A signal is stationary over a given observation scale if its spectrum undergoes no evolution in that scale. This assumption leads Bayram and Baraniuk, (2000) to use Multitaper Spectrograms (MS) for studying the time-dependent features of signals as

$$w_x(t,f) = \frac{1}{K}\sum_{k=0}^{K}\left|\int x(\tau)\mathrm{h}_k(-t)e^{-j2\pi f\tau}d\tau\right|^2 \quad (1)$$

where $\{\mathrm{h}_k(\tau - t), k = 1, \dots, K\}$ stands for the first K Hermite functions, which are used as the short-length windows. Bayram and Baraniuk (2000) used the Hermite functions $\mathrm{h}_k^H(t)$ as the sliding windows since they give the best time-frequency localization and orthonormality in the time-frequency domain. Hermite functions can be obtained recursively, as follows

$$\mathrm{h}_k^H(t) = \pi^{\frac{-1}{4}}\left(2^k k!\right)^{\frac{-1}{2}} e^{\frac{-t^2}{2}} H_k(t) \quad (2)$$

where $\{H_k(t), \mathrm{t} \in \mathrm{N}\}$ represents Hermite polynomials, defined by

$$H_k(t) = 2tH_{k-1}(t) - 2(k-2)H_{k-1}(t) \quad (3)$$

in which $H_0(t) = 1$ and $H_1(t) = 2t$. These family of windows are mutually orthonormal with elliptic symmetry and maximum concentration in the time-frequency domain. To define the global spectrum of signal, we should take the average of MS as (Xiao et al., 2007)

$$\langle w_x(t,f)\rangle_N = \frac{1}{N}\sum_{t=0}^{N} w_x(t,f) \quad (4)$$

For a stationary signal $w_x(t,f)/w_x^{av}(t,f)$ remains almost unchanged at the whole recording window, but in practice fluctuations in this ratio is inevitable. These fluctuations can be defined by a dissimilarity function as

$$c_t^x = D\left(w_x(t,f), w_x^{av}(t,f)\right),\ t = 0, \dots, N \quad (5)$$

The significance of fluctuations can also be assessed by using surrogates (Borgnant et al., 2010). A surrogate is artificially produced in such a way that mimics statistical properties of real data. Isospectral surrogates have identical power spectra as the real signal but with randomized phases (Theiler et al., 1992). Once a collection of J synthesized isospectral surrogates, $\{s_j(t), j = 1, \dots, J\}$, are generated, the dissimilarity between local, $w_{s_j}(t,f)$, and global spectra, $w_{s_j}^{av}(t,f)$, for surrogates can be evaluated by (Borgnant et al., 2010)

$$\left\{c_t^{s_j} = D\left(w_{s_j}(t,f), w_{s_j}^{av}(t,f)\right),\ t = 0, \dots, N,\ j = 1, \dots, J\right\} \quad (6)$$

Borgnat et al., (2010) merged the Kullback-Leibler distance,

$$D_{KL}(A,B) = \int_{\Omega} \left(A(f) - B(f)\right)\log(A(f)/B(f))\,df \quad (7)$$

and log-spectral distance, $D_{KL}(A,B)$,

$$D_{LSD}(A,B) = \int_{\Omega} |\log(A(f)/B(f))|\ df \quad (8)$$

in the following combined form

$$D(A,B) = D_{KL}(A,B).\left(1 + D_{LSD}(\tilde{A},\tilde{B})\right) \quad (9)$$

In these equations A and B are two positive distributions and $\tilde{A}$ and $\tilde{B}$ indicate their normalized versions to the unity over the domain. The dissimilarity function $D(A,B)$ enables us to differentiate an amplitude-modulated or frequency-modulated non-stationary signal from a stationary one. Statistical variance $\Theta_1 = var(c_n^x)_{n=1,\dots,N}$ gives the variance of c_n^xs. Similarly, for each one of J synthesized surrogates we can define a separate variance as

$$\left\{\Theta_0(j) = var\left(c_n^{s_j}\right)_{n=1,\dots,N},\ j = 1, \dots, J\right\} \quad (10)$$

These Θ_0s can be assumed as a set of realizations of Gamma probability distribution with the following description

$$P(x; a, b) = \frac{1}{b^a \psi(a)} x^{a-1} \exp\left(-{}^{x}/_{b}\right) \quad (11)$$

As a null hypothesis original signals is supposed to be stationary but if it violates the predefined threshold γ, null hypothesis is rejected and non-stationarity is assumed, that is

$$\mathcal{J}(x) = \begin{cases} 1 \; if \; \Theta_1 > \gamma: \; non-stationarity \\ \quad 0 \; if \; \Theta_1 < \gamma: \; stationarity \end{cases} \quad (12)$$

The threshold value for γ is considered as a confidence level of 95% for probability distribution under the maximum likelihood sense. By *c*omparing Θ_1 and the estimates of Θ_0, one can define the degree of stationarity. Quantitatively, these difference can be evaluated by index of non-stationarity (INS) (Xiao et al., 2007):

$$\text{INS}= \sqrt{\Theta_1 / \frac{1}{J} \sum_{n=1}^{J} \Theta_0(j)} \quad (13)$$

Further, note the result of stationarity test depends on the window length of spectrogram, T_n. This dependence can be analyzed by the scale of non-stationarity (SNS). It informs us that in which one/ones of considered values for T_n the given threshold in Eq. (10) has been exceeded (Xiao et al., 2007):

$$\text{SNS}= \frac{1}{T} \arg\max_{T_n} \{\text{INS}(T_n)\} \quad (14)$$