Comments from Referees,

The author would indeed prefer to answer these three questions together.

Q (1) It is not clear (in the worst case, not correct) that the long-range correlation of a stationary time series (with 0 < h(2) < 1) can be discerned from a non-stationary process. Stationarity and non-stationarity are different characteristics and such relationship that makes one to be discerned from the other is obscure.

Q (2) Moreover, the authors have not clarified, or not mentioned at all, what type of nonstationarities would affect their data, so that the application of the MFDFA directly would produce misleading results. If the nonstationarities of their data are among those types that MFDFA would be able to deal with, why the pre-processing is proposed in the next sections?

Q (3) It is not clear the concern of the authors in selecting the stationary intervals of signals before using the MFDFA, if the MFDFA is already capable to deal with nonstationarities. Moreover, the authors have not clarified, or not mentioned at all, what type of nonstationarities would affect their data, so that the application of the MFDFA directly would produce misleading results. If the nonstationarities of their data are among those types that MFDFA would be able to deal with, why the pre-processing is proposed in the next sections?

Answer:

In general, there is broad agreement on the **appropriateness** of **MFDFA** in studying multifractal scaling behaviour of **non-stationary** time series, but we wish to draw your attention to the paragraph **"III. ANALYSIS OF SUNSPOT TIME SERIES"** of **Movahed (et al., 2005**) which explains the reasons why further attention must be taken in the analysing the stationarity of signals in the **pre-processing step**, before feeding them into the cycle of Fractal analysis. In connection with this point, we **MUST** make it absolutely clear that **inherent** non-stationarity of signals should **never** be confused with the concept of non-stationarity made by **external** perturbations. Regularly, **fractional Gaussian noises** (fGn) are involved in the **inherently** stationarity process, in contrast, **fractional Brownian motions** (fBm) are linked to the **inherently** non-stationarity process. The stationarity of fGn signals can be characterized by two parameters, σ^2, the variance, and H, the Hurst coefficient, while a fBm process has a time dependent variance. Hence, on the basis of the class to which signals belong, **different** techniques may be required for processing. Failure to match signal class with the appropriate method of fractal analysis results in **serious error** in the estimating H. For instance Dispersional analysis (Disp) is recommended to the analysing the fractionality of fGn signals, while bridge detrended scaled windowed variance analysis (bdSWV) is suitable for fBm signals (Eke et al., 2000). These classes might not be **a-priori known,** so a preliminary interpretation in this regard may be available by **fitting a straight line** of slope $-\beta$ on a log–log plot of the periodogram. Based on this method, signals can be categorized according to the **value** of β, but Eke et al., (2000) placed emphasis on this point that this method is only applicable if β falls into the category $-1<\beta<0.38$ (for an obvious stationary case) or if falls into the range of $1.4<\beta<3$ (for an obvious non-stationary case), but there is no certainty that this method fully comply with the complicated characteristics of signals in the range of $3.8<\beta<1.4$ where stationary and non-stationary **mixed into each other**. Based on this assumption, it is essential to provide another reliable framework for a regular monitoring of stationarity of signals. Signal summation conversion method (SSC) is advised to use as a discriminating method (Eke et al., 2000). An alternative approach for distinguishing fGn signals from fBm signal was proposed by Movahed et al., (2005) who experimentally attempted to examine the feature of Sunspot Time Series by analysing the behaviour of standard deviation of its time series as a function of time scale. There is a growing consensus amongst researchers (Zhong, et al., 2015; Wang et al., 2014) that **Time-Frequency Surrogate Analysis (TFSA)** proposed by Borgnat et al. (2010) can provide a complementary view for testing stationarity of seismic signals in an operational sense. This method is statistically characterized on the basis of a set of **surrogates** which all share the same average spectrum as the analysed signal while being stationarized. With this short introduction, the answers to above-mentioned question can be summarized briefly: We therefore agree with this statement in your question: **Stationarity and non-stationarity are different characteristics**, and this is exactly what we need. In other words, this difference enables us to discern a stationary **fGn** from non-stationary **fBm.** We want to underline the point that nonstationarities from our point of view are those corresponding to inherent non-stationarity which should be known at the pre-processing step before making our choice between fGn and fBm process. TFSA is a rigorous and sufficiently flexible method which not only provides an opportunity for assessing the inherent stationarity/non-stationarity of a signal, but also further it may bring information from their **state** of stationarity. Therefore, another strength of TFSA lies in its capability to constrain the length of stationary of signal by quickly and reliably validating tasks. This would be of utmost importance if our data encompass the microseismic range of frequency (0.1-0.3 Hz) or if data were acquired from the surveys at the vicinity of dome-looking topography features seismic signals appear mostly in the **quasi-stationary** state. In those cases, reproducible seismic signals could fall into the one of the following states: **macroscale**, **mesoscale** or **microscale** state (Fig 1), therefore, more accurate method is needed in order to properly assess the state of this **quasi-stationarity.** In this paper, we do make a point that the **length** of signal directly impacts on the reliability of Long-range autocorrelations assessment. The importance of this factor was previously the subject of other investigations such as Delignieres et al., (2006) and Warlop et al., (2017), but this point has been hidden from view in analysing the fractality of seismic signals. The importance of this issue is strikingly apparent for seismic signals, since the **stationarity** of seismic records at various frequency ranges or temporal **length** are different. The experimental estimates obtained by Gorbatikov & Stepanova, (2008) shows that, at the microseismic range of frequency, signals are mostly **quasi-stationarity,** but this stationarity may not be preserved for very long periods of time. For instance, this interval might be lengthen to the several day or be shorten to **the 1–1.5 h** , while Wang et al., (2014) showed that for frequencies above than 1 Hz signals, the stationarity range of signal is just in the range of **several seconds**. This is where we think we have to focus much of our attention. Obviously, by choosing an extreme short window length users might thereby misunderstand the true origin of non-stationarity, in such a way that lead to results that are non-informative, and potentially misleading. Accordingly, we will be able to adaptively adjust the length of processing in compatible with the stationarity length of signal, if we take full benefit of the advantages of **TFSA**.

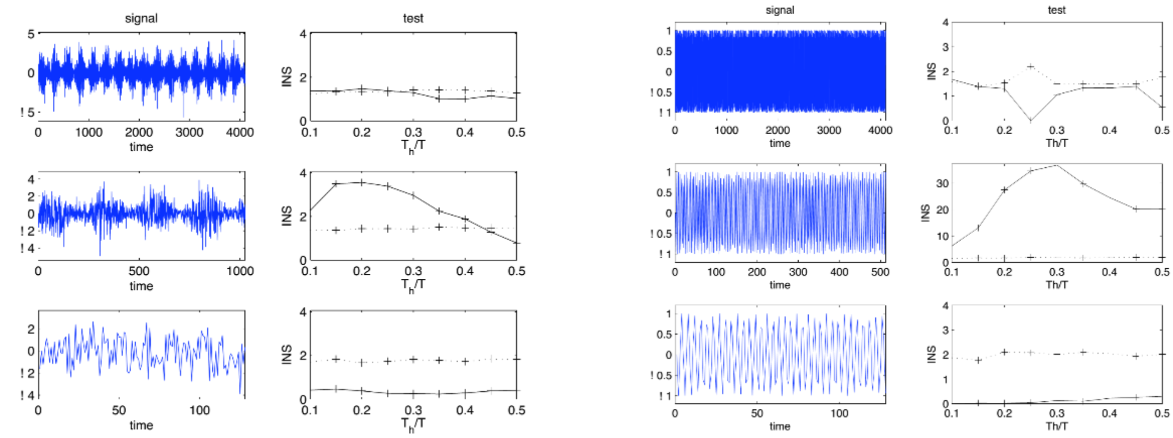

Fig. 1. Amplitude modulated (AM) at left and frequency modulated (FM) signal at right observed over different time intervals (shown at different rows) (Borgnat et al. 2010).

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Comments from Referees

Q (4) The authors state "Existence of a self-affine long range persistence in the seismic noise time series evidences that the current state of system is not in the pure diffused regime and transition from coherent to incoherent motion is still on progress" however not in the Results section nor in the Discussion section it was ever strengthened such

statement on the base of the obtained results, leaving it suspended and without a clear connection with all the performed analysis.

Answer:

Generally speaking, a local perturbation may continuously fluctuate system over time in a complex manner, such that consecutive cycles of a signal exhibit an **interdependency** spanning over long time intervals. Indeed, existence of this type of **long range correlation** (or **self-affine long range persistence**) in the seismic noise wavefield is mostly associated with the existence of **coherent** signals (e.g. P, S, and Surface waves). Matcharashvili, et al. (2013) showed that the **dynamical** features of ambient noise undergo essential changes during preparation**,** and also after triggering the activity of strong local events. At distance far away from a seismic source, the generated waves bounce on several heterogeneities and gradually enter in the **multiple scattering regime** (see Fig. (2)). Therefore, the diffused scattered coda waves overwhelm the direct wave, gradually. In this case, the strength of persistence of signal diminishes through time, due to the attenuation of the coherent wave front. Therefore, the **footprint of a** perturbation should be tracked in **the ambient noises** and the **existence of a self-affine long range persistence** in the seismic noise time series evidences that this system is still in the process of transition, that is, transition from a complete coherent state to a pure diffusive state. In this point of view, the **scattering mean free time,** τ, indicates **the characteristic time** after which such transition is happing. The **scattering mean free time** is the key feature of a medium since it allows knowing in advance the **degree of heterogeneity** of that medium. A medium with low levels of heterogeneity is no longer considered as a **candidate site** for executing advanced process such as seismic interferometry (see for instance, see Wapenaar, 2012 a,b), since the existence of a diffusive wavefield is key precondition for this analysis (Pilz & Parolai 2014). With this short introduction, the answers to above-mentioned question can be summarized briefly:
In short, this sentence "Existence of a self-affine long range persistence in the seismic noise time series evidences that the current state of system is not in the pure diffused regime and transition from coherent to incoherent motion is still on progress" is **evident** from recent studies not directly the conclusion of our research but we **MUST** give the specific reference for this argue. Our paper is actually the continuation of the effort made by Pilz & Parolai (2014), much of our focus was on improving processes, with the enhancing the quality of fractal analysis.

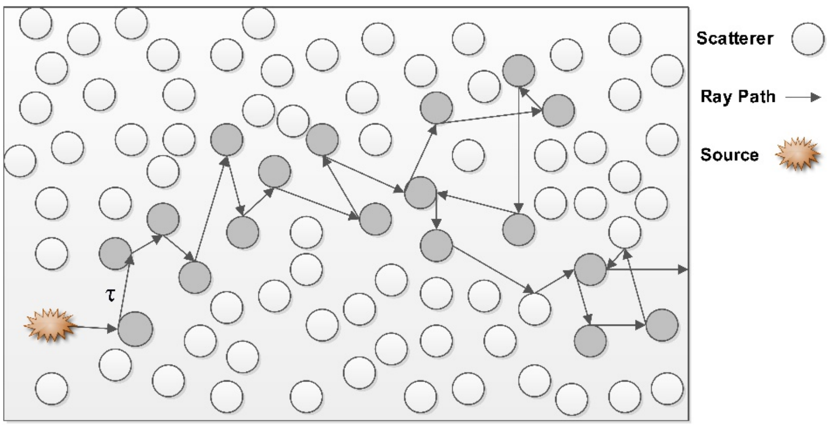

Fig. (2)

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Comments from Referees,

Q (5) The title of subsection 3.1 seems not appropriate, since the fractality of a signal can be detected or identified and not learned.
Answer:

Thanks, we agree. This will be revised definitely.

Comments from Referees

Q (6) It is obscure at all, the summation the authors did in eq. 3; practically they sum over the three different time series, so instead to consider one set of fluctuation functions (depending on q) for each time series, they summed for each q the three fluctuation functions obtained for each time series. I suppose that each time series refers to one direction of the sensor by which the seismic noise was measured, one vertical and two horizontal. Actually, it would has been much more useful and informative of the underlying geophysical process to analyse each direction separately. Probably it would has been better to first calculate the total displacement combining the three time series and then apply the MFDFA on such total displacement.

Answer:

Actually, we followed a process similar to the one outlined in the "Casetra, et al., 2007" provided in Eq. (2). The similar approach has been taken by Pilz & Parolai (2014) in Eq. (2). The rationale behind this suggestion was given by Casetra, et al., (2007) as:

"*Moreover, we consider the 3D soil displacement instead of its three components because we are interested in studying the global soil motion under the effect of seismic noise; considering and comparing the motion in each component separately (H/V spectral ratio, etc.), could be done in a next paper* (Casetra, et al., 2007, p. 259)".

In any case, we feel that your comment is well-founded and that there needs to be reflection on the matter in the advanced processing.

Q (7) Eq (6) is not correct, because the zero-values are of the parabolic fitting function and not of Dq, calculated from the data.

Answer

We exactly do the method introduced by Eq. (9) in Padhy (2016), or by Eq. (21) in Shimizu et al. (2002). Maybe we don't quite understand the question of referee. We would appreciate if you could notify the problem of this method to us.

Q (8) At page 7, the authors say that the "phase structure, which controls the non-stationarity. . ." this is not correct, because the phase are responsible of the non-linearity of a time series. The stationarity/nonstationarity of a time series can be simply verified looking at the power spectrum and its powerlaw shape, which depends solely on the amplitude of the Fourier transform and not on the phase.

Answer

I appreciate for pointing out the mistakes I have made. It will be revised.

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Comments from Referees,

Q (9) In response to questions and comments from referees concerning the TFSA, we try to restate the method in the clear way as follows:

Answer

Maybe I was not quite clear enough in explaining the theoretical aspect of Testing Stationary of Signal, so was maybe not something one would want to do too

A signal is stationary over a given observation scale if its spectrum undergoes no evolution in that scale. This assumption leads Bayram and Baraniuk, (2000) to use Multitaper Spectrograms (MS) for studying the time-dependent features of signals as

$$w_x(t,f) = \frac{1}{K}\sum_{k=0}^{K}\left|\int x(\tau)\mathrm{h}_k(-t)e^{-j2\pi f\tau}d\tau\right|^2 \quad (1)$$

where $\{\mathrm{h}_k(\tau - t), k = 1, \dots, K\}$ stands for the first K Hermite functions, which are used as the short-length windows. Bayram and Baraniuk (2000) used the Hermite functions $\mathrm{h}_k^H(t)$ as the sliding windows since they give the best time-frequency localization and orthonormality in the time-frequency domain. Hermite functions can be obtained recursively, as follows

$$\mathrm{h}_k^H(t) = \pi^{\frac{-1}{4}}\left(2^k k!\right)^{\frac{-1}{2}} e^{\frac{-t^2}{2}} H_k(t) \quad (2)$$

where $\{H_k(t), \mathrm{t} \in \mathrm{N}\}$ represents Hermite polynomials, defined by

$$H_k(t) = 2tH_{k-1}(t) - 2(k-2)H_{k-1}(t) \quad (3)$$

in which $H_0(t)$ =1 and $H_1(t) = 2t$. These family of windows are mutually orthonormal with elliptic symmetry and maximum concentration in the time-frequency domain. To define the global spectrum of signal, we should take the average of MS as (Xiao et al., 2007)

$$\langle w_x(t,f)\rangle_N = \frac{1}{N}\sum_{t=0}^{N} w_x(t,f) \quad (4)$$

For a stationary signal $w_x(t,f)/w_x^{av}(t,f)$ remains almost unchanged at the whole recording window, but in practice fluctuations in this ratio is inevitable. These fluctuations can be defined by a dissimilarity function as

$$c_t^x = D\left(w_x(t,f), w_x^{av}(t,f)\right),\ t = 0, \dots, N \quad (5)$$

The significance of fluctuations can also be assessed by using surrogates (Borgnant et al., 2010). A surrogate is artificially produced in such a way that mimics statistical properties of real data. Isospectral surrogates have identical power spectra as the real signal but with randomized phases (Theiler et al., 1992). Once a collection of J synthesized isospectral surrogates, $\{s_j(t), j = 1, \ldots, J\}$, are generated, the dissimilarity between local, $\mathcal{w}_{s_j}(t, f)$, and global spectra, $\mathcal{w}_{s_j}^{av}(t, f)$, for surrogates can be evaluated by (Borgnant et al., 2010)

$$\left\{c_t^{s_j} = D\left(\mathcal{w}_{s_j}(t, f), \mathcal{w}_{s_j}^{av}(t, f)\right), \, t = 0, \ldots, N, \, j = 1, \ldots, J\right\} \quad (6)$$

Borgnat et al., (2010) merged the Kullback-Leibler distance,

$$D_{KL}(A, B) = \int_{\Omega} \big(A(f) - B(f)\big) \log(A(f)/B(f)) \, df \quad (7)$$

and log-spectral distance, $D_{KL}(A, B)$,

$$D_{LSD}(A, B) = \int_{\Omega} |\log(A(f)/B(f))| \; df \quad (8)$$

in the following combined form

$$D(A, B) = D_{KL}(A, B).\left(1 + D_{LSD}(\tilde{A}, \tilde{B})\right) \quad (9)$$

In these equations A and B are two positive distributions and $\tilde{A}$ and $\tilde{B}$ indicate their normalized versions to the unity over the domain. The dissimilarity function $D(A, B)$ enables us to differentiate an amplitude-modulated or frequency-modulated non-stationary signal from a stationary one. Statistical variance $\Theta_1 = var(c_n^x)_{n=1,\ldots,N}$ gives the variance of c_n^xs. Similarly, for each one of J synthesized surrogates we can define a separate variance as

$$\left\{\Theta_0(j) = var\left(c_n^{s_j}\right)_{n=1,\ldots,N}, \, j = 1, \ldots, J\right\} \quad (10)$$

These Θ_0s can be assumed as a set of realizations of Gamma probability distribution with the following description

$$\mathrm{P(x; a, b)} = \frac{1}{b^a \psi(a)} x^{a-1} \exp\left(-{}^{x}/_{b}\right) \quad (11)$$

As a null hypothesis original signals is supposed to be stationary but if it violates the predefined threshold γ, null hypothesis is rejected and non-stationarity is assumed, that is

$$\mathcal{J}(x) = \begin{cases} 1 \; if \; \Theta_1 > \gamma\text{: } non-stationarity \\ \quad 0 \; if \; \Theta_1 < \gamma\text{: } stationarity \end{cases}$$

(12)

The threshold value for γ is considered as a confidence level of 95% for probability distribution under the maximum likelihood sense. By comparing Θ_1 and the estimates of Θ_0, one can define the degree of stationarity. Quantitatively, these difference can be evaluated by index of non-stationarity (INS) (Xiao et al., 2007):

$$\text{INS} = \sqrt{\Theta_1 / \tfrac{1}{J} \textstyle\sum_{n=1}^{J} \Theta_0(j)} \quad (13)$$

Further, note the result of stationarity test depends on the window length of spectrogram, $\mathrm{T_n}$. This dependence can be analyzed by the scale of non-stationarity (SNS). It informs us that in which one/ones of considered values for $\mathrm{T_n}$ the given threshold in Eq. (10) has been exceeded (Xiao et al., 2007):

$$\text{SNS} = \frac{1}{\mathrm{T}} \arg\max_{\mathrm{T_n}} \{\mathrm{INS(T_n)}\} \quad (14)$$

References:

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Xiao, J., Borgnat, P., Flandrin, P. and Richard, C., 2007, August. Testing stationarity with surrogates-a one-class SVM approach. In *Statistical Signal Processing, 2007. SSP'07. IEEE/SP 14th Workshop on* (pp. 720-724). IEEE.

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Comments from Referees

Q (10) At page 9, N/6 samples correspond to 600 seconds and not 6 seconds.

Answer

Signals have been recorded at 50 sample per second so each one of these signals has $N = 3600 \times 50 = 180000$ samples. By limiting the size of segments into the $h \leq N / 6$ samples, that is, $h \leq 600$ s. Sorry for this mistake in typing.

We also tested the process for different time length, different seasons, different weather conditions, and also night and day times. All of results will be added at the final paper. We confirmed the suitability of this method.