Response to Review of Manuscript:

Complex networks description of the ionosphere

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We would like to thank all the reviewers for their thorough reviews and constructive comments on our manuscript. The comments and suggestions are very helpful for the improvement of our paper. Following these comments, we have conducted a more thorough and rigorous review of our work and made a comprehensive revision. We believe that the manuscript has been seriously revised according to the reviewers' comments. In the document that follows, we describe the associated modifications made to the original version of the paper and address the comments of the reviewers.

I. Responses to Short Comments

(The line numbers referred in the response are those in the manuscript tracking changes. This manuscript is provided as the supplement.)

Short comments 1: *For line 5 in Page 8, the expression* $\log N_B(l_B) \sim -d_B \log l_B$ *should be corrected as* $\lg N_B(l_B) \sim -d_B \lg l_B$

Response 1: Thanks for the suggestions. The corrections have been made in the revised manuscript. The revisions are shown in line 18 of page 10.

Short comments 2: *the world 'which' should be deleted in line 24 in Page 7.*

Response 2: Thanks for the suggestions. The sentence has been revised. The revisions are shown in lines [12-14] of page 8.

Short comments 3: *the world 'the' should be deleted in line 18 in Page 7.*

Response 3: Thanks for the suggestions. The corrections have been made in the revised manuscript. The revisions are shown in line 7 of page 9.

Short comments 4: *'mathmatical definition of information flow' should be corrected as mathematical definition of information flowin line 1 in page 4.*

Response 4: Thanks for the suggestions. That sentence has been deleted.

Short comments 5: *Figure 2 shows the degree distributions of the network of ionosphere. To show the distribution intuitively, I suggest the curve of distribution fitting to be added in the figure.*

Response 5: Thanks for the suggestions. The revisions are made and shown in Figure 2.

Short comments 6: *Subsection 3.3 investigated the small-world structure. For the small world structure caused by the long-range edges, more explanation may be needed.*

Response 6: Thanks for the suggestions. More explanations has been added. The revisions are shown in lines [10-14] of page 8.

Short comments 7: *After careful reading the paper, I think some statements in the article need to be improved, like the subtitle of 2.2 and 3.2. So, I suggest you read more paper about the application of complex network and make an overall revision about the current manuscript.*

Response 7: Thanks for the suggestions. We are sorry about the poor writing. As the anonymous referee 2 suggested, we have invited a native English speaker to proof-read the manuscript. The revisions are tracked using bold face in the manuscript tracking changes.

II. Responses to Anonymous Referee 1

The authors would like to express our sincere gratitude for all the constructive comments on our manuscript. The comments and suggestions are very helpful for the improvement of our paper. In what follows, we present detailed comments in response to the individual points raised by the reviewer and elaborate on how the manuscript has been revised.

(The line numbers referred in the response are those in the manuscript tracking changes. This manuscript is provided as the supplement.)

A. *General Comments*

Comments: *In recent decades, attentions on complex networks have been more and more paid to the field of geoscience as a powerful tool in investigations, particularly, in the study of climatology and seismology. In this paper, the authors firstly introduce this method to construct a directed complex network to investigate the information flow in the ionosphere. Some new results are gained that both the out-degree and in-degree distribution of the ionospheric network are not scale-free. The topological structure of the ionospheric information network is homogeneous. The spatial variation of the ionospheric network shows the connections principally exist between the neighbors in space, indicating that in the ionosphere the information transmission is mainly based on the spatial distance. Since this is the first time that ionospheric data are used to construct such a network, the results are helpful in understanding some special characteristics of the ionosphere.*

Response: We thank the reviewer very much for these positive comments and present the details of responses to the concerns in the following part. In the document that follows, we describe the associated modifications made to the original version of the paper and address the comments of the reviewers separately.

B. *Major Comments*

Issue 1: *Can authors make a simple comparison of results of this paper with other relatively similar earlier published networks like surface temperate data structed networks. Such comparison may provide some useful hints for the further development of the complex network construction.*

Response 1: Many thanks for the comment. As the reviewer suggested, we have made a comparison between the network of this paper with other relatively similar earlier published networks. The details are shown as follows,

Peron et al. (2014) built the temperature network by correlation and regarded the global grid points as nodes. They showed that the network characteristics of the North American region have marked the differences between the eastern and western regions. Such differences were a reflection of the presence of a large network community on the western side of the continent. Correlation is a linear measurement of the dynamics in the climate system. To depict the nonlinearity and uncertainty in the climate, information theory is introduced to construct the complex network. Donges et al. (2009a,b) used complex networks to uncover a backbone structure carrying matter and energy in the global surface air temperature field. They used mutual information (MI) to construct the network which was undirected, because the mutual information is symmetric to measure the dynamical similarity of surface air temperature between regions. Hlinka et. al (2013) investigated the reliability of directed climate networks being constructed by conditional mutual information (CMI), using the dimensionality-reduced surface air temperature data. Compared with MI, CMI is asymmetric and able to build directed networks for the global surface air temperature. However, both MI and CMI are standard bivariate methods, which only describe the interactions between two spatial points without considering the influences of the others. So is the correlation. The revisions are shown in lines [15-27] of page 2.

Issue 2: *Is it possible to explain the results of networks in the real ionospheric features*

Response 2: Many thanks for the comment. We try to explain the results of networks in the real ionospheric features, but the explanations still need to be further verified by observations.

1 To explore the influence of the VTEC's variation over a certain GIM cell, the degree of complex network is employed. As one of the most critical parameters to depict the nodes in a complex network, the degree is the number of edges the node possesses. Concerning ionospheric networks, the degree of a cell can be selected to quantify how many GIM cells display a causal interaction with that given cell in the globe. In other words, cells with large degree can influence large numbers of GIM cells. We have checked the scale-free topology of the ionospheric network by conducting power-law hypothesis testing about the degree distribution. The result shows that the network of the global ionosphere is not scale-free. Thus, there are no visible hub positions for the dynamic process in the ionosphere. The ionospheric network is homogenous. There are no unique positions acting as the sources or sinks of the variations within the ionosphere. This property is completely different from that of the geomagnetic field. The revisions are shown in lines [6-15] of page 5 and lines [9-15] of page 6.

2 As the referee 2 suggested, the distance between connected grid cells is measured in terms of meters instead of degrees. The propagation of the dynamic processes is related to the transmission of energy or particles in the ionosphere. In order to analyze such transport property, the distribution of the edge distances is calculated. The edge distance is defined by the geographical distance between the origin and destination of an edge. The positive signs of the distances represent the directions of edges and can be either eastward or northward. The results show that the propagation is mainly affected by the geospatial distance and almost satisfies the proximity principle in space. Meanwhile, there are also some exceptional long-range edges existing in the ionospheric network. Accordingly, most of the dynamic processes in the ionosphere are locally propagated with some long-range propagation. Such phenomenon indicates the complexity of the inner ionospheric variations in the globe. The proximal propagation may be due to the diffusion effects of charged particles in the ionosphere, while the long-range one may be caused by the geomagnetic field or other global factors. These explanations still need to be further verified by observations. The revisions are shown in lines [2-15] of page 7 and lines [1,5-14] of page 8.

3 We explore the small-world structure of the ionospheric network to check the stability of the ionosphere which is regarded as a dynamical system, because the small-world structure can make the system be stable to react to the abrupt variations. The results indicate that the dynamic processes in the ionosphere present small-world property. Just as the small-world property in the atmosphere, such ionospheric property also results from the teleconnections in the ionospheric network. The teleconnections make the dynamic processes be propagated within the ionospheric network efficiently. If a disturbance is generated somewhere in the ionosphere, the small-world structure of the ionospheric network allows the ionosphere to react quickly and coherently to the variations introduced into the ionosphere. This propagation mechanism of dynamic processes can diffuse local variations, thereby reducing the possibility of prolonged local anomalies and providing more stability for the global ionosphere. Thus, chances of major ionospheric shifts are reduced. The revisions are shown in lines [16-18] of page 8 and lines [2-4] of page 10.

4 The spatial prediction, especially regional prediction, depends heavily on the self-similarity in the ionosphere. Thus, we investigate the self-similar structure in the ionosphere through fractal analysis, which shows the ionospheric network is not self-similar in the current temporal and spatial resolution. Such phenomenon shows that because of the non-fractal property, the predictability of the ionosphere for one year should decrease. Construction of the long-term geospatial model of the ionosphere is still a challenging work. Spatial characteristics within the ionosphere differ complexly and dramatically with the variation of regions for one year. Such complex spatial variations in the current resolution may disrupt the similarity in the ionosphere. To further investigate the self-similarity within the ionosphere, the temporal and spatial resolution of the ionospheric observations should be considered. In our background, the self-similarity in the ionosphere may be detected by the observations of high temporal and spatial resolution. The revisions are shown in lines [11-13] of page 10, lines [11-14] of page 11 and lines [1-2] of page 12.

Issue 3: *The data used in this paper only with one years time span, and time resolution is two hours and ranges from* $-180°$ *to* $180°$ *along the longitude and from* $-87.5°$ *to* $87.5°$ *along the latitude with a revolution of 10 and 5 degree in longitude and latitude. Is the revolution affects the results? For example, in such a revolution, ionospheric equatorial anomaly, small-scaled irregularities are excluded, then how can we say the ionospheric network is not fractal?*

Response 3: Many thanks for the comment.

As the reviewer suggested, the fractal analysis of the ionosphere really depends on the resolution of the observations. The ionosphere is a dynamic system containing complex temporal and spatial variations. As a description of the ionosphere, the construction of the complex network is also influenced by the time window and spatial position of the ionospheric observations. As the reviewer suggested, in the current revolution, ionospheric equatorial anomaly, small-scaled irregularities are excluded. The previous statement of this conclusion is not precise. For rigorous expression, the conclusion should be presented as follows,

”Therefore, in the current temporal and spatial resolution, the ionospheric network does not have self-similar structure, indicating the complexity of the ionospheric temporal and spatial variations. To further investigate the self-similarity within the ionosphere, the time window and space resolution of the ionospheric observations should be considered. In our background, the self-similarity in the ionosphere may be detected by the observations of high temporal and spatial resolution.” The revisions are shown in lines [9-10, 13-14] of page 11 and lines [1-2] of page 12.

III. Responses to Anonymous Referee 2

The authors would like to express our sincere gratitude for all the constructive comments on our manuscript. The comments and suggestions are very helpful for the improvement of our paper. In what follows, we present detailed comments in response to the individual points raised by the reviewer and elaborate on how the manuscript has been revised.

(The line numbers referred in the response are those in the manuscript tracking changes. This manuscript is provided as the supplement.)

A. General Comments

Comments: *This study applies existing methods and concepts to observational gridded data (VTEC). The technical analyses seem to be thorough. However, since there is thus no methodological advance, I would expect some physical / mechanistic motivation for the conducted research, as well as physical interpretations of the results: Why is the network not scale-free? Why was this tested? What are the implications? Similarly, what might be the reason for the small-worldness? And what are the implications of the network not being fractal? What can we learn from the analysis about the physical complex system under study? There are only very vague statements addressing these questions and therefore, in its present state, it seems a bit like the network was constructed and typical network characteristics were determined simply because its possible to do so, and not because of a driving scientific hypothesis. In other words, the results are original, but it is not clear why they are meaningful. I do not recommend rejection of the paper, because the analyses and results seem to be correct, but leave it to the editor to decide to what degree interpretations and discussions of the results are expected for the journal.*

Response: We thank the reviewer very much for these positive comments. The reviewer's general comments are of great value to help us rethink the motivation, physical interpretations and applications of the results. We try to provide clear statements in the manuscript to address the questions above.

Within the global ionosphere, there are interactions among the variations over different positions. Moreover, variations over one position may cause variations over other positions. The motivation of the current study is to explore the causal interactions between the VTEC over different GIM cells within the global ionosphere based on the directed complex network. Hence, we can have a deep understanding of the dynamic processes within the ionosphere. Meanwhile, based on the causal relationship in the ionosphere, we can make a more precise prediction of the

VTEC utilizing the observations obtained at the connected GIM cells in the network. Accurate prediction of the VTEC is valuable to improve the performance of GPS and ionospheric radio propagation. We interpret the dynamic ionospheric processes as the information flow in the directed network and explore the ionospheric characteristics on the global scale. The revisions are shown in lines [33-35] of page 2 and lines [1-4] of page 3.

The detailed explanations are as follows:

1 To explore the influence of the VTEC's variation over a certain GIM cell, the degree of complex network is employed. As one of the most critical parameters to depict the nodes in a complex network, the degree is the number of edges the node possesses. Concerning ionospheric networks, the degree of a cell can be selected to quantify how many GIM cells display a causal interaction with that given cell in the globe. In other words, cells with large degree can influence large numbers of GIM cells. We have checked the scale-free topology of the ionospheric network by conducting power-law hypothesis testing about the degree distribution. The result shows that the network of the global ionosphere is not scale-free. Thus, there are no visible hub positions for the dynamic process in the ionosphere. The ionospheric network is homogenous. There are no unique positions acting as the sources or sinks of the variations within the ionosphere. This property is completely different from that of the geomagnetic field. The revisions are shown in lines [6-15] of page 5 and lines [9-15] of page 6.

2 As the referee 2 suggested, the distance between connected grid cells is measured in terms of meters instead of degrees. The propagation of the dynamic processes is related to the transmission of energy or particles in the ionosphere. In order to analyze such transport property, the distribution of the edge distances is calculated. The edge distance is defined by the geographical distance between the origin and destination of an edge. The positive signs of the distances represent the directions of edges and can be either eastward or northward. The results show that the propagation is mainly affected by the geospatial distance and almost satisfies the proximity principle in space. Meanwhile, there are also some exceptional long-range edges existing in the ionospheric network. Accordingly, most of the dynamic processes in the ionosphere are locally propagated with some long-range propagation. Such phenomenon indicates the complexity of the inner ionospheric variations in the globe. The proximal propagation may be due to the diffusion effects of charged particles in the ionosphere, while the long-range one may be caused by the geomagnetic field or other global factors. These explanations still need to be further verified by observations. The revisions are shown in lines [2-15] of page 7 and lines

[1,5-14] of page 8.

3 We explore the small-world structure of the ionospheric network to check the stability of the ionosphere which is regarded as a dynamical system, because the small-world structure can make the system be stable to react to the abrupt variations. The results indicate that the dynamic processes in the ionosphere present small-world property. Just as the small-world property in the atmosphere, such ionospheric property also results from the teleconnections in the ionospheric network. The teleconnections make the dynamic processes be propagated within the ionospheric network efficiently. If a disturbance is generated somewhere in the ionosphere, the small-world structure of the ionospheric network allows the ionosphere to react quickly and coherently to the variations introduced into the ionosphere. This propagation mechanism of dynamic processes can diffuse local variations, thereby reducing the possibility of prolonged local anomalies and providing more stability for the global ionosphere. Thus, chances of major ionospheric shifts are reduced. The revisions are shown in lines [16-18] of page 8 and lines [2-4] of page 10.

4 The spatial prediction, especially regional prediction, depends heavily on the self-similarity in the ionosphere. Thus, we investigate the self-similar structure in the ionosphere through fractal analysis, which shows the ionospheric network is not self-similar in the current temporal and spatial resolution. Such phenomenon shows that because of the non-fractal property, the predictability of the ionosphere for one year should decrease. Construction of the long-term geospatial model of the ionosphere is still a challenging work. Spatial characteristics within the ionosphere differ complexly and dramatically with the variation of regions for one year. Such complex spatial variations in the current resolution may disrupt the similarity in the ionosphere. To further investigate the self-similarity within the ionosphere, the temporal and spatial resolution of the ionospheric observations should be considered. In our background, the self-similarity in the ionosphere may be detected by the observations of high temporal and spatial resolution. The revisions are shown in lines [11-13] of page 10, lines [11-14] of page 11 and lines [1-2] of page 12.

*B. **Major Issues***

Issue 1: *The manuscript should be proof-read by a native English speaker.*

Response 1: Many thanks for the comment. We are sorry about the poor writing. As the reviewer suggested, we have invited a native English speaker to proof-read the manuscript. The revisions are tracked using bold face in the manuscript tracking changes.

Issue 2: *In its current presentation, the paper is hardly reproducible, because it is not clear how, specifically, the Bayesian Network was constructed. I strongly suggest to add a paragraph where this is explained in detail.*

Response 2: Many thanks for the comment. We are sorry for the unclear explanation. As the reviewer suggested, a paragraph is added to explain the construction of Bayesian network in detail.

The cells in the GIMs are defined as the variables distributed throughout the globe. As the nodes on the network, the variables are separated by their own geospatial locations. The VTEC measurements of each variable are arranged in the form of a time series with the 2-hours' time resolution. Thus, for the year 2012, the length of the observations is 4392 $(12/day \times 366day)$. We employ structure learning algorithm for Bayesian network as a basis for the construction of the ionospheric networks. In our background, the measurements of the 1296 variables are all continuous. To build the directed network, we should determine the existence and directions of edges between any two variables from the holistic perspective instead of just considering the two ones. The Fast Greedy Equivalence Search (FGS) algorithm developed by Joseph Ramsey et al. works well for large numbers of continuous variables to build Bayesian networks. This algorithm utilizes the strategy that, edges are iteratively added starting with an empty network according to maximal increases in BIC score. Here, the variables' distributions are assumed to be Gaussian. We use the implementation of the FGS algorithm in the TETRAD package (Version 5.3.0-2, available at http://www.phil.cmu.edu/projects/tetrad/) and make the penalty discount is 10. TETRAD possesses a convenient user interface to enter preknowledge. The revisions are shown in lines [24-34] of page 4.

Issue 3: *I'm not sure what I should learn from Fig.1*

Response 3: Many thanks for the comment. We are sorry for the unclear presentation.

Fig. 1 is presented for providing some intuitive knowledge about the complex network of the ionosphere. As the ionospheric network includes 1296 nodes and 10,985 directed edges in the globe, it is hard to fully present such a complex network. Here, we exhibit part of the ionospheric network. The revisions are shown in Figure 1 and lines [1-3] of page 5.

Issue 4: *Regarding the spatial variations: The distance between connected grid cells is measured in terms of degrees (lat/lon). However, the spatial distance (in units of meters) between meridians varies with latitude (they are weighted with cos(lat)), and this severely biases the results shown in Fig.3: At high latitudes, a distance of, say, one degree, corresponds to a* *much shorter distance in space than at low latitudes, which produces apparent long-ranged connections if only measured in degrees. Before drawing any conclusions from the apparent asymmetry between latitudinal and longitudinal information transport, the distances should be translated to actual spatial distances, measured in meters!*

Response 4: Many thanks for the comment.

We all approve the reviewer's comment that the spatial distance (in units of meters) between meridians varies with latitude. As the reviewer infers, the results have severely biases. The latitude and longitude spans has been changed to the latitude and longitude distances. The height of the VTEC measurements supplied by CODE is $H = 450km$. As the measurements are on the earth which can be regarded as a sphere, the distances between any two positions can be calculated by the arc lengths on the sphere $d = R\theta$, where $R = R_0 + H$ and R_0 is the earth radius, θ is the corresponding central angle. In order to study the directional characteristics of the propagation of the dynamic ionospheric processes, the distances are mapped to the latitude and longitude directions. The revisions are shown in lines [3-7] of page 7.