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Generation and propagation of stick-slip waves over a fault with rate-independent friction

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Abstract. Stick-slip sliding is observed at various scales in fault sliding and the accompanied seismic events. It is conventionally assumed that the mechanism of stick-slip over geomaterials lies in the rate dependence of friction. However, the movement resembling the stick-slip could be associated with elastic oscillations of the rock around the fault, which occurs regardless of rate properties of the friction. In order to investigate this mechanism, two simple models were considered: a mass-spring model of Burridge and Knopoff type (BK model) and a one-dimensional (1D) model an infinite elastic rod driven by elastic shear spring.

The results show that frictional sliding in the case of BK model demonstrates stick-slip-like motion even when the friction coefficient is constant. The 1D rod model predicts that any initial disturbance moves with a p-wave velocity, that is supersonically with the amplitude of disturbances decreasing with time. This effect might provide an explanation to the observed supersonic rupture propagation over faults.

1 Introduction

Earthquakes can lead to catastrophic structural failures and may trigger tsunamis, landslides and volcanic activity (Ghobarah et al., 2004; Bird and Bommer, 2004). They are generated at faults and are either produced by rapid (sometimes ‘supersonic’) propagation of shear cracks/ruptures along the fault or originated in the stick-slip sliding over the fault. The velocity of rupture propagation is crucial for estimating the earthquake damage. Rupture velocities can be determined by comparison its speed with the speeds of stress waves in the rupturing solid (Rosakis, 2002). There are several types of rupture propagation: supersonic ($V>V_P$), intersonic ($V_S<V<V_P$), subsonic ($V<V_S$), supershear ($V>V_S$), sub-shear ($V_R<V<V_S$) and sub-Rayleigh ($V<V_R$). According to the data of seismic observation of crustal earthquakes, most ruptures propagate with an average velocity that is about 80% of the shear wave velocity (Heaton, 1990). In some cases, however, supershear propagation of earthquake-generating shear ruptures or sliding is observed (Archuleta, 1984; Bouchon et al., 2000, 2001; Dunham and Archuleta, 2004; Aagaard and Heaton, 2004). These observations gave rise to the concept of supershear crack propagation (e.g., Bizzarri and Spudich, 2008; Lu at al., 2009; Bhat et al., 2007; Dunham, 2007). However, there is some debate regarding to the data interpretation (Delouis et al., 2002; Bhat et al., 2007) due to the lack of strong motion recording. For instance, it was suggested that the 2002 Denali Earthquake was propagated at supershear speed about 40 km (Dunham and Archuleta, 2004). This conclusion was based on a single ground motion record. However, the separate inversion of the individual data sets may provide only a partial image of the rupture process of an earthquake. The joint inversion of the combined data sets gives a more robust description of the rupture. The recent studies aimed at deriving kinematic models for large earthquakes have shown the importance of the type of data used. It has been shown that slip maps for a given earthquakes may vary significantly (Cotton and Campillo, 1995; Cohee and Beroza, 1994a).

Sliding over pre-existing fractures and interfaces is one of the forms of instability in geomaterials. It is often accompanied by stick-slip – a spontaneous jerking motion between two contacting bodies, sliding over each over. It is assumed that the

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mechanism of stick-slip lies in intermittent change between static and kinetic friction and the rate dependence of the friction coefficient (Popp and Rudolph, 2004).

The investigation of the friction law on geological faults is the key element in the modelling of earthquakes. Rate- and state-dependent friction laws successfully modelled frictional sliding and earthquake phenomena. These laws were proposed by Dieterich, Ruina and Rice (Dieterich, 1978; Ruina, 1983; Rice, 1983). There are two types of frictional sliding between surfaces, including the tectonic plates. The first type occurs when two surfaces slip steadily ($V=V_0$ condition, where V - is relative velocity, V_0 - is the load point velocity) and is an analogue to the fault creep (Byerlee and Summers, 1975). In the stable state the sliding over discontinuities (faults, fractures) is prevented by friction. However, the faults are continuously subjected to variations in both shear and normal stresses and can produce sliding over initially stable fractures/interfaces. In the Earth's crust the increase in shear stress is obviously a consequence of tectonic movement, while oscillations in the normal stress can be associated with the tidal stresses or seismic waves generated by other seismic events. These can generate the second dynamic state when the sliding occurs jerkily (slip, stick and then slip again). This type is calling "stick-slip" sliding and has cyclic behaviour. Both types of sliding are usually investigated using a simple spring-block model introduced by Burridge and Knopoff in 1967 (Turcotte, 1992).

Modelling of frictional sliding is an important tool for understanding the initiation, the development of rupture, and the healing of faults. Many models and numerical methods were developed to describe seismic activity and the supershear fracture/rupture propagation (Noda and Lapusta, 2013; Lapusta and Rice, 2003; Lu at al., 2009; Lapusta et al., 2000; Sobolev, 2011; Bag and Tang, 1989; Harris and Day, 1993).

In this paper, we however concentrate on the stick-slip-like movement occurring under rate-independent friction due to the eigen oscillations of the fault faces and the associated wave propagation. Also a simple mechanism of unusually high shear fracture or sliding zone propagation is considered. This is the p-sonic propagation of sliding area over a frictional fault. It is based on the fact that the accumulation of elastic energy in the sliding plates on both sides of the fault can produce oscillations in the velocity of sliding even if the frictional coefficient is constant. Brace and Byerlee noticed in 1966 that the stick-slip instabilities in the tectonic plates are associated with the appearance of earthquakes (Feeny et al., 1998; Byerlee, 1970).

2 Single degree of freedom frictional oscillator

This study starts with the self-excited oscillations which may look like stick-slip but occur under constant friction. For this purpose a single degree of freedom block-spring model is used. A block sliding on a rigid horizontal surface is driven by a spring whose other end is attached to a driver moving with a constant velocity (Figure 1). The system consists of mass m, spring of stiffness k and a driver that moves with the constant velocity V_0. Friction is assumed to be cohesionless: $T_{cr}=\mu N$, where T_{cr} is the force at which sliding starts, N is the normal force and μ is the friction coefficient.

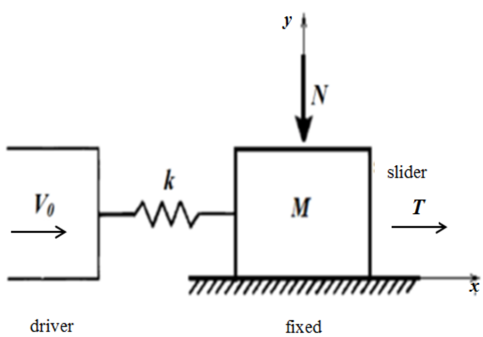

Figure 1: The simple mass-spring model of Burridge and Knopoff type.

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The system of equations representing the motion of the block reads:

$$\begin{cases} m\dot{V} = T - \mathrm{sgn}(V)\mu N \\ \dot{T} = k(V_0 - V) \end{cases} \quad (1)$$

where m is the mass of block, k is the spring stiffness, V_0 is the load point velocity, V is the relative velocity, N is gravity force, T is the shear force, μ is the friction coefficient.

The appearance of the sign function in the system of equations represents the fact that friction always acts against velocity.

Here function sgn(V) is defined as follows:

$$\mathrm{sgn}(V) = \begin{cases} -1 & \text{for } V < 0 \\ 0 & \text{for } V = 0 \\ 1 & \text{for } V > 0 \end{cases} \quad (2)$$

In order to represent the system of equations (1) in dimensionless form, it is convenient to introduce a dimensionless time t^*:

$$t^* = t\omega_0, \quad \omega_0^2 = \frac{k}{m} \quad (3)$$

where ω_0 is the eigen frequency of the block-spring system, m is the block mass and k is the spring stiffness.

The governing system of equations in dimensionless form reads:

$$\begin{cases} \ddot{V} = 1 - V \\ V(0) = V^* \\ \dot{V}(0) = T(0) - \mathrm{sgn}(V^*)\mu N \end{cases} \quad (4)$$

Here the dot represents the derivative with respect to dimensionless time t^*, $\ddot{V} = \dot{T}$.

2.1 Behaviour of the system under different initial conditions

In order to demonstrate the behaviour of the system under different initial conditions leading to the steady sliding and stick-slip-type regimes we assume velocity V >0 and consider the block sliding under the following two sets of initial conditions:

$$\begin{aligned} &V(0) = 1,\ \dot{V}(0) = -\mu N; \quad V(0) = 0,\ \dot{V}(0) = -\mu N \\ &V(0) = 1,\ \dot{V}(0) = 0; \quad\quad\ \ V(0) = 0,\ \dot{V}(0) = 0 \\ &V(0) = 1,\ \dot{V}(0) = \mu N; \quad\ \ V(0) = 0,\ \dot{V}(0) = \mu N \end{aligned} \quad (5)$$

Figure 2 represents the corresponding two types of behaviour of the system (dimensinless velocity vs. dimensionless time).

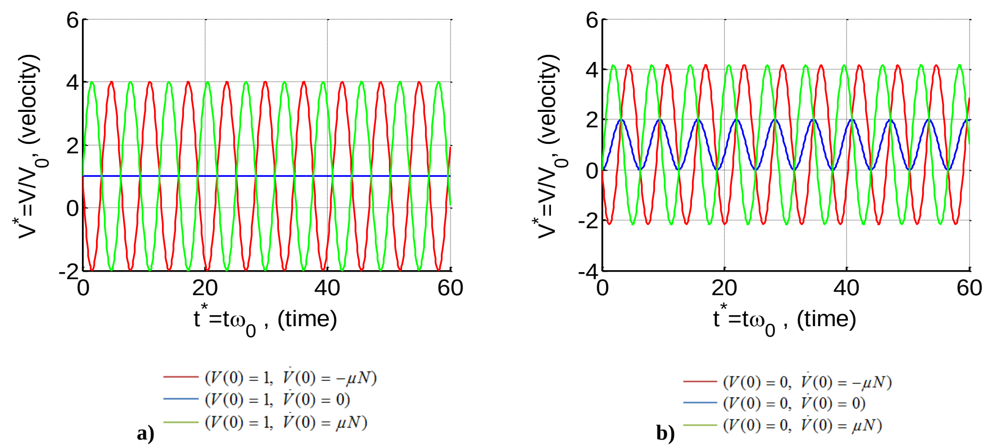

Figure 2: Block sliding under different initial conditions.

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It is seen that the system exhibits self-excited oscillations even with constant friction coefficient, which somewhat resemble the stick-slip-type sliding. This is a harmonic motion with the frequency is equal to the eigen frequency of the system. The friction coefficient only affects the initial conditions. In more detail the behaviour of such a system is investigated in our previous works (Karachevtseva et al., 2014; Karachevtseva et al., 2014).

3 Stress wave propagation in frictional sliding (generalisation 1D solid)

The previous section shows the stick-slip-like motion occurring even when the friction coefficient is constant. Now this understanding will be generalised to slide over a fault where a stick-slip phenomenon is traditionally flagged as a mechanism of earthquakes. For this purpose, following Nikitin (1998) we consider the simplest possible 1D model of fault sliding, which takes into account the rock elastic response and the associated dynamic behaviour, shown in Figure 3.

To this end, an infinite elastic rod of height (thickness) *h*, per unit length in the direction normal to the plane of drawing in Figure 3 and linear density ρ sliding over a stiff surface is considered. The stiff surface can be though of as a symmetry line, such that instead of the (horizontal) fault only the upper half of it is considered. The rod is connected to a stiff layer moving with a constant velocity V_0. The connection is achieved through a series of elastic shear springs. Both the elastic rod and the elastic springs model the elasticity of the rock around the fault, Figure 3. We assume that the system is subjected to a uniform compressive load σ_N such that the friction stress is kept constant; it is assumed equal to $\tau_{fr} = k\mu\sigma_N = const$.

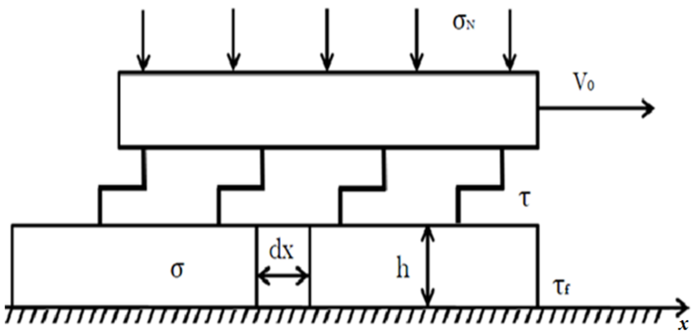

Figure 3: The model of infinitive elastic rod driven by elastic shear spring.

Let the longitudinal (normal) stress in the rod be σ and the contact shear stress be τ, friction stress τ_f and the load point velocity V_0. The equation of movement of the rod reads:

$$\frac{\partial\sigma}{\partial x} + \frac{1}{h}(\tau - \tau_{fr}) = \rho\frac{\partial V}{\partial t} \tag{6}$$

where $V(x,t)$ is the velocity of point x of the rod at time t, Figure 3.

If the Young's modulus of the rod is E, then the Hooke's law gives $\sigma = E\frac{\partial u}{\partial x}$, where $u(x,t)$ is the displacement. After differentiating the Hooke's law is expressed as:

$$\frac{\partial\sigma}{\partial t} = E\frac{\partial V}{\partial x} \tag{7}$$

The elastic reaction of the shear springs is expressed through the following equation

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$$\frac{\partial \tau}{\partial t} = k(V - V_0) \tag{8}$$

where k is the spring stiffness relating stress and displacement discontinuity (the difference between the rod displacement and the zero displacement of the base). In the usual way system of equations (6)-(8) produces the wave equation:

$$\frac{\partial^2 \Delta V}{\partial t^2} = c^2 \frac{\partial^2 \Delta V}{\partial x^2} - \omega^2 \Delta V \quad \Delta V = u \tag{9}$$

where $c = \sqrt{Eh/\rho}$ is the velocity of the longitudinal wave (p-wave), $\omega = \sqrt{k/(h\rho)}$ is what can be regarded as eigenfrequency of the system consisting as a unit length of the rod considered as a lamp mass on the shear springs.

It is seen that despite frictional sliding between the rod and the stiff surface the waves propagate with the p-wave velocity determined by the Young's modulus and density of the rod. So according to the terminology described in Introduction the wave should be named *p-sonic wave*. It should be emphasizes that while such waves look like the shear waves they are in fact compressive waves propagation along the rod, hence the p-wave velocity.

In order to analyse the way the pulse propagates, equation (9) is complemented by initial conditions:

$$u(x,t) = f(x); \quad \frac{du}{dt} = F(x) \tag{10}$$

Solution of wave equation (9) can be found by using the Riemann method (e.g., Koshlyakov, 1964).

$$u(x,t) = \frac{1}{2}[f(x-at) + f(x+at)] + \frac{1}{2}\int_{x-at}^{x+at} \Phi(x,t,z)dz \tag{11}$$

where

$$\Phi(x,t,z) = \frac{1}{\sqrt{a^2t^2 - (z-x)^2}} \varphi(x,t,z) \tag{12}$$

The integral from (11) can be found by using the Chebyshev-Gauss method

$$I(x,t) = \int_{x-at}^{x+at} \Phi(x,t,z)dz \approx \frac{\pi}{n}\sum_{j=1}^{n} \varphi(x,t,\ x+\xi_j at), \quad \xi_j = \cos\left(\frac{2j-1}{2n}\pi\right) \tag{13}$$

where

$$\varphi(x,t,z) = \frac{1}{a}F(z)J_0\left(\frac{b}{a}i\sqrt{a^2t^2-(z-x)^2}\right)\sqrt{a^2t^2-(z-x)^2} + btf(z)\left(\frac{1}{i}\right)J_0'\left(i\frac{b}{a}\sqrt{a^2t^2-(z-x)^2}\right) \tag{14}$$

3.1 Propagation of an initial disturbance

Figures 3-5 represent the propagation of initial disturbance under the different initial conditions. Particularly, a triangular displacement impulse and zero velocity were used as initial conditions for Figure 3. For Figure 4 linear and harmonic functions were used for displacement and velocities as initial conditions.

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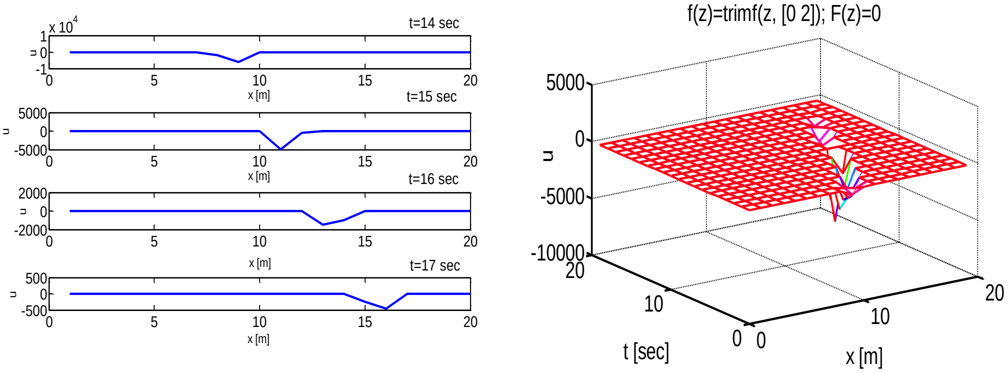
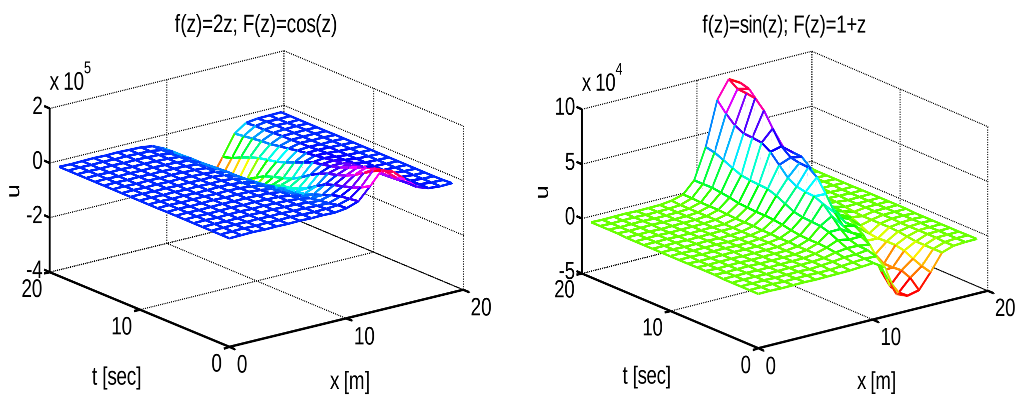


Figure 3: Propagation of initial disturbance (f (z) = trimf (z, [0 2]); F(z)=0).



a) f(z)=2z; F(z)=cos(z); **b) f(z)=sin(z); F(z)=1+z;**

Figure 4: Propagation of initial disturbances.

It is seen that the initial disturbance (impulse) propagating with p-wave velocity keeps its width but the amplitude reduces with time. Obviously as the impulse propagates it looses energy which goes to increasing the energy of shear springs. Figure 5 shows the peak of initial disturbance changing with time (here the triangular displacement and zero velocity were set as initial conditions).

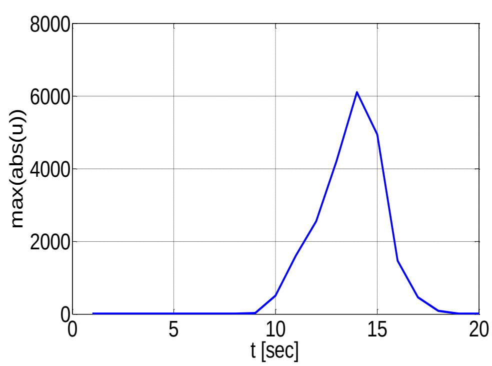


Figure 5: Maxima of initial disturbance.

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4 Conclusions

The accumulation of elastic energy in the sliding plates on both sides of the fault can produce oscillations in the velocity of sliding even when the friction is constant. These oscillations resemble stick-slip movement, but they manifest themselves in terms of sliding velocity rather than displacement. The sliding exhibits wave-like propagation over long faults. Furthermore, an infinite elastic rod model shows that the zones of disturbances propagate along the fault with the velocity of p-wave. The mechanism of such fast wave propagation is the normal (tensile/compressive) stresses in the neighbouring elements (normal stresses on the planes normal to the fault surface) causing a p-wave propagating along the fault rather than the shear stress controlling the sliding. This manifests itself as a p-sonic propagation of an apparent shear rupture.

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