Study on Topological structure of sandstone 3D pore networks

Guannan Liu [1,2*], Xiaopeng Pei [3], Jishan Liu [1,2], Dayu Ye [3], Feng Gao [1,2], Jianguo Wang [1,2]

[1]Mechanics and Civil engineering Institute, China University of Mining and Technology, Xuzhou city, Jiangsu province 221000, China
[2] State Key Laboratory for Geomechanics and Deep Underground Engineering, China University of Mining and Technology, Xuzhou city, Jiangsu province 221000, China
[3] Chemical engineering Institute, China University of Mining and Technology, Xuzhou city, Jiangsu province 221000, China
[*]correspondence to Guannan Liu (guannanliu@cumt.edu.cn)

Abstract: There are a large number of pores and throats in rock, the size and shape of which are different, and the connection status is complicated. Based on complex network theory and X ray CT scanning techniques, we take sandstone as an example to investigate the topological characteristics of sandstone pore networks. The results show that the pore network of sandstone is similar to that of scale-free network. The average path length increases with the increase of network size (number of nodes). It is concluded that a few special pore nodes play an important role in the overall connectivity of the percolation network, and the nodes are ranked by the importance in the network. By analyzing the pore network modularity, it is concluded that the sandstone pore network has a homogeneous community structure, which means there is no dominant communities in the networks. The paper try to provide a new perspective for the study of the mechanism of fluid storage and transport in rock and other porous materials.

Key words: Complex network theory sandstone network topology Seepage distribution of degree

Introduction:

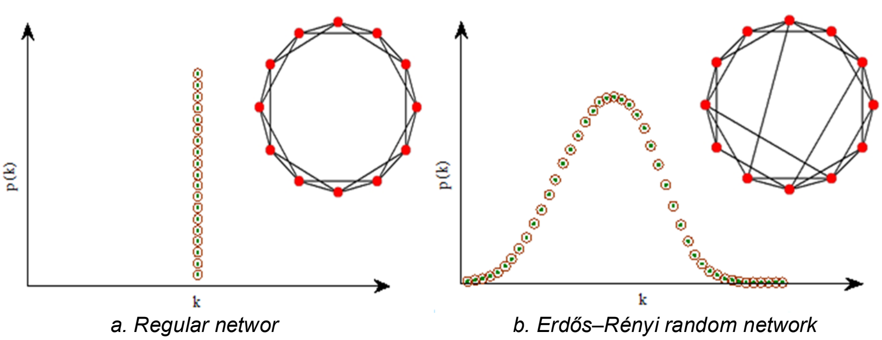
The pore morphology in rocks are different, cross-scale distributed, and the connection structure is complex[1-3].It has great theoretical and practical significance to study the characteristics of rock for the accurate and quantitative evaluation of rock permeability, revealing the mechanism of oil and gas accumulation and migration, and enhancing oil recovery[4-6].

In recent years, a lot of research on the statistical characteristics and evolution model of rock pore structure have been done by many scholars. Blunt and Raeini developed a generalized network extraction method for three-dimensional digital core to reduce the uncertainties of modeling[7] . Using a focused ion beam scanning electron microscope (FIB-SEM), Shaina K. [8] obtained a microscale shale digital core and compared it with conventional scanning electron microscopy in terms of porosity, organic content, and pore connectivity. Using statistical method, Hajizadeh took the CT scan image data of sandstone as a sample, and coupled the continuous two-dimensional multi-point statistical simulation to the multi-scale data extraction program, and proposed a random porous media reconstruction technique[9]. As in the traditional Darcy law is no longer suitable for dense rock, Civan[10] used the modified Darcy law to describe the gas migration in dense shale, in which the apparent permeability is a function of intrinsic permeability and porosity. Considering the basic concept of pore network model, a modeling method based on pore volume and throat searching is proposed by Zhang[11] , and combined with percolation theory, the permeability of different models is calculated. Yang J. [12] used fractal control function to describe the complex morphology of rock pore structure, and proposed a fractal reconstruction model of rock pore structure with improved simulated annealing algorithm.

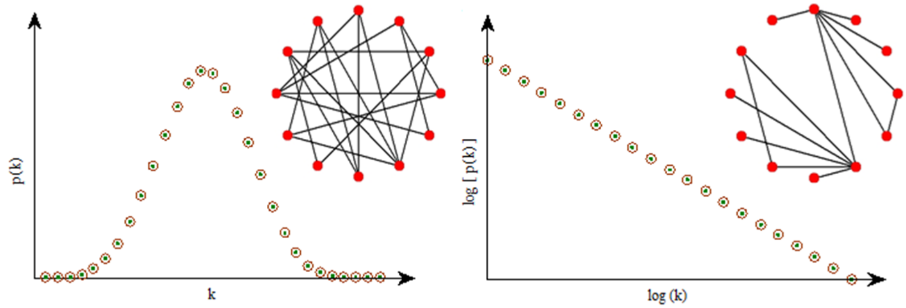
At present, the characterization of pore connectivity structure is mainly studied from macro average perspective or by Eular number. The comprehensive topological characteristics of the porous network such as the degree of clustering and the choice of percolation path are still need to be studied. In this paper, from the point of view of complex network theory, aiming at the real sandstone pore network, the topological characteristics and correlation properties of pore throat network are analyzed to provide the basis for revealing the microscopic mechanism of porous rock seepage.

1. Type of network structure

Regular network, ER random network and WS small world network are three classical models to describe network structure. Regular networks (Fig.1.a) are equal in degrees for each node, and have minimal average path length (average of the minimum number of edges among all node pairs) in all networks with the same number of nodes . In ER stochastic networks, the degree distribution satisfies the Poisson distribution, and the average path length is larger. As a transition from regular network to stochastic network, the degree of BA small world network (Fig.1.c) can be approximately represented by Poisson distribution, especially for the case where the number of nodes is large, and the average path length is very small. As a model for heterogeneous networks, the degree of BA scale-free network model obeys power-law distribution , and there is no obvious eigenvalue, which can describe the real network topology better. Empirical study [13-16] shows that many real networks are heterogeneous networks.



a. Regular networ b. Erdős–Rényi random network

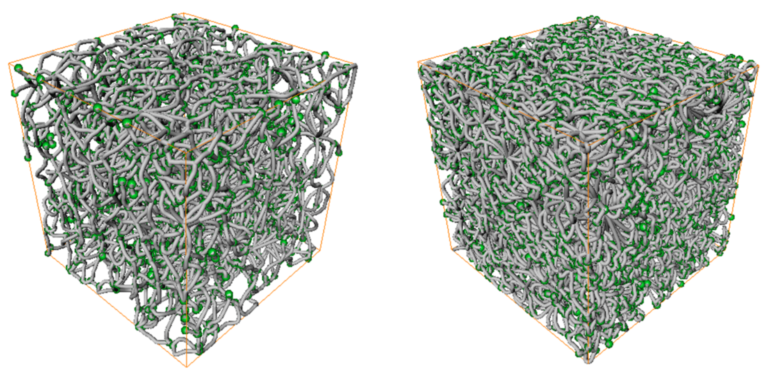


c. WS small world network *d. BA scale-free network*

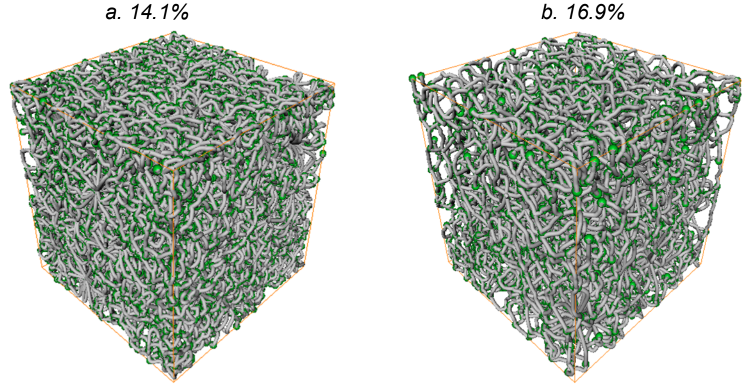
Fig.1 Degree distribution of different network models

2. Sandstone pore networks structure

The data of sandstone core constructed by Professor Blunt of Imperial College London is used in our work. Image analysis uses Micro-CT produced by German Phoenix company was used to analysis the image, which equipped with a 1 m focus system with a view of 512 * 512 pixels with a 8 inch 16 bit detector. Four sandstone samples with porosities of 4.1%, 16.9%, 17.1% and 24.6% are used in our work. The three-dimensional sandstone pore network of the four digital cal is shown in Fig. 2[17-18].



a. 14.1% *b. 16.9%*



c. 17.1% *d. 24.6%*

Fig. 2 Sandstone three-dimensional pore network (Bounding Box volume: 300 μ m by 300 μ m by 300 μ m. The pore network is extracted by the skeleton algorithm in the digital rock reconstructed from the X ray CT scanner)

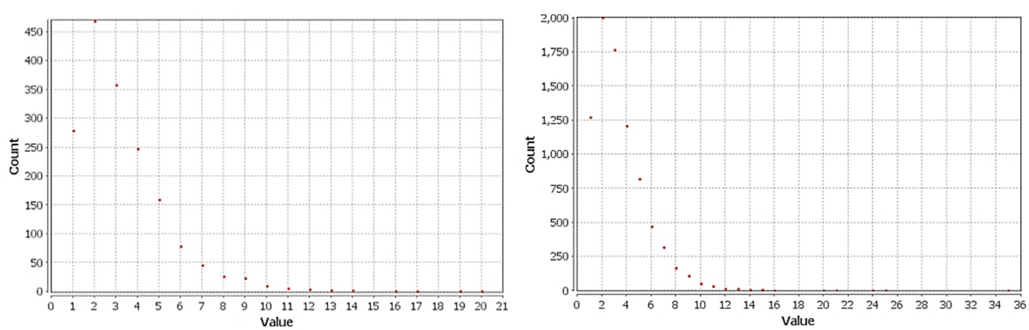
Because the isolated pores do not influence the seepage process, they were removed during the pore network construction of sandstone. Based on the complex network theory, the basic information of the pore network structure of the simplified sandstone was analyzed, as shown in table 1. As shown in Table 1, the pore network properties of sandstone are mainly related to network size (nodal number) rather than sandstone porosity, When the scale of the network increases, the diameter of the network and the average path length show a general trend of growth.

3. Results and discussion

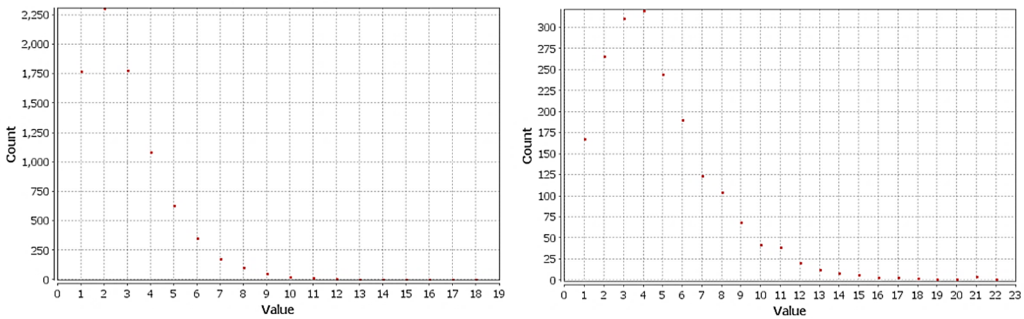
Table 1 Basic information of pore network

Porosity ϕ	Node Number N	Edge Number M	Average Degree $\langle k \rangle$	Network Diameter d	Average Path length L	Network Density D
14.1%	1717	2824	3.289	25	10.151	0.002
16.9%	8301	14381	3.465	46	16.469	0
17.1%	8542	12432	2.962	50	19.428	0
24.6%	1945	4697	4.83	22	8.466	0.002

First, the degree of distribution of nodes in the network is analyzed, as shown in figure 3. The results show that when the porosity is 14.1%, 16.9% and 17.1%, the nodes which degree are 2 have a moderate proportion, and when the porosity is 24.6%, the highest proportion of nodes are the nodes which degree are 4. When the degree exceeds these values, the number of nodes decreases abruptly as the degree increases. The porosity of sandstone network with different porosity has no obvious characteristic value. Therefore, the sandstone seepage network belongs to a class of scale-free networks [19-20].



a. 14.1%   b. 16.9%



c. 17.1% d. 24.6%

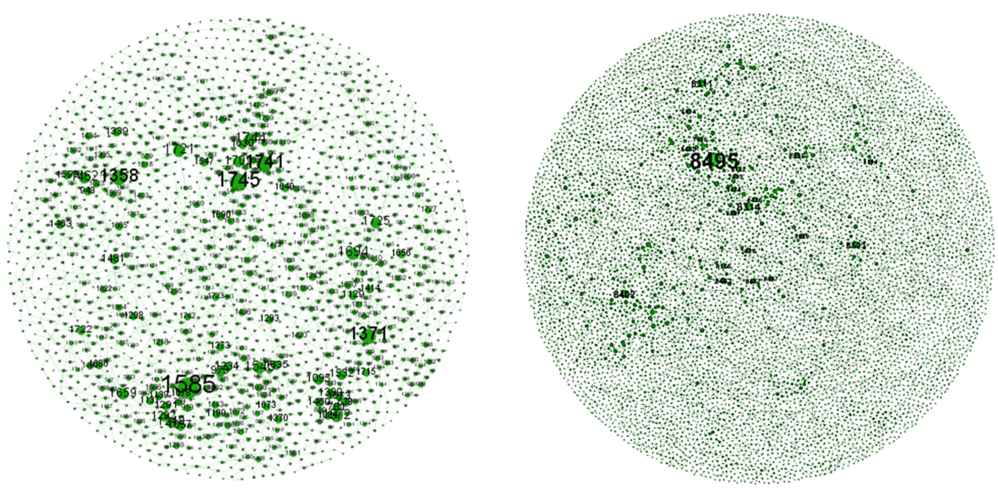
Fig. 3 Degree distribution of sandstone three-dimensional pore network

In order to analyze the importance of individual nodes in sandstone pore networks, the eigenvector centrality is used to compute the network nodes. Make x_i as the index value of node i, and A_{ij} as the adjacency matrix of the network. For node i, the centrality index is proportional to the exponential sum of all nodes connected to it。That is:

$$x_i = \frac{1}{\lambda} \sum_{j \in M_i} x_j = \frac{1}{\lambda} \sum_{j=1}^{N} A_{i,j} x_j$$

Where M_i is the set of nodes that are connected to node i, λ is constant.

To facilitate the intuitive understanding, the Fruchterman Reingold algorithm is used to distribute the pore nodes(Fig.2) uniformly in the circular surface. And the node centrality is sorted as shown in Fig.4. Where the node size represents the node centrality, and the number represents the node number.



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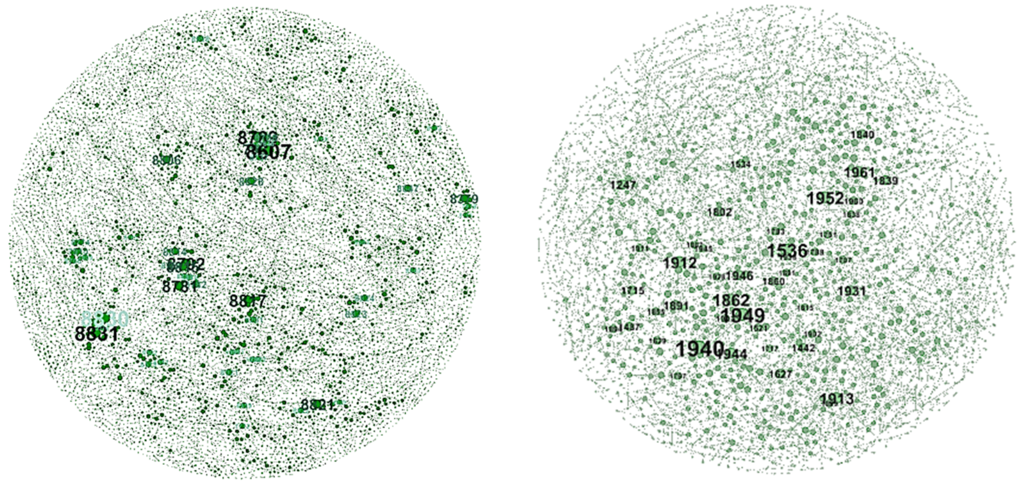


c. 17.1% *d. 24.6%*

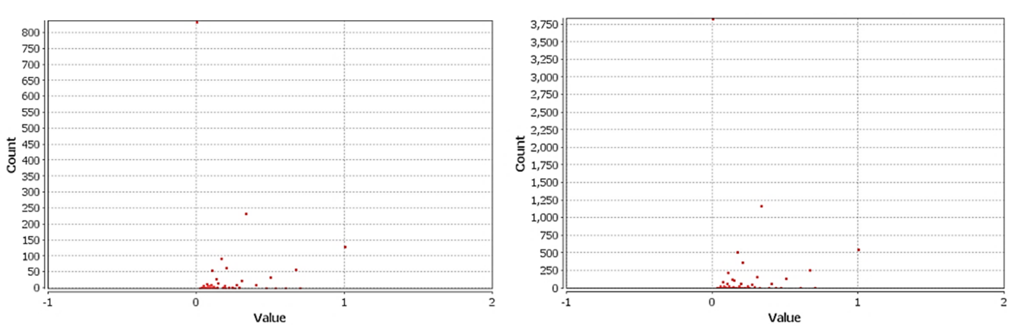
Fig. 4 centrality sorting of sandstone pore nodes

The clustering coefficient is the coefficient that indicates the degree of node aggregation in a graph. In the network of reality, the higher the clustering coefficient, the more tend the nodes tend to be more closely connected. In the network, if the node i is connected to the node $i+1$ and the node $i+1$ is connected to the node $i+2$, then the node $i+2$ may be connected to the i. This phenomenon reflects the connection density between nodes. In an undirected network, the clustering coefficients can be expressed as:

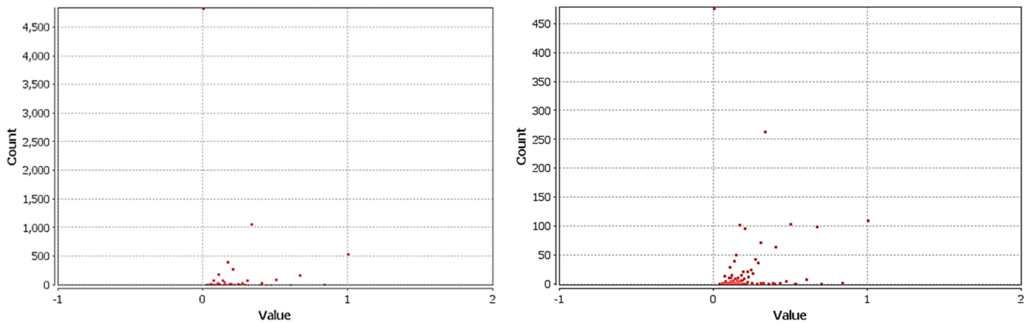
$$C = \frac{n}{C_k^2} = \frac{2n}{k(k-1)}$$

Where k Represents the number of adjacent nodes of the node i; n represents the number of edges that are connected among all adjacent nodes of the node i.

The clustering coefficient distribution of sandstone pore network is shown in Fig. 4.



a. 14.1% *b. 16.9%*



c. 17.1% d. 24.6%

Fig. 5 clustering coefficient distribution of sandstone three-dimensional pore network

Fig.5 shows that the clustering coefficients of different sandstone pore networks are mainly distributed between 0 and 0.5. The nodes with the clustering coefficient of 0 have the largest proportion, and the number of nodes decreases as the clustering coefficient increases. It shows that sandstone pore network has less connectivity than the global coupling network which clustering coefficient is 1. According to figure 2, the main reason is that the main pore connected to the sandstone pore is the peripheral pore node. In general, as the distance increases, the probability of connection between two nodes become weaker.

The degree of aggregation of the entire network can be represented by the average value of the clustering coefficients of all nodes, as shown in Fig. 6. Networks with higher average clustering coefficients have smaller average distances in different nodes. Fig. 6 shows that the average clustering coefficient is the largest when the porosity is 24.60% in the sampling sandstone, and as shown in Table 1, the average path length is minimum, that is, the network has the best connectivity among the four sandstone sample.

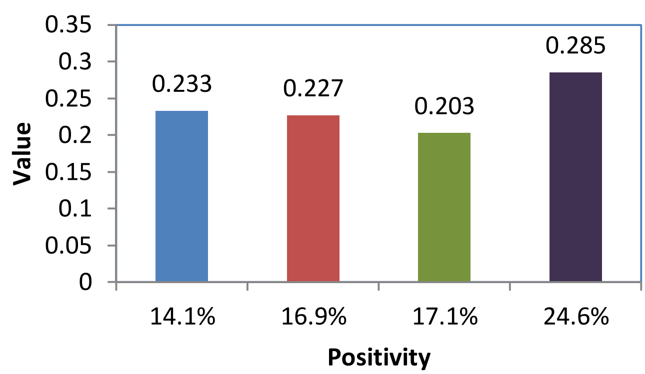


Fig. 6 Average clustering coefficient of sandstone pore network

The closer connections between adjacent nodes in the network, the larger the density between them, and the relatively independent node modules may form in the process of flow. The modular algorithm proposed by Blondel is used to analyze sandstone pore networks for community properties. We calculate the density of each sub network after partition, and then calculate the density of the sub network under the condition of random connected. There is a difference between these two densities, which represents the degree in which the sub network deviates from a random situation. The greater the value, the more dense the network is, compared with the random network.

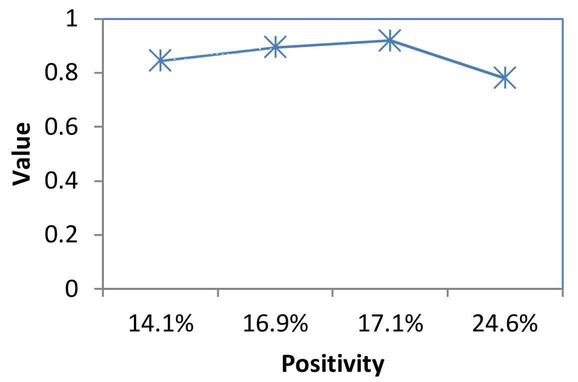
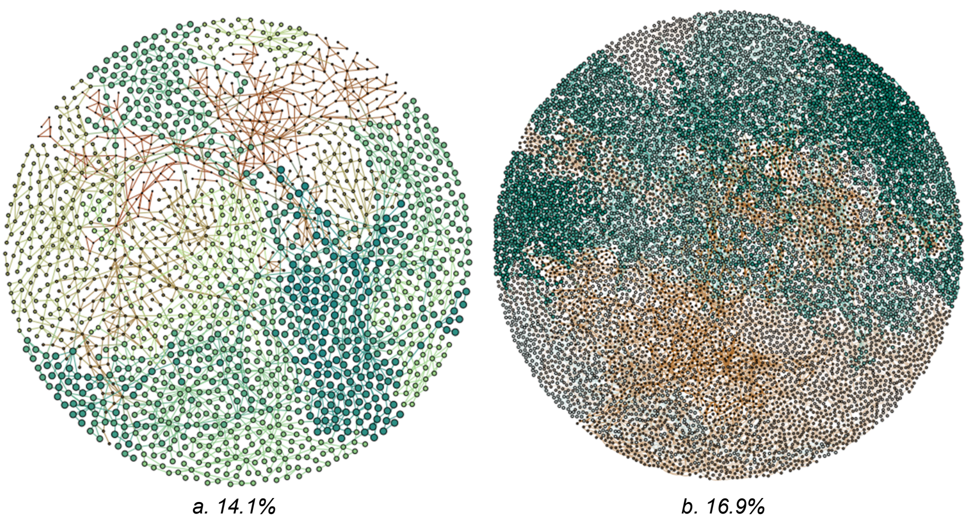
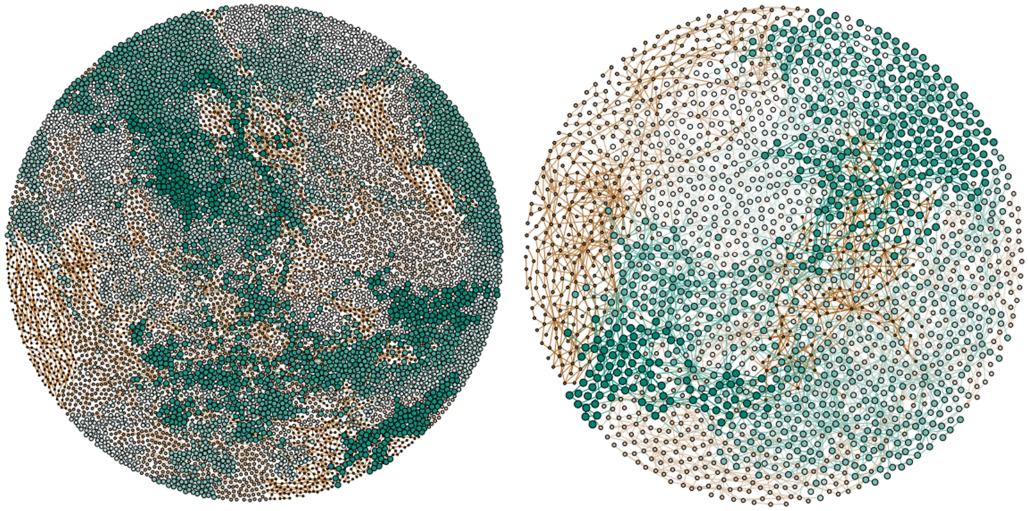


Fig. 7 Sandstone pore network modularity

The sum of all subnets included in a network is the modular degree of this complex network. The calculation results are shown in Fig. 7. When the porosity is 17.1%, the pore network module of sandstone is approximately equal to 0.919, and the pore size of different sandstone pore networks is close to 1. It shows that sandstone pore network has good community structure. The nodes are grouped according to different societies, and the results are shown in Figure 8, where the same community nodes are of the same color and size.



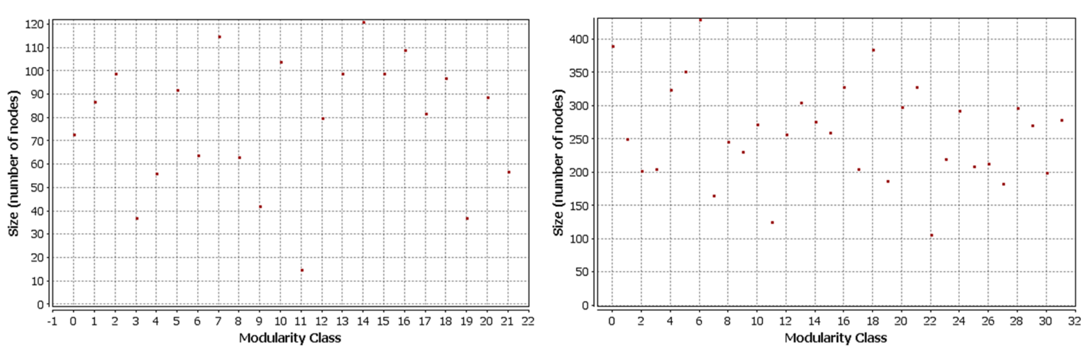
a. 14.1% *b. 16.9%*



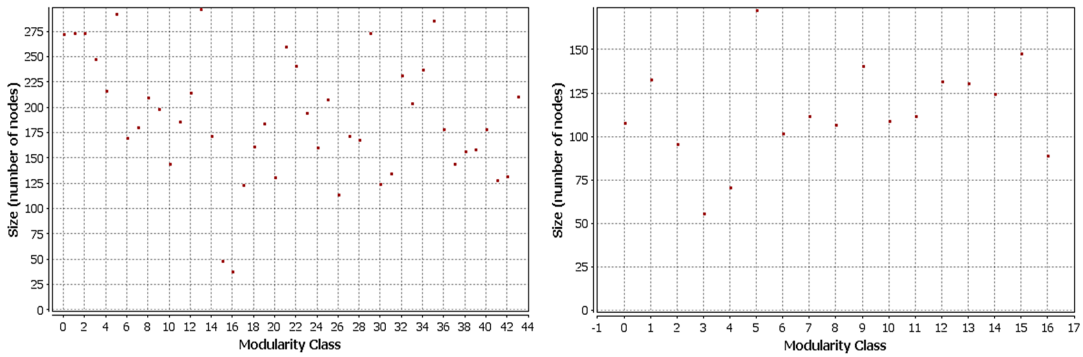
c. 17.1% d. 24.6%

Fig. 8 Association structure of sandstone pore network

The number of each network community contains are shown in figure 9. As shown in Figure 9, when the porosity is 14.1% and 24.6%, the number of nodes in the pore network is less, the number of nodes contained in each node community is between 50 and 130. When the porosity is 16.9% and 17.1%, the number of nodes in a single community increases obviously. In the four samples, when the porosity is 16.9% and the Modularity Class is 6, the maximum number of nodes in the network community is about 450, about 1/20 of the total number of nodes in the network. It shows that sandstone pore network distribution is uniform, and there is no phenomenon that minority nodes occupy a dominant position.



a. 14.1% b. 16.9%



c. 17.1% *d. 24.6%*

Fig.9 Distribution of node numbers in network community of sandstone

4. Conclusions

The topological structure of sandstone percolation network is the result of its physical and chemical properties, and the external environment. The calculation results show that the scale-free network can presents the topological structure of the sandstone percolation network. When the porosity is 24.6%, the clustering coefficient is the largest, and the connectivity of the network is the best. From the properties of scale-free networks, it can infer that a few nodes play the role of "Hub" in network connectivity. Therefore, it can be predicted that if the number of large degree pores increases, even if the number is small, the connectivity of sandstone pores may be greatly altered, thus greatly enhancing permeability. Then we sorted the nodes by the importance of the sandstone percolation network. In addition, we get sandstone pore network with large modularity, and can be divided into several similar small communities. Although we do not know the quantitative relationship between the average path length and the absolute permeability of the key nodes to the porous sandstone, the conclusions of this paper may provide a reference for the exploitation of underground oil and gas resources from a new angle.

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