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Nonlinear Processes in Geophysics
Discussions
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Interactive comment on "A general theory on frequency and time-frequency analysis of irregularly sampled time series based on projection methods. I. Frequency analysis" *by* Guillaume Lenoir and Michel Crucifix

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We are very grateful to the reviewer for the constructive comments and suggestions. We provide below a point-by-point reply to the reviewer's comments. In addition, during the review process, we got comments from other people that we judged pertinent to include in this revised version. The related minor changes are listed afterwards.
When we mention a section or an equation, we refer to the new version of the manuscript.

1 How long can the "gaps" in the time series be? How irregular can the time series be? This should be discussed.

The gaps can a priori be of any length. Indeed, the algorithms are constrained to avoid as much as possible the artifacts caused by aliasing, which are themselves caused by the gaps. This is explained in Sect. 4.5.4 of paper I and Sect. 3.8 of paper II. In paper II, we show that the algorithms do a good job at rejecting areas where artifacts are caused by big gaps, which is probably the most problematic case. The criteria presented in these sections give a basis which could be improved in subsequent studies.

2 Eq. (13): The notation of the SDE does not make sense to me. White noise is not differentiable. Wouldn't it make sense to write the MA part as a memory kernel?

This is now Eq. (14). We agree that the white noise is, strictly speaking, not differentiable. Things are now better explained: we still keep the same notation in Eq. (14), like in classical references in the field, e.g. Jones and Ackerson (1990) or Brockwell (2016, Sect. 11.5), but we explain that is interpreted through an Itô differential equation, which makes sense. We also define the white noise from the Brownian motion, in order to keep things clear for the specialists. All of this new material comes from the classical reference of Brockwell and Davis (2016, Sect. 11.5). Here is the modified part of the manuscript:
A CARMA(p,q) process is simply the extension of an ARMA(p,q) process to a continuous time[1]. A zero-mean CARMA(p,q) process $y(t)$ is the solution of the following

[1] A CARMA(p,q) process sampled at the times of an infinite regularly sampled time series is an ARMA(p,q) process.

stochastic differential equation:

$$\frac{\mathrm{d}^p y(t)}{\mathrm{d}t^p}+\alpha_{p-1}\frac{\mathrm{d}^{p-1}y(t)}{\mathrm{d}t^{p-1}}+...+\alpha_0 y(t)=\beta_q\frac{\mathrm{d}^q\epsilon(t)}{\mathrm{d}t^q}+\beta_{q-1}\frac{\mathrm{d}^{q-1}\epsilon(t)}{\mathrm{d}t^{q-1}}+...+\epsilon(t), \quad (1)$$

where $\epsilon(t)$ is a continuous-time white noise process with zero mean and variance σ^2. It is defined from the standard Brownian motion $B(t)$ through the following formula:

$$\sigma \mathrm{d}B(t)=\epsilon(t)\mathrm{d}t \quad (2)$$

The parameters α_0, ... , α_{p-1} are the autoregressive coefficients, and the parameters β_1, ..., β_q are the moving average coefficients. $\alpha_p=\beta_0=1$ by definition. When $p>0$, the process is stationary only if $q<p$ and the roots $r_1,...,r_p$ of

$$\sum_{k=0}^{p}\alpha_k z^k=0, \quad (3)$$

have negative real parts. Strictly speaking, the derivatives of the Brownian motion $\frac{\mathrm{d}^k B}{\mathrm{d}t}$, $k>0$, do not exist, and we therefore interpret Eq. (1) as being equivalent to the following measurement and state equations

$$y(t)=bw(t), \quad (4)$$

and

$$\mathrm{d}|w(t)\rangle = A|w(t)\rangle\mathrm{d}t+\mathrm{d}B(t)|e\rangle, \quad (5)$$

where $|b\rangle=[\beta_0,\beta_1,...,\beta_q,0,...,0]'$ is a vector of length p, $|e\rangle=[0,0,...,0,\sigma]'$, and

$$A=\begin{pmatrix} 0 & 1 & 0 & \dots & 0\\ 0 & 0 & 1 & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \dots & 1\\ -\alpha_0 & -\alpha_1 & -\alpha_2 & \dots & -\alpha_{p-1}\end{pmatrix}. \quad (6)$$

Equation (5) is nothing else but an Itô differential equation for the state vector $|w(t)\rangle$.

3 Does your significance test also test for trend significance?

There is no significance test for the trend. We rather build periodograms (Eq. (58), (59) or (64)) that are blind to the parameters of the trend (i.e. they are invariant with respect to these parameters).

4 All references in the manuscript are in the style (Author, year) but in many cases it should be Author (year). This should be corrected.

Corrected.

5 Page 18, line 7: “This it” should be “This is”

Corrected.

Other changes

- Notations: All is bra-ket now, instead of a mix between bra-ket and bold symbols. For example, $\overline{\mathrm{sp}}\{\mathbf{a}\}$ is changed to $\overline{\mathrm{sp}}\{|a\rangle\}$.
- The angular frequency in the model for the data is now denoted Ω (instead of ω), in order to make the difference between this frequency and the probed frequency by the periodogram which is denoted ω. See Sect. 3.1.
- In Fig. 11b, the periodograms are now normalized according to Eq. (58), which is more rigorous when comparing them. Visually, the results with the

ODP1148 data set are very similar as without this normalization, so that the discussion/interpretation remains unchanged.

References

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