



Lifetime estimate for plasma turbulence

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Abstract. A method is proposed to experimentally determine the intrinsic time scale or a decay rate of turbulent fluctuations. The method is based on the assumption that the Breit-Wigner spectrum model with a non-Gaussian frequency broadening is valid in the data analysis. The decay rate estimate is applied to the multi-spacecraft magnetic field data in interplanetary space, yielding that the result that the decay rate on spatial scales of about 1000 km (about 10 times larger than the ion inertial length) is higher than the theoretical predictions from the random sweeping time scale of the eddy turnover time. The faster decay of fluctuation components in interplanetary space is interpreted as a realization of plasma physical (and not fluid mechanical) process.

1 Introduction

Turbulent fluctuations appear in various fluid or gaseous media, and exhibit both temporally developing and spatially varying fields. The fluctuation properties are characterized by correlations between two spatial points and between two different times. An Fourier representation of the two-time, two-point correlation is the energy spectrum in the wavenumber-frequency domain.

Out experimental access to the wavenumber-frequency spectrum is limited. One can easily obtain the frequency spectra from time series data using a single probe, whereas one has to determine the wavenumber spectra using multiple probes simultaneously. Moreover, the Fourier transform cannot be performed if the number of probes is small (say, below 10). For this reason, Taylor's frozen-in flow hypothesis (Taylor, 1938) is widely implemented to the measurement of the turbulence energy spectra. Taylor's hypothesis assumes that fluctuating fields are "frozen-in" into the high-speed flow such that the time series data of fluctuations are interpreted as spatial structures swept by the flow and passing by the probe. In other words, the spectral energy is assumed to lie on the Doppler shift, $\omega = k_s U_0$, where ω denotes the frequency in the Eulerian frame, k_s the streamwise wavenumber (the wavenumber in the direction to the flow), and U_0 the mean flow speed.

Time evolution is neglected during the time period of measurements when one uses Taylor's hypothesis, so the intrinsic time scale information cannot be estimated from single-point measurements. Of course, Taylor's hypothesis breaks down whenever the frozen-in condition of the fluctuating field is violated, for example, when the flow speed has large-scale variations (the ran-



dom sweeping model), or when propagating waves exist in the turbulent fluctuations. Is it then ever possible to experimentally determine the intrinsic time scale in turbulence studies?

Here we develop a method to experimentally determine the lifetime of fluctuation components in plasma turbulence. The task of measuring the wavenumber-frequency spectrum is achieved by using multi-point magnetic field measurements by Cluster spacecraft in interplanetary space (Escoubet et al., 2001) and a high-resolution interferometric spectral estimator (Narita et al., 2011). The spatial scales of consideration is 1000 km (about 10 times larger than the ion inertial length). The broadening of the energy spectrum around the Doppler shifted frequencies (sideband formation) is interpreted as a realization of temporally decaying waves in turbulence. To the goal of measuring the lifetime, the concept of Breit-Wigner spectrum is introduced and applied to the spacecraft data. The measure of the frequency broadening around the Doppler shift also serves as a test for Taylor's frozen-in hypothesis in the spectral domain Taylor (1938).

2 Breit-Wigner spectrum

One may extend the frequencies from the real numbers (as oscillatory part), to the complex numbers by including an imaginary part for a temporal damping. We combine the excitation frequency ω_0 and the decay rate γ as

$$\omega_0 \rightarrow \omega_0 - i\frac{\gamma}{2} \quad (1)$$

The wave field Φ (e.g., flow velocity, density, or magnetic field) is expressed as

$$\Phi(t) = \Phi(0) \exp \left[-i \left(\omega_0 - \frac{i\gamma}{2} \right) t \right] \quad (2)$$

where $\Phi(0)$ is the initial wave amplitude. The wave energy is estimated by the square amplitude:

$$P(t) = |\Phi(t)|^2 = |\Phi(0)|^2 \exp[-\gamma t]. \quad (3)$$

The lifetime of wave excitation τ is an inverse of the decay rate

$$\tau = \frac{1}{\gamma}. \quad (4)$$

The decay rate is associated with the frequency broadening around the peak frequency in the Breit-Wigner spectrum.

The picture under consideration is that turbulent fluctuations are decaying while being excited intermittently or continuously at different wavenumbers to sustain the energy cascade balance. It is assumed that the fluctuations are excited and subject to decay with a rate equivalent to the frequency broadening γ .



The Fourier transform of the wave field (which turns out to be the same as the Laplace transform with respect to the decay rate γ):

$$\tilde{\Phi}(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \Phi(t) \exp[i\omega t] dt \quad (5)$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \Phi(0) \exp\left[\left(i(\omega_0 - \omega) + \frac{\gamma}{2}\right)t\right] dt \quad (6)$$

$$= \frac{1}{\sqrt{2\pi}} \Phi(0) \frac{1}{i(\omega_0 - \omega) + \gamma/2}. \quad (7)$$

The frequency broadening is a measure of wave lifetime. According to the Breit-Wigner formula in nuclear resonance phenomena of the compound-state formation (Breit and Wigner, 1936; Bohr, 1936),

$$P(\omega) = |\tilde{\Phi}(\omega)|^2 \quad (8)$$

$$= \frac{1}{2\pi} |\Phi(0)|^2 \frac{1}{(\omega_0 - \omega)^2 + (\gamma/2)^2} \quad (9)$$

$$= \frac{1}{\gamma} |\Phi(0)|^2 f_{\text{BW}}(\omega, \gamma) \quad (10)$$

where

$$f_{\text{BW}}(\omega, \gamma) = \frac{1}{\pi} \frac{\gamma/2}{(\omega_0 - \omega)^2 + (\gamma/2)^2} \quad (11)$$

is the Breit-Wigner distribution widely applied in the field of nuclear resonance phenomena. The Breit-Wigner distribution has a higher probability than that of a Gaussian distribution at a position or a variable value x larger than the standard deviation of the distribution (Fig. 1). The peak of f_{BW} at the resonant frequency ω_0 is given as $f_{\text{BW}} = \frac{2}{\pi\gamma}$. The half-value of the peak is $\frac{1}{\pi\gamma}$, and The half-value width is realized when the decay rate γ satisfies the condition

$$\frac{1}{\pi\gamma} = \frac{1}{\pi} \frac{\gamma/2}{(\Delta\omega/2)^2 + (\gamma/2)^2}, \quad (12)$$

namely,

$$\Delta\omega = \gamma. \quad (13)$$

Thus, the decay rate can be estimated by measuring the half-value-width frequency of the spectral peak. The decay rate γ is directly associated with the lifetime of the excited wave component as $\tau = \gamma^{-1}$, and justifies Eq. (4).

The shape of the Breit-Wigner spectrum depends on two parameters: the spectral peak frequency ω_0 and the half-value-width (or the decay rate) γ . A fitting procedure is needed to estimate the two parameters from the measured spectrum. In this work, the Levenberg-Marquardt algorithm for the least square fitting (Levenberg, 1944; Marquardt, 1963) is used with a detailed numerical implementation in Press (1992).

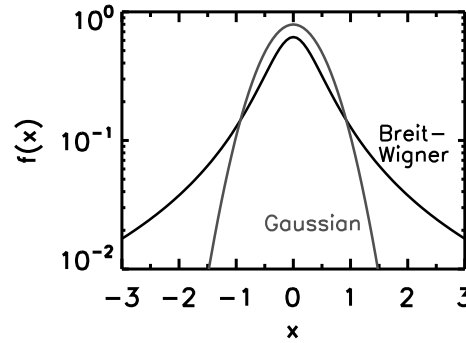


Figure 1. Breit-Wigner distribution for a half-value-width of 0.5 ($\omega_0 = 0, \omega \rightarrow x, \gamma/2 = 0.5$ in Eq. (11)) in black and Gaussian distribution with a standard deviation of $\sigma = 0.5$ (in gray).

3 Application to space plasma

Observational setup

The Breit-Wigner spectrum is tested against turbulent fluctuations in interplanetary space. Four Cluster spacecraft (Escoubet et al., 2001) encounter the solar wind ahead of the Earth bow shock. The observational setup is illustrated in Fig. 2 for the time interval on 17 March 2005, 1000–1200 UT. The Cluster spacecraft are in the solar wind and form a nearly regular tetrahedron with an inter-spacecraft distance of about 1000 km. For reference, the ion inertial length is on the order of 100 km in the solar wind. The solar wind plasma streams radially away from the sun at a mean speed of about 373 km/s, measured by the electrostatic ion analyzer on board the Cluster (Rème et al., 2001). The interplanetary magnetic field is almost in the plane spanning the sunward and the duskward directions, and is highly inclined downward. The mean magnetic field is: $B_x = 2.8$ nT, $B_y = -4.2$ nT, and $B_z = 0.6$ nT in the GSE (geocentric solar ecliptic) coordinate system. The mean ion number density is about 13 cm^{-3} . The ion temperature is 0.18 MK and 0.15 MK parallel and perpendicular to the mean magnetic field, respectively. The observed time interval represents a low-speed plasma stream of the solar wind, and exhibits a large-scale pressure-balanced structure between the magnetic pressure and the ion thermal pressure, indicated by an anti-correlation of the field variations between the magnetic field magnitude and the ion number density in Fig. 3. The observed time interval is suited to testing for the Breit-Wigner spectrum for the nearly constant mean plasma flow speed over the interval.

Streamwise wavenumber-frequency spectrum

The streamwise wavenumber-frequency spectrum for the magnetic field fluctuations is displayed in Fig. 4. The spectrum exhibits an extended structure from the origin of the spectral domain (zero wavenumber and zero frequency) to a wavenumber of xxx and a frequency of xxx. The spectral extension is almost linear, and the slope of the extension (the propagation speed estimated by from the ratio of the frequency to the wavenumber) roughly agrees with the mean flow speed, xxx km/s. The

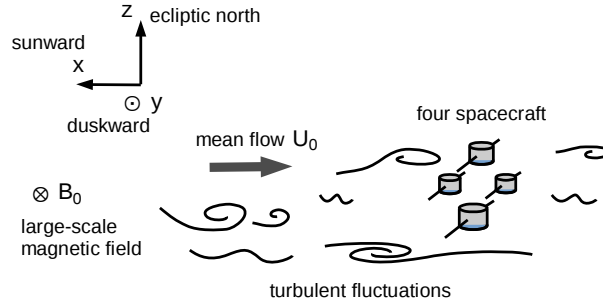


Figure 2. .

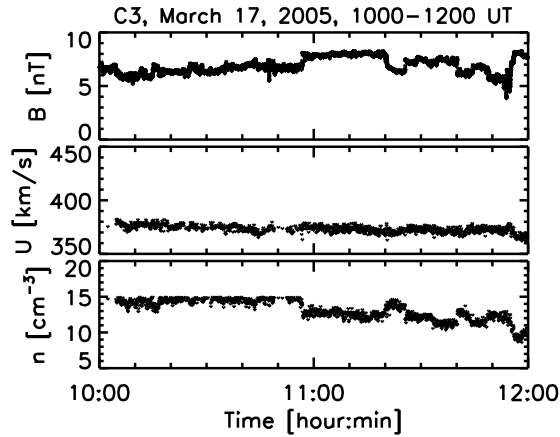


Figure 3. Time series data.

spectral extension shows a broadening, which can be interpreted either as a broadening over the wavenumbers or that over the frequencies. We perform the data analysis by interpreting that the spectral broadening appears along the frequency axis and the broadening is a function of the wavenumbers.

5 Fitting to the Breit-Wigner spectrum

The wavenumber-frequency spectrum is obtained for the magnetic field data of Cluster (Balogh et al., 2001) using the wave telescope projection technique with an MSR extension (multi-point signal resonator) to eigenvalue-based projection (Narita et al., 2011). The spectrum is displayed in Fig. 5, and is analyzed to determine the peak frequency and the broadening frequency using the Breit-Wigner form of spectrum. To perform the fitting, the energy spectrum is sliced over the frequencies at a given value of streamwise wavenumber. Three examples of the frequency slice are displayed as one-dimensional spectra in Fig. 5 (curved lines in black) at wavenumbers of 0.0010 rad/km, 0.0015 rad/km, and 0.0020 rad/km.

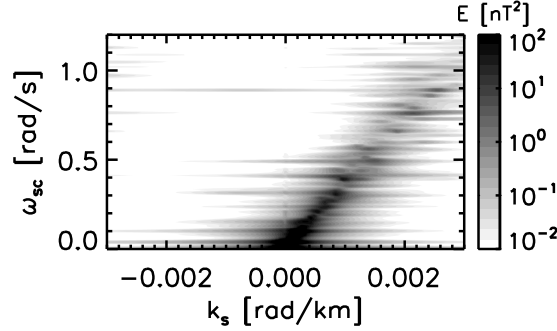


Figure 4. Eulerian (spacecraft-frame) energy spectrum for magnetic field fluctuations in the solar wind in the streamwise wavenumber-frequency domain. The measurements of Cluster spacecraft on a time interval of 17 March, 2005, 1000–1200 UT are used.

For the least-square fitting procedure, the Levenberg-Marquardt algorithm is applied to each frequency slice on the logarithmic scale of the spectral energy. The one-dimensional slices of the spectra and a fluctuation level of the spectra are used as inputs to the least-square fitting. An fluctuation amplitude of 0.2 nT^2 is used in the least-square fitting, which comes from the measurement of the fluctuation level of the spectrum. The peak frequency ω_0 (at which the spectrum maximizes) and the broadening frequency γ (which is a measure of the decay rate) are obtained as fitting outputs. We obtain, for example, $(\omega_0, \gamma) = (0.308 \pm 0.010 [\text{rad/s}], 0.056 \pm 0.002 \text{ rad/s})$ at a wavenumber of 0.0010 rad/km , $(\omega_0, \gamma) = (0.465 \pm 0.010 [\text{rad/s}], 0.186 \pm 0.003 \text{ rad/s})$ at a wavenumber of 0.0015 rad/km , and $(\omega_0, \gamma) = (0.635 \pm 0.020 [\text{rad/s}], 0.463 \pm 0.002 \text{ rad/s})$ at a wavenumber of 0.0020 rad/km , respectively. The best-fitted spectra are over-plotted in Fig. 5 in gray curves.

The increasing sense of the peak frequencies and the half-value-widths at larger wavenumbers are quantitatively tracked by repeating the fitting procedure at various wavenumbers (Fig. 6). The increasing sense of the peak frequencies reasonably agrees with the Doppler shift (dotted line), represented by $\omega = k_s U_0$, which justifies the picture of ideal convection (Taylor's hypothesis) in the lowest-order picture (talking only about the peak frequencies).

The measured half-value-widths γ are interpreted as the decay rate associated with the respective peak frequencies, and are compared to the estimate for the random sweeping (dotted line), represented by $\Delta\omega = \sqrt{k_s^2 \delta U^2} \propto k_s$, and that for the eddy turnover time (dashed line), represented by $\Delta\omega = \sqrt{k_s^2 u^2} \sim \sqrt{k^2 (\epsilon^{1/3} k_s^{-2/3})^2} \sim \epsilon^{1/3} k_s^{2/3}$. Here the value of the energy transfer rate ϵ is fitted to the measured curve of the decay rate to $\epsilon = 27 \text{ rad km}^{-2} \text{ s}^{-3}$ at wavenumbers below 10^{-3} rad/km . At wavenumbers above 10^{-3} rad/km the decay rate γ becomes increasingly larger at smaller wavenumbers. The measured increase rate is faster than the prediction from the random sweeping model and that from the eddy turnover time. In other words, the lifetime of the fluctuation components is shorter than the that in fluid turbulence. The increasing sense of the half-value-widths indicates that the mapping quality from the wavenumbers onto the frequencies becomes diminished at larger wavenumbers; the picture of ideal convection (Taylor's hypothesis) breaks down in the second-order picture.

Interpretation as fluid picture. Doppler shift and Breit-Wigner type broadening
 rough estimate of mapping quality of Taylor's hypothesis

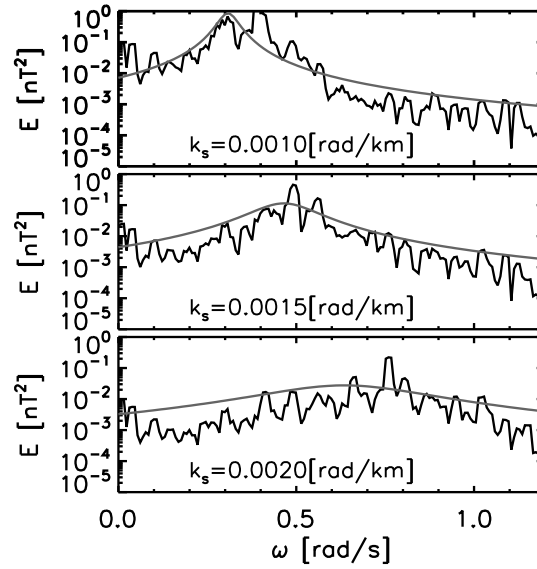


Figure 5. One-dimensional slice of the energy spectrum over the spacecraft-frame frequencies at a streamwise wavenumber of 0.015 [rad/km] from Fig. 4 (in black) and a fitted curve by the Breit-Wigner distribution (in gray) for $\omega_0 = 0.49$ [rad/s] and $\gamma = 0.08$ [rad/s].

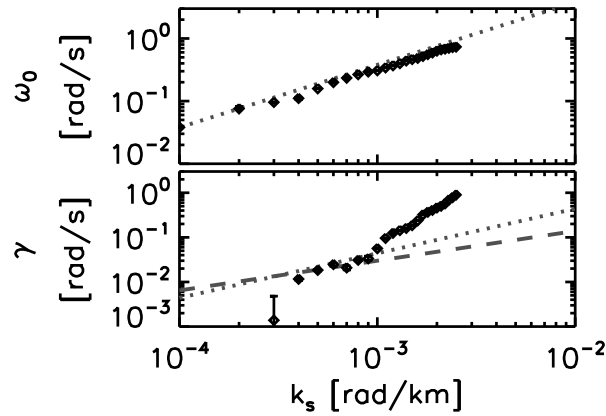


Figure 6. Peak frequency ω_0 (upper panel) as a function of the streamwise wavenumbers with comparison to the Doppler shift (dotted line) and half-value-width γ (lower panel) with comparison to the random sweeping (dotted line) and eddy turnover time (dashed line).



4 Discussion and Outlook

Decay rate is not constant over the spatial scales, but becomes increasingly larger (faster decay) toward smaller spatial scales. The increasing sense of decay at larger wavenumbers agrees with the picture of fluid turbulence, but the decay rate becomes increasingly larger than anticipated from the fluid turbulence models (random sweeping model, eddy turnover model). The larger values of the decay rate should be interpreted as a nature of collisionless plasma turbulence.

Various scenarios are possible to explain the deviation of the decay rate from the estimates based on fluid turbulence (random sweeping time or eddy turnover time). First, wave-wave interactions are in operation and generate other fluctuation types such as electric field or density fluctuations, transferring the magnetic field fluctuation energy into the other energy types. Second, wave-particle interactions are in operation, too, such as coherent scattering (Landau or cyclotron resonance) or incoherent scattering (pitch angle scattering). It is interesting to note that the sudden increase of the decay rate is not directly associated with the ion kinetic scales, since the wavenumber for the ion inertial length (assuming protons) is about 0.016 rad/km. Whether the sudden increase of the decay rate is due to a magnetohydrodynamic effect or an ion-kinetic effect remains an open question.

To conclude the manuscript, we raise several items that should be studied more elaborately to strengthen (or disprove) the use of the Breit-Wigner spectrum in plasma turbulence research.

1. Invariance of the spectral index of one-dimensional energy spectra between the wavenumber domain $E(k)$ and the frequency domain $E(\omega)$. The invariance holds in the case of Gaussian frequency distribution. For the Breit-Wigner frequency distribution, the invariance of the spectral index is not net guaranteed. Also, error estimate for Taylor's hypothesis needs to be recalculated for a non-Gaussian shape of frequency distributions. For a Gaussian frequency distribution, the error of Taylor's hypothesis can conveniently expressed by the Gauss error function.
2. The spectral peak may break up into multiple branches when different linear mode waves are excited in the turbulent field at once. The frequency slice of the spectrum will show a convolution of different branches (with different peak frequencies and different half-value-widths).
3. Another way to express the non-Gaussian shape distribution is to introduce higher-order moments. The fourth-order moment (in a non-trivial manner by measuring the deviation from a Gaussian shape, e.g., kurtosis or flatness index) is particularly suited to such a task. Perhaps there is a relation between the decay rate and the fourth-order moment of the frequency spread around the spectral peak.

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