups: (A,B,C) depending on the thickness of the stratified layer and D for simulation of ISW reflection from a vertical wall in the laboratory experiment (Stamp and Jacka, 1995). The waves in Table 1 can be categorized according to the values of the parameters $\mathrm{Fr}_{\max}$ and $\mathrm{Ri}_{\min}$ (Maderich et al., 2015) into three classes: (i) the weakly nonlinear waves without trapped cores at 1<$\mathrm{Ri}_{\min}$, $\mathrm{Fr}_{\max}$ <1; (ii) the stable strongly nonlinear waves with trapped cores at $0.15 < \mathrm{Ri}_{\min} < 1$, $1 < \mathrm{Fr}_{\max} < 1.3$; (iii) the unstable strongly nonlinear waves with trapped cores at $\mathrm{Ri}_{\min}$<0.1; $\mathrm{Fr}_{\max} \approx 1.35$. The boundary conditions on the surface include the kinematic condition for the free surface. At the ~~lateral and bottom~~ rest of boundaries, the free-slip conditions are used. No flux condition for density deviation was applied at all boundaries. For large ε, ~~this allows~~ these allow for the simulation of the interaction of mode-1 ISWs with trapped core, propagating in stratified layers near the surface and the ISWs interaction near the bottom, as considered here, and the interaction of mode-2 ISWs, assuming symmetry in the Boussinesq approximation around the horizontal midplane (Maderich et al., 2015). ~~No flux condition for salinity was applied at all boundaries. The~~ The numerics of model is described in detail in (Kanarska and Maderich, 2003; Maderich et al., 2012). A total of ~~35~~ 40 runs were performed in Series A-D. Most of the runs were performed ~~in a two-dimensional setting~~ with a grid resolution of 3000×400 (length and height, respectively), whereas several ~~three-dimensional runs were~~ runs (waves A11-A13) were also carried out with a grid resolution of ~~$2000 \times 200 \times 45$ (length, height and width, respectively)~~6000×800 to verify effect of grid resolution on the wave interaction and to make the finest structure clearer. Comparison of the main and doubled grid resolution showed the equivalence of the calculated fields, with the exception of wave A13 for which 6000×800 resolution was used.

3 Results and discussion

3.1 Interaction of waves of equal amplitudes without trapped cores

The interaction of ISWs of equal amplitude $\alpha_L = \alpha_R = 0.81$ (case (A2; A2)) is shown in Fig. 2a. These waves belong to the class of weakly nonlinear waves without trapped cores ($\mathrm{Fr}_{\max}$=0.71, $\mathrm{Ri}_{\min}$ = 52). After a collision, waves retain their profile and lose amplitude mainly due to the viscous effects. Before and after collision, the wave profiles are similar to those of the weakly nonlinear BO solitons even if the wave amplitudes are not small (Fig. 3). The collision for this class of waves is not fully elastic~~, as seen in Fig. 4, where the relative excess~~. For two layer stratification, in which the outer layer is assumed to be infinite deep, a weakly-nonlinear theory (Matsuno, 1998) predicts excess $\Delta\alpha$ of the maximum runup ~~amplitude α_m~~ α_m over the sum of the amplitudes of equal interacting waves ~~$\Delta\alpha$ (runup excess) and the phase shift $\Delta\theta$ are plotted versus α. Here, $\alpha = \alpha_L = \alpha_R$, $\Delta\alpha = \alpha_m/\alpha - 2$ and $\Delta\theta$ is the phase shift normalized at the characteristic time scale τ_0~~α as $\Delta\alpha \sim \alpha^2$ whereas normalized to characteristic time τ_0 temporal phase shift $\Delta\theta$ is $\Delta\theta \sim \alpha$. The presence of a phase shift due to the collision of mode-2 waves for $\alpha = 0.98$ was confirmed in a laboratory experiment (Honji et al., 1995). We estimated temporal phase shift by comparing trajectories of the wave crests with and without collision. As seen in Fig. 4 the relative runup excess ~~$\Delta\alpha$ and~~ $\Delta\alpha/\alpha$ and normalized temporal phase shift $\Delta\theta$ increase as the interacting wave amplitude α increases at small and moderate α as weakly nonlinear theory predicts.

3.2 Interaction of waves with a trapped core and moderate amplitude

The head-on collision between ISWs of equal moderate amplitude with trapped cores $\alpha = \alpha_L = \alpha_R = 1.6$ (case (A5; A5)) is characterized by special features, as seen in Fig. 2b. These waves, belonging to class (ii), i.e., stable strongly nonlinear waves with trapped cores ($\mathrm{Fr}_{\max}$=1.11, $\mathrm{Ri}_{\min}$=1.1), carried fluid in the cores before collision. The trapped fluid slowly leaks from rear of trapped bulge similarly to the laboratory experiments (e. g. Maderich et al., 2001; Brandt and Shipley, 2014). However, after collision, the waves lost all fluid trapped by the wave cores. This fluid slowly collapsed in the viscous and diffusive-viscous regimes (Galaktionov et al., 2001). The profile of the incident wave at $\alpha = 1.6$ (as well as other characteristics (Maderich et al., 2015)) essentially differs from the predictions made by using the weakly nonlinear theory (Fig. 5a). The amplitudes of ~~transmitted~~ interacting waves ($\mathrm{Fr}_{\max}$=1.0, $\mathrm{Ri}_{\min}$ =1.2) decrease after collision. They propagate as weakly nonlinear BO solitons (Fig. 5b). This kind of head-on collision occurs in the range of approximately $1 \leq \alpha \leq 1.6$. Notice that in the numerical study Terez and Knio (1998), the wave lost trapped fluid in the process of interaction even at $\alpha = 2.1$. As shown in Fig. 4, the normalized excess of the maximum amplitude $\Delta\alpha$ almost linearly increases in the range $1 \leq \alpha \leq 2$, whereas the increase in the phase shift $\Delta\theta$ slows down.

3.3 Interaction of internal waves with stable trapped cores

The large amplitude ISWs with $1.2 \lesssim \mathrm{Fr}_{\max} \lesssim 1.3$ and $0.15 \lesssim \mathrm{Ri}_{\min} < 1$ are characterized by stable long-lived cores. Fig. 6a shows the collision of waves with equal amplitude $\alpha = \alpha_L = \alpha_R = 3.3$ (case (A9; A9)) with the parameters $\mathrm{Fr}_{\max}$=1.28 and $\mathrm{Ri}_{\min}$=0.25. As seen in the figure, the volumes of dyed fluid in the trapped core collide together with the waves. The cores did not mix during the collision, which was also observed in a laboratory experiment (Honji et al., 1995). Then, the ~~transmitted~~ outgoing waves captured the cores and carried the dyed fluid in the opposite directions with little mass loss. Some mass exchange between waves that occurred in the mode-2 experiment (Stamp and Jacka, 1995) was, perhaps, the result of a ~~small offset pycnocline~~ slight displacement of the pycnocline in the vertical direction, which is often observed in laboratory experiments (Carr et al., 2015).

The interaction process is described in Fig. 7 in more detail. Here, the velocity and vorticity fields are shown together with an isopycnal distribution. At the beginning of collision (Fig. 7a), the trapped cores almost touch. They form a pair of vortices carrying trapped fluid upward. The next snapshot (b) corresponds to the time when the potential energy of the interacting waves reaches a maximum and the kinetic energy reaches a minimum. Unlike waves of class (i), at this moment in time, the kinetic energy of the waves does not vanish because the flows in the trapped cores change sign ~~when the colliding waves pass through each other~~due to the collision. This process is also different from the process of the formation of waves with captured cores due to the collapse of the mixed region, which was initially in a state of rest. Then, the fluid in the cores is entrained by the ~~transmitted~~ outgoing waves with some mixing, as represent in a 2-D context, arising due to instability~~, resulting~~ . This results in the slight loss of mass from the trapped cores and a decrease in the phase velocity of 8 % (Figs. 7c and 6a). As shown in Fig. ~~4~~4a, for stable waves of class (ii), the runup excess ~~$\Delta\alpha$~~ $\Delta\alpha/\alpha$ still almost linearly increases in the range $2.3 \leq \alpha \leq 4.6$, whereas the increase in the phase shift $\Delta\theta$ is substantially slowed down when $\alpha > 1$, and then $\Delta\theta$ tends towards a constant value at $\alpha \geq 4$. The distributions of $\Delta\alpha$ and $\Delta\theta$ in Fig. 4 for stable waves were approximated ~~based on the~~ by linear and exponential ~~dependences~~dependencies, respectively, ~~as~~ which were based on the weakly-nonlinear asymptotics $\Delta\alpha/\alpha \sim \alpha$ and $\Delta\theta \sim \alpha$ (Matsuno, 1998) for small α and obtained in numerical experiments almost constant distribution of $\Delta\theta$ at large α. These fitted curves are

$$\Delta\alpha = 0.116\alpha, \quad \Delta\theta = 4.1[1 - \exp(-1.33\alpha)]. \tag{8}$$

The behaviour of mode-2 ISWs during reflection off a solid vertical wall is similar to that of the collision of two waves of equal amplitude. A comparison with the simulated reflection of ISWs off a vertical wall (case D1) in a laboratory experiment (Stamp and Jacka, 1995) is given in Fig. 8. The parameters of the experiment were as follows: density difference $\Delta\rho/\rho_0$=0.05, pycnocline thickness $h = 0.0025$m and $\alpha = 2.2$. The incident wave belongs to the class (ii) ~~ISWs (ii)~~of ISWs: $\mathrm{Fr}_{\max}$=1.18, $\mathrm{Ri}_{\min}$=1.05. The calculated density isopycnals in a vertical cross-section along the flume at $t = 16$s in Fig. 8a agree with the density isopycnals visualized in the experiment by water insoluble droplets of different densities in Fig. 8b. The interaction process is similar to that shown in Fig. 6a, where after collision, some instability and mixing are observed in the rear of the trapped core. In simulated case D1, the corresponding values of $\mathrm{Fr}_{\max}$ and $\mathrm{Ri}_{\min}$ after reflection are 1.1 and 1.21, respectively. The simulated and observed trajectories of the wave crests, as shown in Fig. 8c, are similar. The corresponding simulated runup excess $\Delta\alpha$=0.28 and phase shift $\Delta\theta$= 3.8. These values agree with the other values of $\Delta\alpha$ and $\Delta\theta$ in Fig. 4. The experimentally observed (Stamp and Jacka, 1995) phase shift values are also given in Fig. 4b. They demonstrate large scatter due to difficulties encountered in the experiment, as indicated by (Stamp and Jacka, 1995).

3.4 Interaction of internal waves with unstable trapped cores

The large amplitude ISWs with $\mathrm{Fr}_{\max} \approx 1.3$ and $\mathrm{Ri}_{\min} \lesssim 0.1$ belong to class (iii), which is characterized by a local wave-induced shear instability resulting in the appearance of the Kelvin-Helmholtz (KH) billows (Maderich et al., 2015); globally, however, this wave/self-generated shear system can be stable, as noted by (Almgren et al., 2012). The waves carry out trapped fluid, but the cores gradually lose trapped fluid to the wake through KH billows shifting to the wave rear and through recirculation in trapped core (Terez and Knio, 1998; Maderich et al., 2001; Lamb, 2002). Fig. 6b shows the collision of waves with equal amplitude $\alpha = \alpha_L = \alpha_R = 6.4$ (case (A13; A13), with the parameters $\mathrm{Fr}_{\max}$=1.31 and $\mathrm{Ri}_{\min}$ =0.06)~~for a 2D setting~~. Unlike that shown in Fig. 6a (case (A9; A9)), the collision of trapped cores was accompanied by billows, resulting in mixing. The divergent waves remained locally unstable, again forming KH billows in the wave aft. The amplitude of diverging waves gradually decreased due to the loss of mass of the trapped cores. ~~The mixing process caused by the destruction of the KH billows is essentially three-dimensional; therefore, 3D structures should be important for the transport of trapped cores (Deepwell and Stastna, 2016). The wave collision in a 3D setting for the same case (~~

In the ocean and in the most of the laboratory experiments the Schmidt number is about 700-800. The used grid does not allow the whole range of inhomogeneities in salinity (density) to be resolved. Therefore, it is important to evaluate the effect of molecular diffusion of salinity on the dynamics of waves and to verify the possibility that diffusion can be neglected in the wave collision for large Sc. Two cases for large amplitude waves were considered (A9; A9) and (A13; A13)~~is shown in Fig.~~ . We performed runs for Sc=1; 10 and 1000. In the collision case (A9;A9) the behaviour of colliding waves are the same, whereas the difference between runs for Sc=1 and Sc=1000 was less than 1% of $\Delta\alpha/\alpha$ and $\Delta\theta$ values. The comparison of the density snapshots during collision in case (A13; A13) for different Schmidt numbers is shown in Fig. 9. ~~The interacting ISWs are visualized by a density isosurface with ρ~~Figure clearly depicts difference between structure of interacting waves for cases Sc=~~1005 $kg\ m^{-3}$ at different times. In the side plans, the distributions of the density are shown. Initially,~~ 1 and Sc=10. The corresponding values of $\Delta\alpha/\alpha$ and $\Delta\theta$ differ by 5% and 0.6%, respectively. This was in agreement with the results by Deepwell and Stastna (2016), where it was shown essential effect of molecular diffusivity on the mass transport by mode-2 ISW in the range $1 \leq \mathrm{Sc} < 20$. At the same time, the results of calculations at Sc=10 and Sc=1000 in Fig.9b and 9c practically coincide, which indicates that molecular diffusion may not be taken into account when studying the global properties of colliding waves. This conclusion agrees with Terez and Knio (1998) as they estimate that the value of Sc=100 was "sufficiently high for density diffusion to be ignored during simulation period" and the ~~development of instability was two-dimensional (Fig. 9a), resulting in the development of KH billows similar to that in the 2D simulation in Fig. 6b.~~ results by Deepwell and Stastna (2016) simulation, according to which the mass transfer is virtually independent of Sc already at Sc>20. However, ~~overturning the KH billows resulted in the appearance of spanwise structures. These processes were enhanced by the interaction of waves and their trapped cores, causing spanwise instability and turbulence (Fig~~diffusion can be important for small scale mixing processes in tiny density structures (see e.g. (Galaktionov et al., 2001)) forming in result of instability and turbulent cascade processes (Deepwell and Stastna, 2016) and persisting over time in a wake behind moving bulge of trapped fluid (Terez and Knio, 1998). These subgrid scale structures in our simulations were smashed by numerical diffusion which did not affect larger scale due to use of second order total variation diminishing (TVD) scheme for advective terms in transport equation. ~~9b, c). The diverging waves after the collision remained shear-unstable (Fig. 9d). The comparison of side plans for the 2D setting and for the cross-section averaged distribution of density for the 3D setting are given in Figs. 9e and 9f, respectively. As seen in the figure, 3D instability results in greater mixing comparative with the 2D case. Therefore, KH billow visible in 2D setting (Fig.9e) disappeared in cross averaged 3D distribution in Fig. 9f. However, differences between 2D and 3D estimations of $\Delta\alpha$ and $\Delta\theta$ in Fig. 3 did not exceed 2.5% and 1.2%, respectively.~~

3.5 Interaction of internal waves with trapped cores and different amplitudes

The collision process is modified for waves of different amplitudes by the exchange of trapped fluid between colliding waves due to the conservation of momentum (Stamp and Jacka, 1995). This process is shown in Fig. 10 for two cases. In the first case, two stable strongly nonlinear waves with trapped cores collide ($\alpha_L = 3.3$ with $\mathrm{Fr_{max}}$=1.28, $\mathrm{Ri_{min}}$=0.25; α_R =2.15 with $\mathrm{Fr_{max}}$=1.16, $\mathrm{Ri_{min}}$ =0.4). As seen in Fig. 10a, the part of the trapped core fluid from the wave of larger amplitude (blue colour) is merged with the trapped fluid from the smaller wave (red colour) without noticeable mixing. The circulation inside the core of the larger ~~transmitted~~ outgoing wave results in the stirring of the fluid in such a way that the smaller core fluid eventually ends up inside the fluid from the larger wave.

The collision of an ISW of small amplitude (class (i)) with a stable wave of large amplitude (class (ii)) was considered for case (A11; A1) to study the possibility of triggering instability in the wave of large amplitude via a small disturbance, similar to the waves of mode-1 in a two-layer fluid (Almgren et al., 2012). The simulation results are shown in Fig. 10b. As seen in the figure, the small amplitude ISW ($\alpha = 0.5$; $\mathrm{Fr_{max}}$=0.33, $\mathrm{Ri_{min}}$ =81) triggered instability in the ISW with an amplitude that was ten times larger than that of the small wave. Notice that the large amplitude wave has parameters ($\alpha = 4.6$; $\mathrm{Fr_{max}}$=1.3, $\mathrm{Ri_{min}}$ =0.15) that are close to critical for the development of instability. The amplitude of the small wave essentially decreased during the interaction process due to the interaction and due to the viscous attenuation at the low Reynolds number of the wave ($\mathrm{Re_m}$ =45.1). Unlike the head-on collision of large amplitude mode-1 and weakly nonlinear mode-2 ISWs (Stastna et al., 2015), the ~~transmitted~~ outgoing wave of small amplitude did not degenerate. Spatiotemporal diagrams for the paths of two ISWs of different amplitudes colliding head-on are shown in Fig. 11 for cases (A9; A7) and (A1; A11). As seen in Fig. 11a, the trajectories of waves of larger amplitude propagating from left to the right were less subject to changes due to the collision, whereas the phase shift and the decrease of phase velocity for the smaller waves propagating from right to the left were essentially greater.

3.6 Estimation of the energy loss due to collision

We defined a relative energy loss due to the wave collision (ΔE_{loss}) as the difference between the total loss of energy E_{tot} due to the collision and the loss of energy by two single waves due to the viscous decay or instability ΔE_{dis}

$$\Delta E_{loss} = \Delta E_{tot} - \Delta E_{dis}. \tag{9}$$

The relative loss of energy due to the collision of ISWs can be calculated as the normalized difference in energy of ~~incoming waves and transmitted waves~~

$$\Delta E = \frac{PSE_L^{(in)} + PSE_R^{(in)} - PSE_L^{(tr)} - PSE_R^{(tr)}}{PSE_L^{(in)} + PSE_R^{(in)}},$$

~~where $PSE_L^{(in)}$ and $PSE_R^{(in)}$~~ waves before and after collision

$$\Delta E_{tot} = \frac{PSE_L^{(bf)} + PSE_R^{(bf)} - PSE_L^{(af)} - PSE_R^{(af)}}{PSE_L^{(bf)} + PSE_R^{(bf)}}, \tag{10}$$

$$\Delta E_{dis} = \frac{PSE_L^{(bf)} + PSE_R^{(bf)} - \widetilde{PSE}_L^{(af)} - \widetilde{PSE}_R^{(af)}}{PSE_L^{(bf)} + PSE_R^{(bf)}}, \tag{11}$$

where $PSE_L^{(bf)}$ and $PSE_R^{(bf)}$ are the pseudo-energies of the ~~incoming waves~~ waves before collision at the cross-sections x_L and x_R, respectively, and ~~$PSE_L^{(tr)}$ and $PSE_R^{(tr)}$~~ $PSE_L^{(af)}$ and $PSE_R^{(af)}$ are the pseudo-energies of the ~~transmitted waves~~ waves after collision at the cross-sections x_L and x_R, respectively, whereas $\widetilde{PSE}_L^{(af)}$ and $\widetilde{PSE}_L^{(af)}$ are the energies of the outgoing waves without interaction at cross sections x_L and x_R, respectively. The pseudo-energy is the sum of the kinetic and available potential energies (Shepherd, 1993) of ~~incident and transmitted waves~~ waves before and after collision. The method for estimation of the available potential energy and energy fluxes was given in (Scotti et al., 2006; Lamb, 2007). A detailed description of the procedure of the pseudo-energy calculation was presented by ~~(Maderich et al., 2010). We define the energy loss due to the wave collision (ΔE_{loss}) as the difference between the total loss of energy E_{tot} (6) and the loss of energy by single waves due to the viscous decay or instability ΔE_{dis}~~

$$\cancel{\Delta E_{loss} = \Delta E_{tot} - \Delta E_{dis}.}$$

Maderich et al. (2010).

From dimensional arguments $\Delta E_{loss} = \Phi(\alpha, \mathrm{Re_m}, \mathrm{Sc})$, where Φ is function of three arguments. Assuming complete similarity on the $\mathrm{Re_m} \to \infty$ and $\mathrm{Sc} \to \infty$ consider dependence ΔE_{loss} on α. As seen in Fig. 12, ~~the dependence of the relative loss of energy on the dimensionless wave amplitude~~ this dependence given for symmetric collisions ($\alpha = \alpha_L = \alpha_R$) is not ~~monotonous~~monotonic and is not universal, changing depending on the series of calculations. It can be divided into three different ranges. In range I ($0 \leq \alpha \leq 1$), the energy loss due to the interaction increases as the wave amplitude increases. This range coincides with the range of weakly nonlinear waves without trapped cores. In range II ($1 < \alpha \leq 1.75$), the relative energy losses reach a maximum. The range coincides with the range in which colliding waves lose trapped cores in the process of interaction. This fact can explain the relative maximum of energy loss as the loss of potential energy of the cores. In range III ($1.75 \leq \alpha$), the behaviour of the loss of energy is also ~~non-monotonous~~ non-monotonic and non-similar. At first, in the zone of stable large amplitude collisions, the loss of energy decreases, but as the amplitudes of collided waves increase, the interaction is accompanied by the development of instability; therefore, the loss of energy increases. Finally, for unstable waves, the energy losses due to the interaction increase monotonically with increasing amplitude. ~~The Euler equations for stratified fluid do not contain non-dimensional parameters if the thickness of~~

The absence of complete self-similarity on the Reynolds and Schmidt numbers also means that the Euler equations do not describe the wave interaction processes in deep water even for the ~~stratified layer h, the phase velocity of the long linear waves c_0 and the characteristic time τ_0 are used for the equation scaling. Therefore, it should be expected that the dimensionless relations in Figs. 4 and 12 will be similar for different h. However, they demonstrate a lack of complete similarity due to the potential influence of other parameters. It was suggested by (Maderich et al., 2015) that this is a result of the incomplete similarity on the wave Reynolds number $\mathrm{Re_m}$, representing the effect of viscosity.~~ range of stable waves. As shown in Table 1, the parameter $\mathrm{Re_m}$ varies in Series A-C several times for waves of the same dimensionless amplitude α. ~~Another factor of the incomplete similarity can~~ The incomplete similarity scaling following (Barenblatt, 1996) results in relation: $\Delta E_{loss} \sim \Psi(\alpha)\mathrm{Re_m}^{\mathrm{m}}\mathrm{Sc}^{\mathrm{n}}$, where Ψ is function, m and n are exponents. However, this rescaling also did not result in universal dependence. We conclude that it is due to the different mechanisms governing collision process in ranges I-III: nonlinear wave interaction, collapse of collided trapped masses and instability. Another factor influencing the interaction may be the diffusivity effect (Deepwell and Stastna, 2016), which is described by the Schmidt number~~$\mathrm{Sc} = \nu/\chi$~~. However, in ~~our study~~these experiments, the Schmidt number was large and constant.

4 Conclusions

The dynamics and energetics of a head-on collision of internal solitary waves (ISWs) with trapped cores propagating in thin pycnocline were studied numerically within the framework of the Navier-Stokes equations for a stratified fluid. The peculiarity of this collision is that it involves the trapped masses of a fluid. The interaction of ISWs ~~differs~~ was different for three classes of waves: (i) weakly nonlinear waves without trapped cores, (ii) stable strongly nonlinear waves with trapped cores, and (iii) shear unstable strongly nonlinear waves with trapped cores. The simulations showed that the wave phase shift grew as the amplitudes of the interacting waves increased for interacting waves of classes (i) and (ii) and remained almost constant for those of class (iii). The excess of the maximum runup amplitude over the sum of the amplitudes of colliding waves almost linearly increased as the amplitudes of the interacting waves belonging to classes (i) and (ii) increased. However, it decreased somewhat for those of the unstable class (iii). The dependence is similar to the interaction of the mode-1 waves in a two-layer stratification (Terletska et al., 2017), with the difference being that the phase shift continues to grow for the collision of interfacial waves of mode-1. The waves of class (ii) with a normalized thickness of the pycnocline amplitude α fully lost fluid trapped by the wave cores in the approximate range of $1 \leq \alpha \leq 1.75$. The interacting stable waves of higher amplitude captured cores and carried trapped fluid in the opposite directions with little mass loss. The collision of locally shear unstable waves of class

(iii) was accompanied by the development of ~~three-dimensional instabilityand turbulence.~~ instability. We concluded that this kind of interaction reduced the capacity for an ISW to transport mass. The dependence of energy loss on wave amplitude was not ~~monotonous~~monotonic. Initially, the energy loss due to the interaction increased with increasing wave amplitude. Then, the energy losses reached a maximum due to the loss of potential energy of the cores upon collision and then started to decrease. With further amplitude growth, the collision was accompanied by the development of instability, and the loss of energy increased. The collision process was modified for waves of different amplitudes because of the exchange of trapped fluid between colliding waves due to the conservation of momentum. Merging of the trapped fluid due to the collision of stable waves belonging to class (ii) occurred through the stirring mechanism without noticeable mixing. Similar to waves of mode-1 in a two-layer fluid (Almgren et al., 2012; Terletska et al., 2017), the interaction of a wave of large amplitude with a wave of small amplitude can trigger local wave instability of the large amplitude wave if the parameters of this wave ~~are~~ were close to critical for the development of instability. The obtained results can be applied to the interaction dynamics of subsurface trapped core formed within a shoaling large amplitude internal waves (Lien et al., 2012). Notice, however, that the destruction of the KH billows is essentially three-dimensional process, therefore, 3D high-resolution simulation is necessary to predict turbulence development (Arthur and Fringer , 2014; Deepwell and Stastna, 2016). This is the subject of a separate study, whereas the interaction of the colliding waves as a whole can be described in 2D setting.

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Table 1. Summary of parameters of interacting ISWs: pycnocline thickness parameter h, wave amplitude a, wavelength $\lambda_{0.5}$, ratio ε, dimensionless ISW amplitude α, Froude number $\mathrm{Fr}_{\mathrm{max}}$, minimum Richardson number $\mathrm{Ri}_{\mathrm{min}}$, Reynolds number Re_{m} and ISW class.

Wave	h (cm)	a (cm)	~~λ~~ $\lambda_{0.5}$ (cm)	ε	α	$\mathrm{Fr}_{\mathrm{max}}$	$\mathrm{Ri}_{\mathrm{min}}$	Re_{m}	Wave class
A1	0.5	0.25	3.15	92	0.5	0.33	81	45.1	i
A2	0.5	0.4	2.35	92	0.81	0.71	52	~~86.59~~ 86.6	i
A3	0.5	~~0.575~~ 0.58	1.9	92	1.15	0.82	14	132	i
A4	0.5	~~0.675~~ 0.68	2	92	1.35	0.98	11	~~166.05~~ 166	i
A5	0.5	0.8	2.2	92	1.6	1.11	1.1	223	ii
A6	0.5	0.94	2.4	92	1.88	1.12	0.8	277.4	ii
A7	0.5	~~1.075~~ 1.08	2.65	92	2.15	1.16	0.4	338.4	ii
A8	0.5	1.3	3.15	92	2.6	1.25	0.35	~~443.4~~ 443	ii
A9	0.5	1.7	3.65	92	3.3	1.28	0.25	~~683.0~~ 683	ii
A10	0.5	1.9	4.25	92	3.38	1.29	0.19	~~785.2~~ 785	ii
A11	0.5	2.3	4.75	92	4.6	1.3	0.15	1075	ii
A12	0.5	2.5	5.35	92	5	1.35	0.12	1242	iii
A13	0.5	3.2	6.35	92	6.4	1.31	0.06	1681	iii
B1	1	0.63	5.21	46	0.63	0.51	24	~~153.4~~ 153	i
B2	1	0.85	4.23	46	0.85	0.68	11.5	~~225.5~~ 225	i
B3	1	1.25	3.61	46	1.25	1.02	2.4	~~388.0~~ 388	ii
B4	1	1.95	5.25	46	1.95	1.16	0.38	~~765.5~~ 765	ii
B5	1	2.68	6.65	46	2.68	1.22	0.18	~~1225.~~ 1225	ii
B6	1	2.86	7.1	46	2.86	1.22	0.13	~~1345.~~ 1345	ii
B7	1	3.56	8.6	46	3.56	1.23	0.11	~~1839.~~ 1839	iii
C1	2	0.42	14	23	0.21	0.19	52	~~161.4~~ 161	i
C2	2	0.76	10.4	23	0.38	0.30	25	~~291.2~~ 291	i
C3	2	1.2	7	23	0.6	0.50	3.1	~~511.68~~ 511	i
C4	2	1.7	6.1	23	0.85	0.69	1.1	~~669.1~~ 669	i
C5	2	2.02	6.21	23	1.01	0.84	0.45	~~881.6~~ 881	i
C6	2	2.6	6.25	23	1.3	0.9	0.23	~~1159.~~ 1159	i
C7	2	2.9	7.22	23	1.45	1.01	0.22	~~1483.~~ 1483	ii
C8	2	3.3	7.9	23	1.65	1.08	0.18	~~1851.~~ 1851	ii
C9	2	3.5	8.5	23	1.75	1.147	0.15	~~2030.~~ 2030	ii
C10	2	4.1	9.2	23	2.05	1.17	0.13	~~2521.~~ 2521	ii
C11	2	4.56	10.82	23	2.28	1.23	0.12	~~2812.~~ 2812	ii
C12	2	4.846	12.44	23	2.42	1.24	0.09	~~3171.~~ 3171	iii
C13	2	5.28	13.4	23	2.64	1.25	0.07	~~3478.~~ 3478	iii
C14	2	5.94	15.31	23	2.97	1.29	0.05	~~3884.~~ 3884	iii
D1	0.25	0.55	12.5	56	2.2	1.18	1.05	329	ii

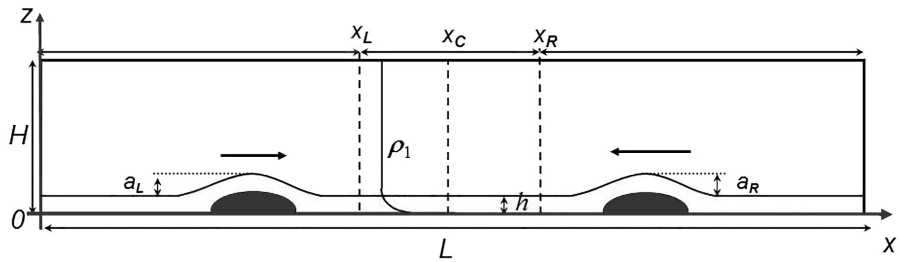

Figure 1. Configuration of the experiment exploring the interaction of ISWs with trapped cores.

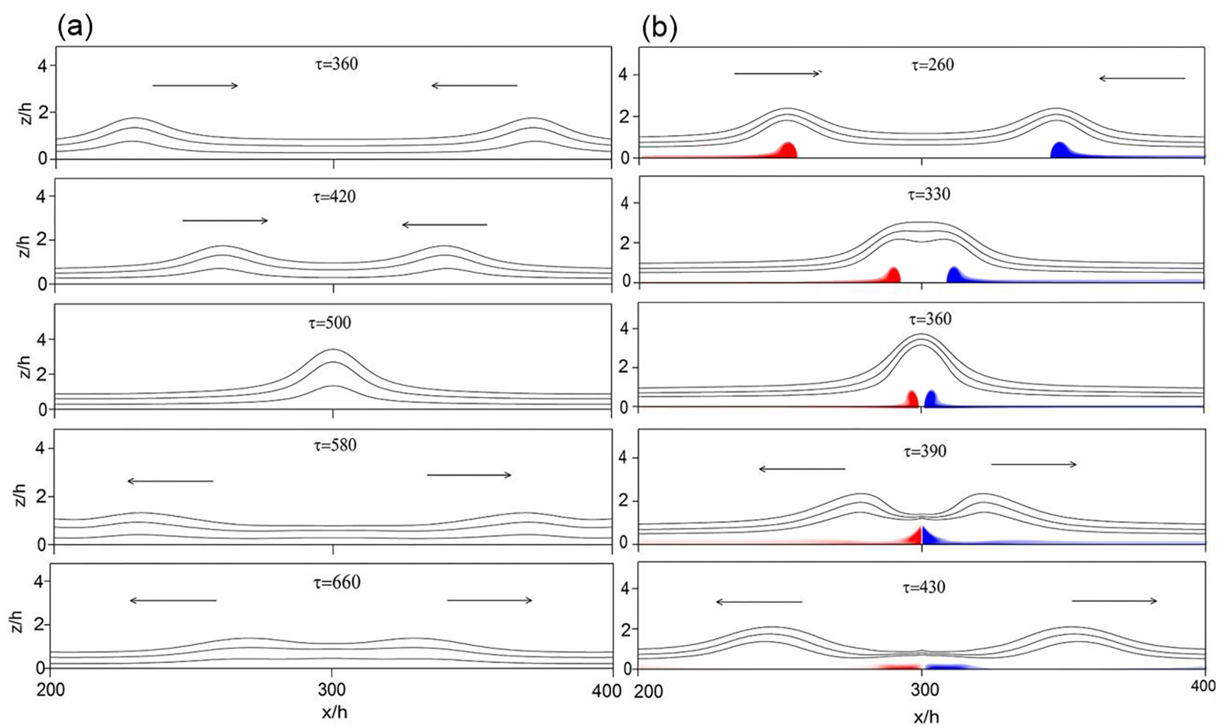

Figure 2. Snapshots of the density isopycnals during the collision of ISWs~~in a 2D setting~~. (a) Case (A2; A2). (b) Case (A5; A5). The trapped cores are visualized by dyed fluid.

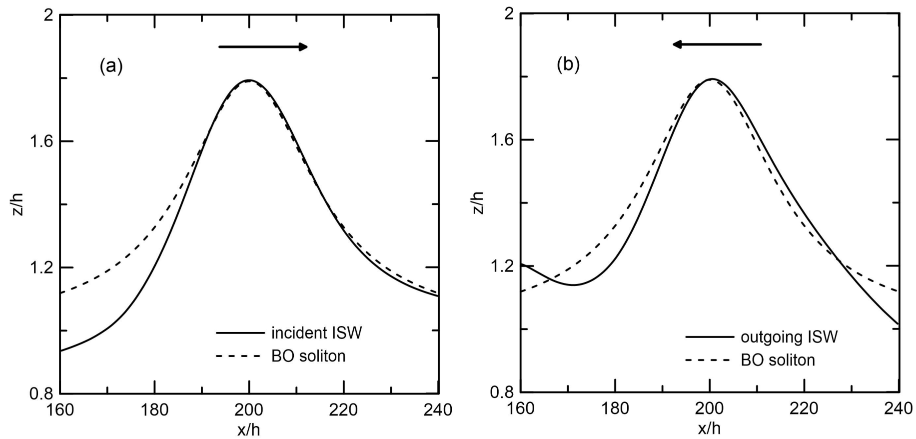

Figure 3. Wave profile of ~~incident~~ wave ~~in section x_L~~ before (a) and ~~wave profile of transmitted wave~~ after (b) collision in section ~~x_R~~ x_L (b) for ~~$\alpha = 1.6$~~ $\alpha = 0.81$ (case ~~A5~~A2; ~~A5~~A2). These profiles are compared with the profile of the BO soliton.

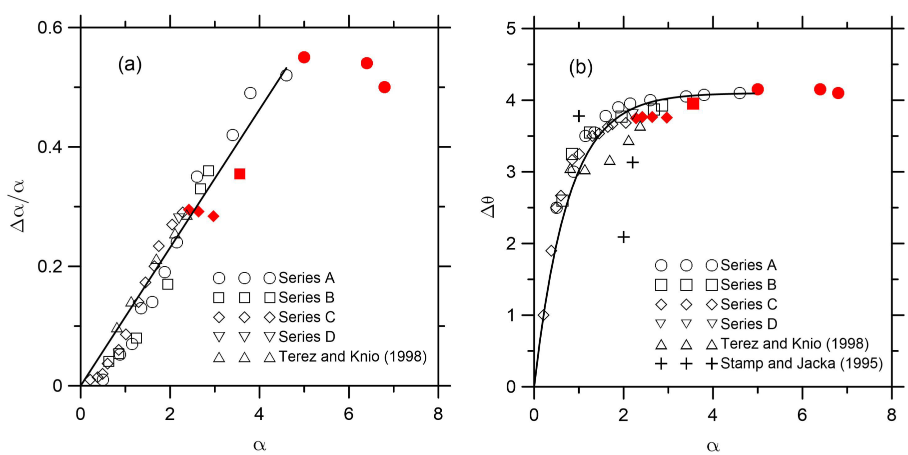

Figure 4. Relative runup excess $\Delta\alpha$ (a) and phase shift $\Delta\theta$ (b) of the interacting symmetric ISWs versus the normalized amplitude of the wave α. The filled symbols correspond to the cases with KH instability. The fits, done by using a straight line in (a) and an exponential function in (b), are shown.

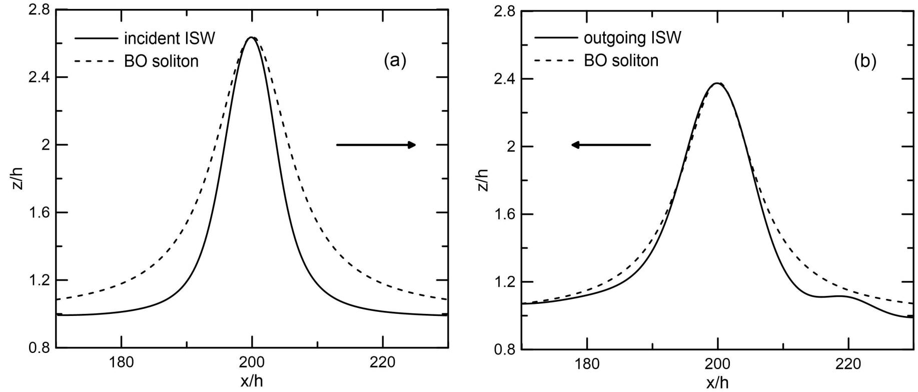

Figure 5. Wave profile of ~~incident~~ wave ~~in section x_L~~ before (a) and ~~wave profile of transmitted wave~~ after (b) collision in section ~~x_R~~ x_L (b) for $\alpha = 1.6$ (case A5; A5). These profiles are compared with the profile of the BO soliton.

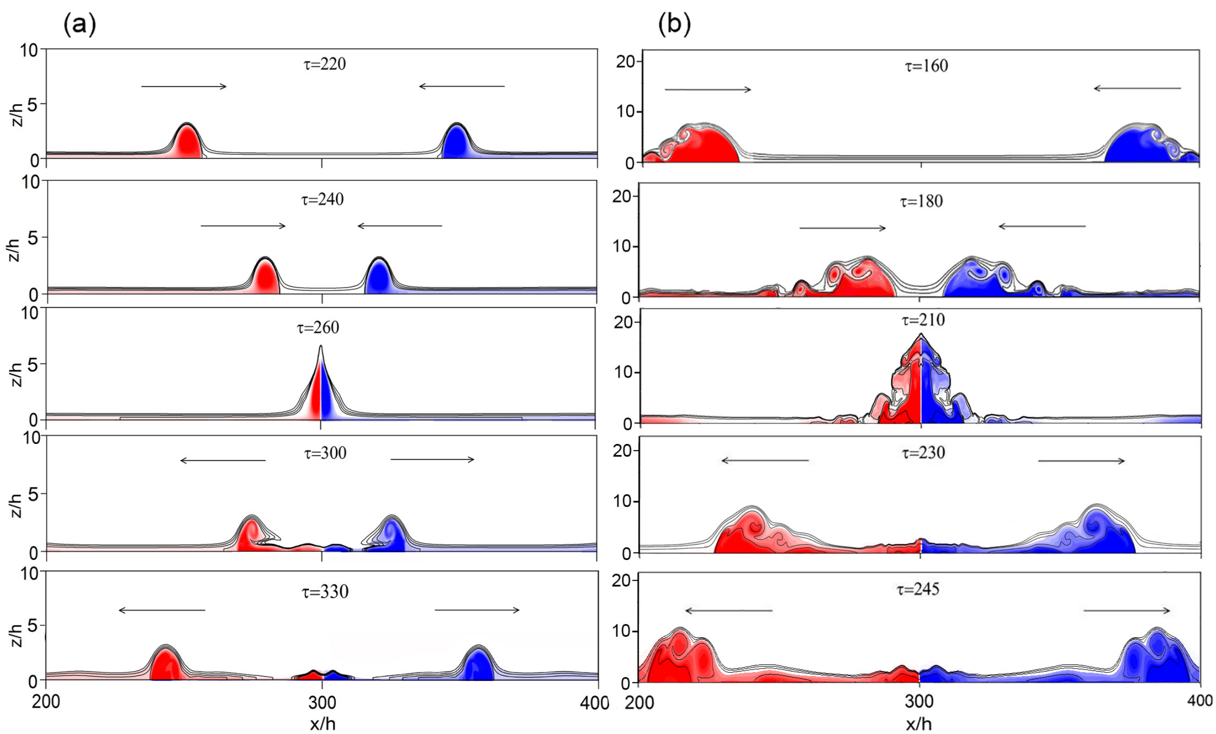

Figure 6. Snapshots of the density isopycnals during the collision of ISWs~~in a 2D setting~~. (a) Case (A9; A9). (b) Case (A13; A13). The trapped cores are visualized by dyed fluid.

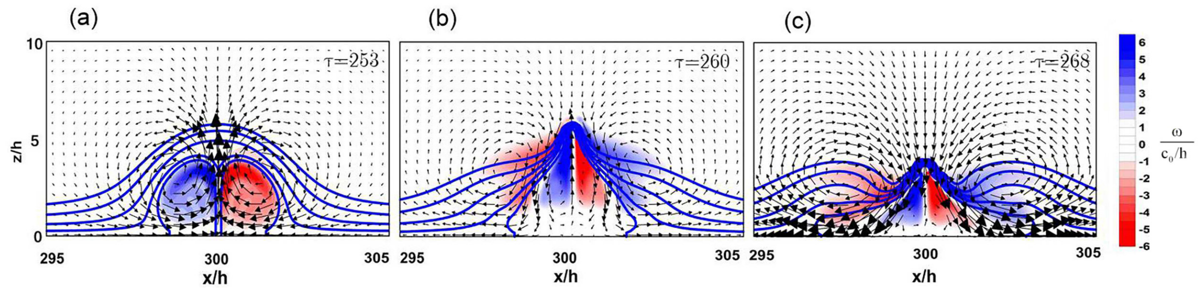

Figure 7. Details of the interaction of waves of equal amplitude $\alpha = 3.3$ at different times (case (A9; A9)) in a 2D setting. The velocity, vorticity ω and isopycnals are shown.

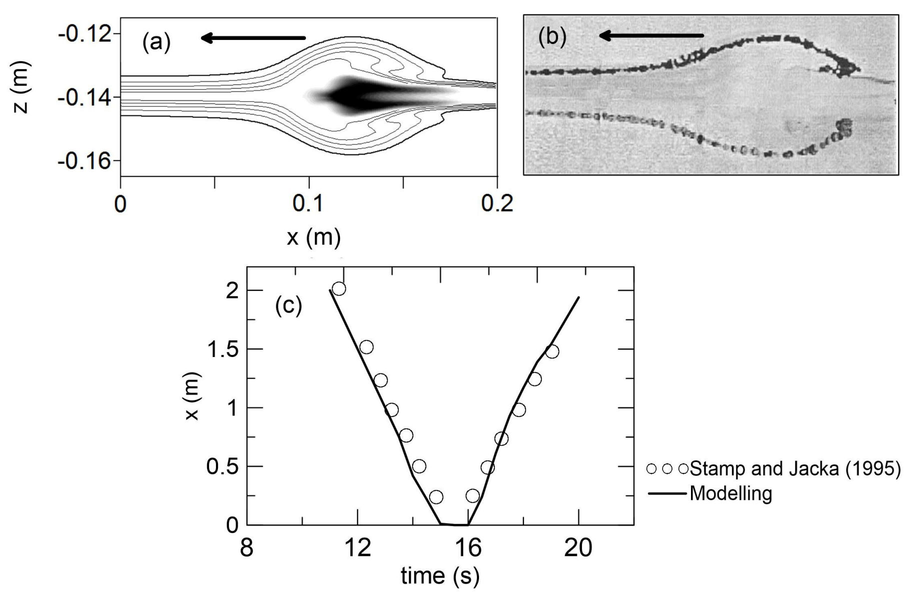

Figure 8. Comparison of the simulated reflection of ISWs off a vertical wall (case D1) in a 2D setting with a laboratory experiment (Stamp and Jacka, 1995). (a) Snapshot of the calculated density isopycnals, visualized by a black tracer trapped fluid at $t = 16$ s. (b) Density isopycnals in the experiment, visualized by water insoluble droplets of different densities. (c) Spatio-temporal diagrams of the path of an ISW during reflection off a wall.

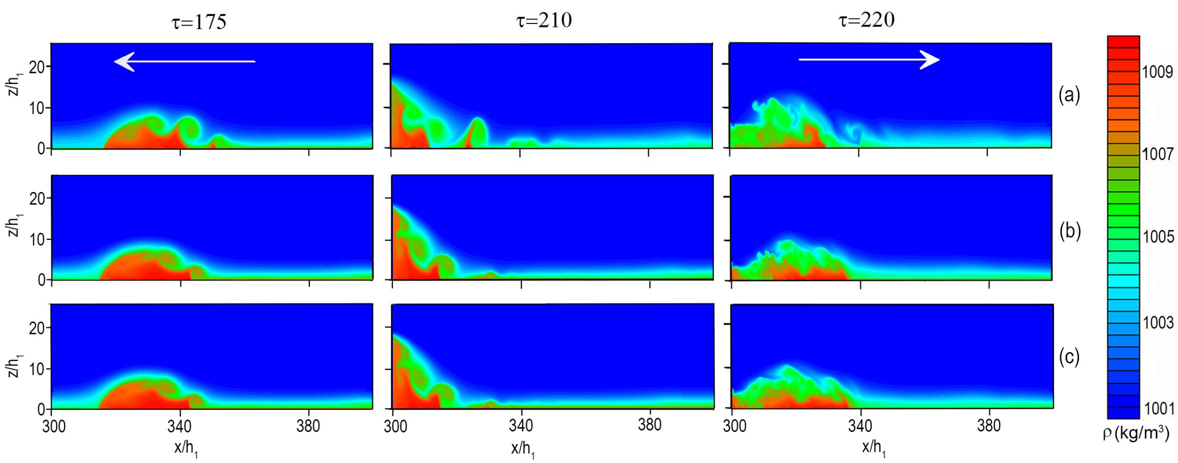

Figure 9. ~~Three-dimensional evolution~~ Comparison of the density snapshots during collision of ISWs ~~for~~ in case (A13; A13) ~~(a-d)~~for different Schmidt numbers. ~~The wave is visualized by~~ (a~~density isosurface with ρ~~) Sc=~~1005 kg m^{-3}. The side plan shows the distribution of density. The comparison of side plans for a 2D setting and for the cross-section averaged distribution of density for a 3D setting at $\tau = 250$ are given in~~ 1. (~~e~~b) ~~and~~ Sc=10. (~~f~~c) ~~, respectively~~Sc=1000. The right half of the numerical flume is shown due to the symmetry of the interaction process.

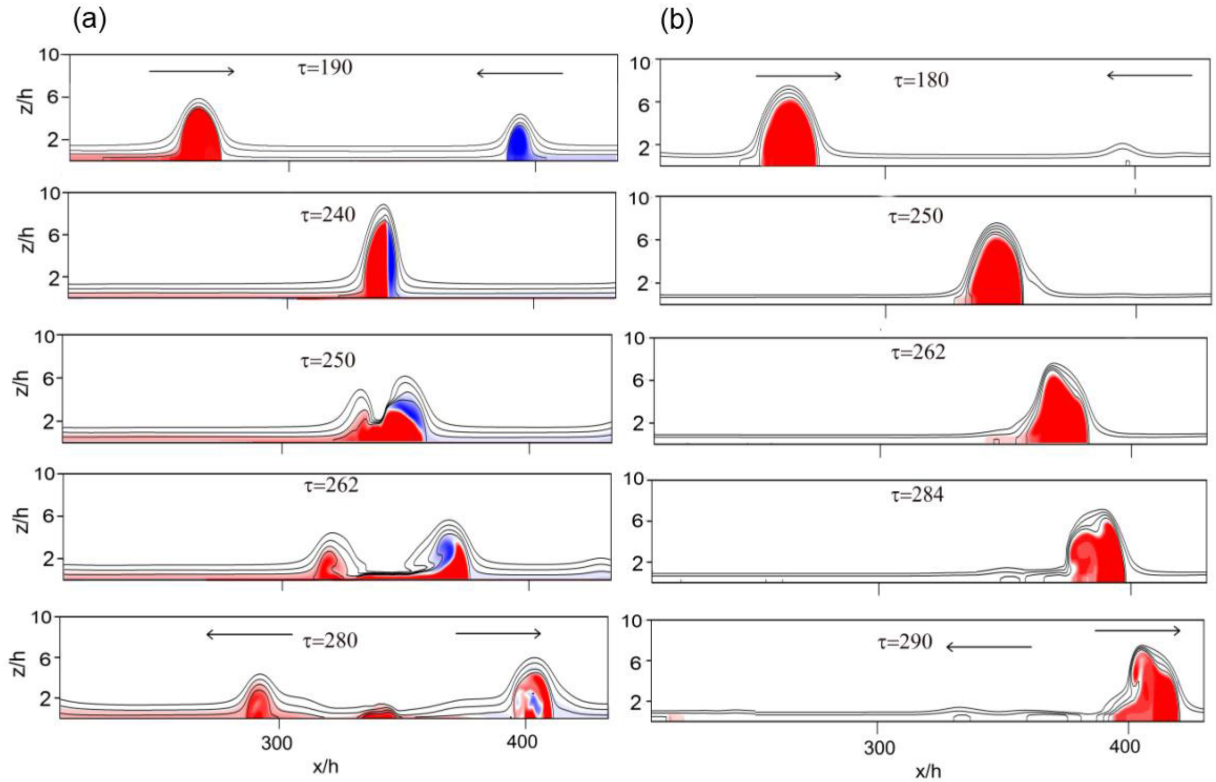

Figure 10. Snapshots of the density isopycnals during the collision of ISWs with different amplitudes for case (A9; A7) (a) and case (A11; A1) (b). The trapped cores are visualized by dye.

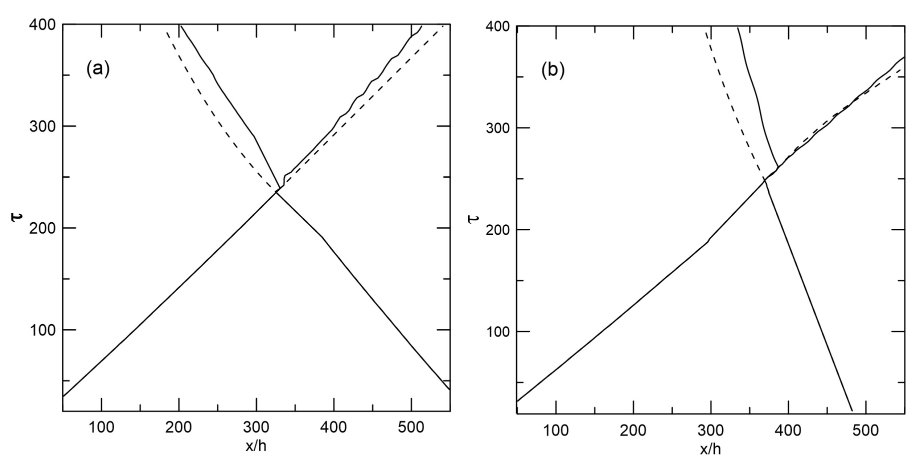

Figure 11. Spatio-temporal diagrams for paths of two ISWs of different amplitudes colliding head-on. (a) Case (A9; A7)~~;~~. (b) ~~case~~ Case (A11; A1). The diagrams for the waves without interaction are shown by dashed lines.

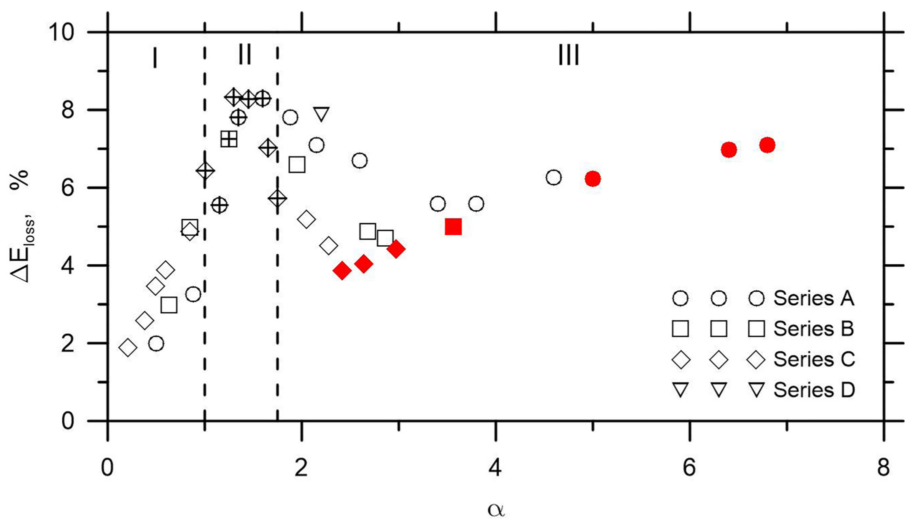

Figure 12. Plot of the energy loss versus the amplitude of the equal colliding waves. The filled symbols correspond to the cases with KH instability. The crossed symbols correspond to the cases where colliding waves lost trapped cores in the process of interaction.