

Interactive comment on “Fractional Brownian motion, the Matérn process, and stochastic modeling of turbulent dispersion” by Jonathan M. Lilly et al.

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We would like to thank the reviewer, Peter Ditlevsen, for his careful review of our paper. We believe we have addressed all of his comments.

Major points

1. *This is a well written review of the Fractional Brownian motion (fBm) and the less well known Matérn process. The model is applied to the case of particle dispersion in 2D turbulent flow. The review is clear and with the relevant degree*

C1

of detail to be readable by non-experts in stochastic processes.

Thank you for these positive comments. We are especially glad you found it suitable for non-experts in stochastic processes, as this was one of our main objectives.

2. *Concepts of short and long memory and of diffusive, sub-diffusive and super-diffusive processes are well described as is the whole concept of diffusivity for stochastic processes, which is less known in the “time series community”. The treatment of diffusivity, auto-covariance and corresponding spectra is illuminating. The introduction of the concepts of diffusive, sub-diffusive, and super-diffusive as pertaining to types of processes is one of the main contributions of this paper. We are very glad to hear you found this illuminating.*
3. *The paper is very long, so I propose that the authors perhaps consider rewriting into two back-to-back papers, with the second being the application to turbulent dispersion... I leave that to the authors to decide and recommend publication with pleasure.*

We believe it should remain as one long contribution. The main point of the paper is to connect the physical intuition with statistical concepts, and therefore we believe an integrated approach is preferable. Thank you for leaving this to our judgment.

4. *Furthermore, for better overview, some of the results, especially in section 4, could be moved to Appendices.*

We have done so. There are now four new Appendices: Appendix B, Diffusivity in terms of the spectrum, takes material from the previous section 2.3; Appendix C, Diffusiveness and memory, takes material from the previous section 2.4; Appendix D, The Rihaczek distribution of fractional Brownian motion, takes material from Section 3.3; and Appendix H, The Matérn impulse response function, takes material from Section 4.4.

C2

Minor points

1. *P10, L25-30: seems to be irrelevant paragraph, including 4 self-references.* The point is actually important, but it has been greatly shortened and combined with the previous paragraph.
2. *P14, Figure 3: It would be useful to indicate the slope of the “Power Law line” and indicate the relation to the turbulence scaling theory (if any).*
The power law line has been added. We are not aware of any turbulence theory for 2D frequency (as opposed to wavenumber) spectra; this has been noted in the text, as follows: “The spectral form is seen to provide an excellent match to the observed Lagrangian velocity spectra over roughly eight decades of structure. The high-frequency slope is seen to be roughly $|\omega|^{-8}$, a very steep slope. We are not aware of any physical theory to account for this, nor for the value of the damping parameter λ . Despite the fundamental role that the Eulerian *wavenumber* spectrum of velocity plays in turbulence theory, the Lagrangian *frequency* spectrum has received relatively little attention. Attempting to connect the observed form of this spectrum to physical principles is, however, outside the scope of the present paper.”
3. *P19, L3: faveraging \rightarrow averaging.*
Fixed.
4. *P20, L27: It would be useful with a definition of a Gaussian process in this context, and a little more explanation of why the “original process is Gaussian as well as zero mean”.*
Done.
5. *P26, L6: To me it seems more logical to say “globally white” rather than “locally white”, in the sense that points separated in time by $T \gg \lambda^{-1}$ are independent.*

C3

The term “locally white” means that the spectrum is constant, or white, within some frequency range, but not everywhere. The meaning of this usage has been clarified.

6. *P27, L3: This is unclearly written: It is $d^2/d\omega^2(\ln S) = 0$ for $|\omega| = \lambda$. P33, L16: $\Delta \rightarrow \Delta t$.*
We were inconsistent in using Δ or Δt for the sampling interval. Now, we have consistently used Δ , without the t . Thank you for noticing this.

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