Summary of changes

(Reviewer #1)

First, we would like to thank the reviewer for his interest in our work and for helpful comments that will drastically improve the paper. As indicated below, we have checked all comments provided by the reviewer and have addressed necessary changes accordingly to his feedback.

Below are reviewer's comments and our responses:

C1: "There is no "discussion" in the work, where it would be appropriate to discuss in detail the non-linear effects of disturbance propagation obtained in the work and their links to the processes in nature".

R1: The discussion part has been added into the paper. There the non-linear effects are discussed.

C2: In Parts 2 and 3 all the variables and constants used in equations should better be listed once in a single table instead of repeating the terms in different equations with different meanings.

R2: We would like to thank the reviewer for this comment. The variables are now listed in Table 1.

C3: In Part 3 the simplest 1D case is considered, so, a disturbance, once emerged, can propagate only along the rod, and the law of its propagation is defined by the parameters E and ρ, which means that the disturbance can only propagate at the velocity of p-wave, because no other motion is possible.

A3: Yes, it is a 1D case, but the shear motion is allowed as well. So, it is not immediately obvious why it should be just p-wave velocity. To emphasise the point we modified the first sentence in the para after (9), which now reads "It is seen that despite the presence of shear springs and friction between the rod and the stiff surface the waves propagate with the p-wave velocity determined by the Young's modulus and density of the rod."

C4: The captions should be revised to make them more substantial, clarifying and informative.

A4: Thank you for your comment. It has been done.

Less important remarks:

C5: "Raw 38. Cohee and Beroza, 1994a → Cohee and Beroza, 1994"

A5: Thank you. It has been done.

C6: "Raws 48-49. "However, the faults ... can produce sliding over initially stable fractures/interfaces" – a citation is needed".

A6: Thank you. It has been done

C7: "Raw 64. The citations should better be replaced by (Brace & Byerlee, 1966)".

A7: Thank you. It has been done.

C8: "Raw 82, Eq. 2. As a matter of fact, this equation defines the rule of the frictional force action. When V=0 the frictional force can act on a body only provided that the shear force is not zero. In the presented system this condition is not true".

A8: We agree with the reviewer; it was a misprint. The system of equations has been corrected.

C9: "Raw 93, Eq.5. If all the variables are dimensionless, it is unclear, why the relation μN appears? It misses in the plots presented in Fig.2".

A9: Thank you. The Fig. 2 has been replaced.

C10: "Raw 95, Fig.2. Under the action of a frictional force constant modulo, the energy should dissipate, but it doesn't. This fact should be explained".

A10: Thank you. This has been added into the paper. Please see below.
"The energy in the system does not change with time, obviously due to the constant energy influx by velocity V_0 whose excess is dissipated by friction".

C11: Raw 105. Fig.2 presents harmonic oscillations, but not the regime of "stick-slip".

A11: These oscillations resemble stick-slip movement, but they manifest themselves in terms of sliding velocity rather than displacement.

C12: "Raw 114. τfr = kμσN. What is k"?

A12: Wrong formula was used. It has been corrected.

C13: "Raw 115, Fig.3. There is τf in the figure, but not τfr".

A13: Typo was in Eq.6. It has been corrected.

C14: "Raw 126, Eq.6. It is unclear, what is k – the stiffness of a single spring, of all the springs, or the specific stiffness of springs per unit length? Attention should be paid to Eq.1, where the same notation is used".

A14: We agree with the reviewer. It has been changed. The details are in the table 1.

C15: "Raw 129, Eq.9. The formula is presented in a faulty way. If one supposes that ΔV=u is a re-introduced new value, it appears that the increment of velocity equals to displacement, which is impossible".

A15: Awkward notation was used. *U* was not to be displacement. It has been changed.

C16: "Raws 137-145. Equations 11-14. All the constants and variables should be clarified".

A16: It has been done. Please see table 1.

C17: "Raw 145. Eq.14. What is the function J0, what are the coefficients i u b, and what is the difference between the Bessel functions J0 and J0'"?

A17: *i* – is imaginary unit; J_0 – is Bessel function; J_0'– is derivative of Bessel function. Please see table 1.

C18: "Part 3.1. Since the results are presented in the form of time series of dimensional variables, parameters of the model should be designated, which were used in calculations. The visual presentation of results is not pictorial enough. To my mind, the grid is too coarse. The dimensionality of Y-axis is not mentioned".

A18: Thank you for suggestion. We have modified the paper structure and data presentation.

C19: "Raw 152. Fig.3 (right). It is better to plot all the curves using a single X-axis, and one and the same scale of the Y-axes (may be, it's better to use the logarithmic scale)".

A19: Thank you for your suggestion. A confusing figure was used. It has been deleted.

C20: "Raw 152. Fig.3 (left). Propagation of the disturbance is not seen at all. The Y-axis should be inverted, or even better, re-calculated for the disturbance when $u(t, x) > 0$.
The function of pulse shape is specified in a poorly comprehensible way. It's better to give it in a standard mathematical form".

A20: Thank you for your suggestion. The Fig.3 was corrected. A standard mathematical formula was added, please see equation 15.

C21: "Raw 155, Fig.4 (left). The disturbance is not seen in the area of big t. The viewing angle should be changed. No need in the inscriptions in the plot".

A21: It has been done.

C22: "Raw 162, Fig.5. The amplitude of the disturbance is maximal at the initial moment and reduces with time (raw 158). But, in the figure the amplitude is zero in the range of 0-9 s, then it increases in the range of 10-14 s, and then it decreases. What really shown in the figure"?

A22: A confusing figure was used. It has been deleted.

Summary of changes

(Reviewer #2)

First, we would like to thank the reviewer for his interest in our work and for helpful comments that will drastically improve the paper. As indicated below, we have checked all comments provided by the reviewer and have addressed necessary changes accordingly to his feedback.

C1: "The paper does a poor job of placing the work in a context with previous work that relates fault slip behavior to elastic oscillations of the rock surrounding the fault. Addressing this comment will make the paper more readable to a wide earth science audience and place it in better context to other work that has been done on a similar topic".

A1: Thank you for your suggestion. The additional literature review part has been added.

C2: "An application of simple models like the Burridge-Knopoff model and 1D model of an infinite elastic rod driven by elastic shear spring for the declared purpose should be substantiated in details".

A2: The original BK model consists of an assembly of blocks, where each block is connected via the elastic springs to the next block and to the moving plate. In the present paper, we simulate the simple one-dimensional version of BK model, which consists from one block.

Additional details and description of these models were added into the paper.

C3: “The constant friction factor used in the models instead of generally accepted rate-and-state friction law has to be grounded and supported by lab results and field observations”.

A3: We do not advocate constant friction. We just demonstrated that even with constant friction a stick-slip like behaviour is possible. We now added discussion where we analyse the effect of rate-dependent friction.

C4: “A discussion section of the manuscript is required for an analysis and comparison of the numerical results and drawn conclusions with published data obtained under laboratory and natural conditions”.

A4: We agree with the reviewer. The discussion part has been added.

C5: “Moreover, I realized that the English writing is not good enough, some parts of the text are difficult for understanding, there are some syntax and spelling errors, and I strongly recommend reviewing the text by a native English speaker”.

A5: Thank you for your suggestion. This has been done.

List of all relevant changes

1. Discussion part has been added
2. Table 1 has been added
3. The text was modified: “It is observed that despite the presence of shear springs and friction between the rod and the stiff surface, the waves propagate with the p-wave velocity determined by the Young’s modulus and density of the rod.”
4. The captions in the paper have been modified
5. Raw 38. Cohee and Beroza, 1994a → Cohee and Beroza, 1994” has been modified
6. *“However, the faults ... can produce sliding over initially stable fractures/interfaces” – a citation is needed”.* Citation has been added
7. *“Raw 64. The citations should better be replaced by (Brace & Byerlee, 1966)”.* The citation has been replaced.
8. Equations 1-4 have been corrected.
9. Figure 2 has been replaced.
10. Additional para has been added: “Furthermore, the energy in the system does not change with time, obviously due to the constant energy influx by velocity V_0, where the excess of the V_0 is dissipated by friction”.
11. *“Raw 114. $\tau fr = k\mu\sigma N$. What is k”?* Formula has been corrected.
12. Equation 6 has been corrected
13. Awkward notations were used in eq. 8-14. *U* was not to be displacement. It has been changed.
14. The paper structure and data presentation have been modified.
15. Figures 1, 2, 3, 4 have been modified.
16. A standard mathematical formula was added, please see equation 15.
17. Additional literature review has been added.
18. The references part has been modified.
19. Additional details and description of present models have been added into paper.

Generation and propagation of stick-slip waves over a fault with rate-independent friction

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Abstract. Stick-slip sliding is observed at various scales in fault sliding and the accompanied seismic events. It is conventionally assumed that the mechanism of stick-slip over geomaterials lies in the rate dependence of friction. However, the movement resembling the stick-slip could be associated with elastic oscillations of the rock around the fault, which occurs irrespective~~regardless~~ of the rate properties of the friction. In order to investigate this mechanism, two simple models are considered in this paper~~were considered~~: a mass-spring model of self-maintaining oscillations ~~Burridge and Knopoff type (BK model)~~ and a one-dimensional (1D) model of wave propagation through an infinite elastic rod~~an infinite elastic rod driven by elastic shear spring~~. The rod slides with friction over a stiff base. The sliding is resisted by elastic shear springs. The results show that the frictional sliding in the mass-spring model generates oscillations that resemble the stick-slip motion~~case of BK model demonstrates stick-slip-like motion even when the friction coefficient is constant~~. Furthermore, it was observed that the stick-slip-like motion occurs even when the frictional coefficient is constant. The 1D wave propagation~~rod~~ model predicts that despite the presence of shear springs the frictional sliding waves move with the p-wave velocity, denoting the wave as intersonic~~any initial disturbance moves with a p-wave velocity, that is supersonically with the amplitude of disturbances decreasing with time~~. It was also observed that the amplitude of sliding is decreased with time. This effect might provide an explanation to the observed intersonic~~supersonic~~ rupture propagation over faults.

1 Introduction

Earthquakes can lead to catastrophic structural failures and may trigger tsunamis, landslides and volcanic activities~~activity~~ (Ghobarah et al., ~~2004~~2006; Bird and Bommer, 2004). The earthquakes~~They~~ are generated at faults, and are either produced by rapid (sometimes 'supersonic') propagation of shear cracks/ruptures along the faults, or originated in the stick-slip sliding over the fault. The velocity of rupture propagation is crucial for estimating the earthquake damage. The ~~Rupture~~ rupture velocities can be ~~determined~~ classified by comparison its speed with the speeds of stress waves in the rupturing solid (Rosakis, 2002). There are several types of rupture propagation: supersonic ($V>V_P$), intersonic ($V_S<V<V_P$), subsonic ($V<V_S$), supershear ($V>V_S$), sub-shear ($V_R<V<V_S$) and sub-Rayleigh ($V<V_R$). According to the data obtained from ~~of~~ the seismic observation of crustal earthquakes, most ruptures propagate with an average velocity that is about 80% of the shear wave velocity (Heaton, 1990). However, i~~I~~n some cases, ~~however,~~ supershear propagation of earthquake-generating shear ruptures or sliding is observed (Archuleta, 1984; Bouchon et al., 2000, 2001, 2010; Dunham and Archuleta, 2004; Aagaard and Heaton, 2004). The above~~se~~ observations ~~gave rise~~introduced ~~to~~ the concept of supershear crack propagation (e.g., Bizzarri and Spudich, 2008; Lu at al., 2009; Bhat et al., 2007; Dunham, 2007). However~~,~~, due to the lack of strong motion recording ~~there~~, there ~~is~~ are still some debates regarding ~~to~~ the data interpretation (Delouis et al., 2002; Bhat et al., 2007) ~~due to the lack of strong motion recording~~. For instance, it was suggested that the 2002 Denali Earthquake was propagated at a supershear speed of about 40 km (Dunham and Archuleta, 2004). ~~This conclusion~~However, the data was based on a single ground motion record. ~~However, the separate inversion of the individual data sets may provide only a partial image of the~~

~~rupture process of an earthquake.~~ The joint inversion of the combined data~~-~~ set~~s~~ ~~gives~~ provides a more robust description of the rupture. The recent studies, which are aimed at deriving the kinematic models for large earthquakes, have shown the importance of the type of data used. It has been shown that slip maps for a given earthquakes may vary significantly (Cotton and Campillo, 1995; Cohee and Beroza, 1994a).

The analytical (e.g., Burridge, 1973) and numerical (e.g., Das and Aki, 1977) research in fracture dynamics indicate that only the Mode II rupture (shear-induced slip occurring in the direction perpendicular to the crack front) can propagate with intersonic velocity ($V_s<V<V_p$) for short durations, as long as the prestress of the fault is high compared to both failure and residual stresses (Dunham, 2007). Intersonic Mode II crack propagation was first confirmed in laboratory by Rosakis et al. (1999).

Sliding over pre-existing fractures and interfaces is one of the forms of instability in geomaterials. It is often accompanied by stick-slip – a spontaneous jerking motion between two contacting bodies~~,~~ sliding over each over. It is assumed that the mechanism of stick-slip lies in intermittent change between static and kinetic friction and the rate dependence of the frictional coefficient (Popp and Rudolph, 2004).

The investigation of the friction law on geological faults is the key element in the modelling of earthquakes. Rate- and state-dependent friction laws proposed by Dieterich, Ruina and Rice (Dieterich, 1978; Ruina, 1983; Rice, 1983) have successfully modelled frictional sliding and earthquake phenomena. ~~These laws were proposed by Dieterich, Ruina and Rice (Dieterich, 1978; Ruina, 1983; Rice, 1983).~~ There are two types of frictional sliding between surfaces that include the~~, including the~~ tectonic plates. The first type occurs when two surfaces slip steadily ($V=V_0$ condition, where V - is relative velocity, V_0 - is the load point velocity) and is ~~an analogue~~ analogous to the fault creep (Byerlee and Summers, 1975). In the stable state, the sliding over discontinuities (faults and~~,~~ fractures) is prevented by friction. Modelling of the frictional sliding is an important tool for understanding the initiation and the development of rupture, and also, the healing of the faults. Many models and numerical methods are developed to describe seismic activities and the supershear fracture/rupture propagation (Noda and Lapusta, 2013; Lapusta and Rice, 2003; Lu at al., 2009; Lapusta et al., 2000; Sobolev, 2011; Bagk and Tang, 1989; Harris and Day, 1993).

T~~ However, t~~he faults are continuously subjected to variations in both shear and normal stresses, and can produce sliding over initially stable fractures or ~~/~~interfaces (Boettcher and Marone, 2004). ~~-~~In the Earth's crust, the increase in shear stress is ~~obviously a~~an obvious consequence of tectonic movement, while oscillations in the normal stress can be associated with the tidal stresses or seismic waves generated by other seismic events. These can generate the second dynamic state when the sliding occurs jerkily ~~jerkily~~ (slip, stick and then slip again). This type of sliding is called~~ing~~ "stick-slip" sliding ~~and has~~which exhibit cyclic behaviour. Brace and Byerlee supposed that the stick-slip instabilities in the tectonic plates are associated with the appearance of earthquakes (Brace and Byerlee, 1966). Both types of sliding are usually investigated using a~~a simple~~ spring-block model introduced by Burridge and Knopoff in 1967 (Turcotte, 1992). The BK model consists of an assembly of blocks, where each block is connected via the elastic springs to the next block and to the moving plate.

In the present paper, we firstly simulate a single element block model, which is one block undergoing frictional sliding on a stiff base. The movement is caused by a spring attached to the block. The other end of the spring moves with a constant velocity. The paper begins with considering stick-slip-like movement occurring under rate-independent friction due to the eigen oscillations of the fault faces and the associated wave propagation. This demonstrates that the rate dependence of friction is not necessarily a controlling phenomenon. We also analyse a simple mechanism of unusually high shear fracture or sliding zone propagation, also referred as the p-sonic propagation of sliding area over a frictional fault. The analysis is based on the fact that accumulation of elastic energy in the sliding plates on both sides of the fault can produce oscillations in the velocity of sliding even if the frictional coefficient is constant. We note that Walker and Shearer (2009) found evidence of the intersonic rupture speeds close to the local P-wave velocity by analysing the Kokoxili and Denali earthquakes seismic data. This paper considers a highly simplified 1-D rod model where many properties of real fault system have been neglected. (Considerable fault geometry simplification is in use in analysing intersonic ruptures, e.g., Bouchon et al., 2010.)

~~Modelling of frictional sliding is an important tool for understanding the initiation, the development of rupture, and the healing of faults. Many models and numerical methods were developed to describe seismic activity and the supershear fracture/rupture propagation (Noda and Lapusta, 2013; Lapusta and Rice, 2003; Lu at al., 2009; Lapusta et al., 2000; Sobolev, 2011; Bag and Tang, 1989; Harris and Day, 1993).~~

~~In this paper, we however concentrate on the stick-slip-like movement occurring under rate-independent friction due to the eigen oscillations of the fault faces and the associated wave propagation. Also a simple mechanism of unusually high shear fracture or sliding zone propagation is considered. This is the p-sonic propagation of sliding area over a frictional fault. It is based on the fact that the accumulation of elastic energy in the sliding plates on both sides of the fault can produce oscillations in the velocity of sliding even if the frictional coefficient is constant. Brace and Byerlee noticed in 1966 that the stick-slip instabilities in the tectonic plates are associated with the appearance of earthquakes (Feeny et al., 1998; Byerlee, 1970).~~

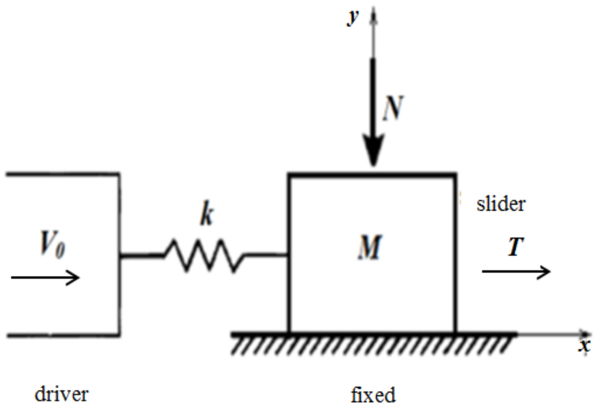
2 Single degree of freedom frictional oscillator

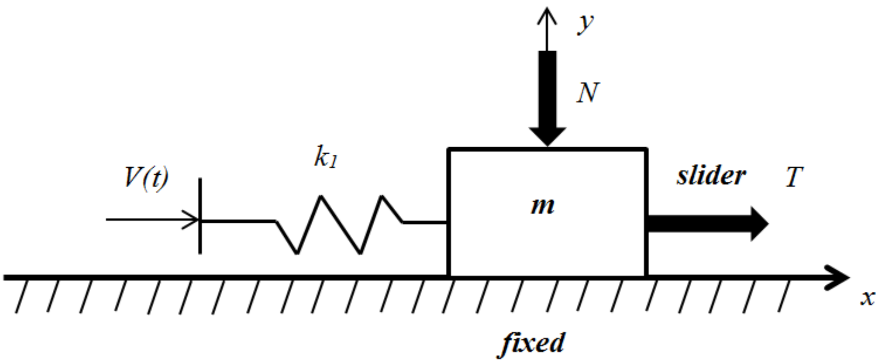
~~This study~~ We start~~s~~ with the self-excited oscillations, which ~~which may~~ resembles ~~look like~~ the stick-slip-like motion, but occurring under constant friction. ~~For this purpose a~~A single degree of freedom block-spring model is used for this purpose. A block sliding on a rigid horizontal surface is driven by a spring whose other end is attached to a driver moving with a constant velocity (Figure 1). All variables and constants used in equations are listed below in Table 1.

Table 1: The list of variables and constants

Symbol	Meaning	Symbol	Meaning
V_0	load point velocity	τ	shear stress
V	relative velocity of block	τ_f	friction stress
k_1	single spring stiffness	E	Young's modulus
m	block mass	c	velocity of longitudinal wave (p=wave)
N	gravity force	ω	eigen frequency
T	shear force	k_2	the spring stiffness relating stress and displacement discontinuity (the difference between the rod displacement and the zero displacement of the base)
μ	friction coefficient	J_0	Bessel function of order 0
ω_0	eigen frequency	J_0'	derivative of Bessel function
t	time	i	imaginary unit
h	thickness of an infinite rod	ξ	independent variable
ρ	volumetric rod density	z	integration variable
σ_N	uniform compressive load	f, g	arbitrary functions
σ	longitudinal stress		

Friction is assumed to be cohesionless: $T_{cr}=\mu N$, where T_{cr} is the force at which sliding starts. The system consists of mass m, spring of stiffness k and a driver that moves with the constant velocity V_0. Friction is assumed to be cohesionless: $T_{cr}=\mu N$, where T_{cr} is the force at which sliding starts, N is the normal force and μ is the friction coefficient.





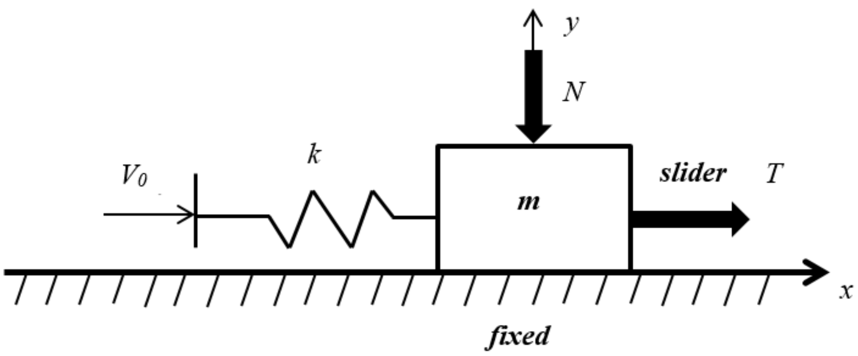


Figure 1: The simple single block model.
mass-spring model of Burridge and Knopoff type.

The system of equations representing the motion of the block reads:

$$\begin{cases} m\dot{V} = T - \mathrm{sgn}(V)\mu N \\ \dot{T} = k(V_0 - V) \end{cases} \qquad \begin{cases} m\dot{V} = f(T, \mu N) \\ \dot{T} = k_1(V_0 - V) \end{cases} \qquad (1)$$

The appearance of the $f(T,\mu N)$ function in the system of equations represents the fact that $V \geq 0$.

The function $f(T,\mu N)$ is defined as:

$$f(T,\mu N)=\begin{cases} T-\mu N, & T>\mu N \text{ and } V>0 \\ 0, & T<\mu N \text{ or } V<0 \end{cases} \qquad (2)$$

where m is the mass of block, k is the spring stiffness, V_0 is the load point velocity, V is the relative velocity, N is gravity force, T is the shear force, μ is the friction coefficient.

The appearance of the sign function in the system of equations represents the fact that friction always acts against velocity. Here function sgn(V) is defined as follows:

$$\operatorname{sgn}(V)=\begin{cases} -1 & \text{for } V<0 \\ 0 & \text{for } V=0 \\ 1 & \text{for } V>0 \end{cases} \qquad (2)$$

In order to represent the system of equations (1) in dimensionless form, it is convenient to introduce a dimensionless time t^*:

$$t^*=t\omega_0, \quad \omega_0^2=\frac{k_1}{m} \qquad (3)$$

where ω_0 is the eigen frequency of the block-spring system, m is the block mass and k_1 is the spring stiffness.

The governing system of equations in dimensionless form readsis defined as:

$$\begin{cases} \dot{V}=f(T^*,\mu N^*) \\ \dot{T}=1-V^* \end{cases} \qquad (4)$$

$$\begin{cases} \ddot{V}=1-V \\ V(0)=V^* \\ \dot{V}(0)=T(0)-\operatorname{sgn}(V^*)\mu N \end{cases}$$

(4)

Here where the dot represents the derivative with respect to dimensionless time t^*, $\ddot{V}=\dot{T}$ and V^*, T^* and N^* are the dimensionless velocity, shear force and gravity force respectively.

$$V^*=\frac{V}{V_0},\ T^*=\frac{T}{mV_0\omega_0}\ N^*=\frac{N}{mV_0\omega_0}.$$

2.1 Behaviour of the system

under different initial conditions

In order to demonstrate the behaviour of the system at stick-slip-type regime, we consider the block sliding under the following set of initial conditions:

$$V(0)=0,\ \dot{T}(0)=0 \qquad (5)$$

Figure 2 represents the corresponding behaviour of the system (dimensinless velocity vs. dimensionless time).

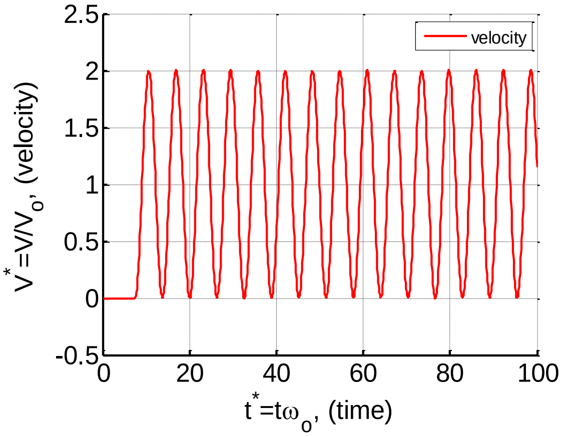


Figure 2: Block sliding with constant friction coefficient.

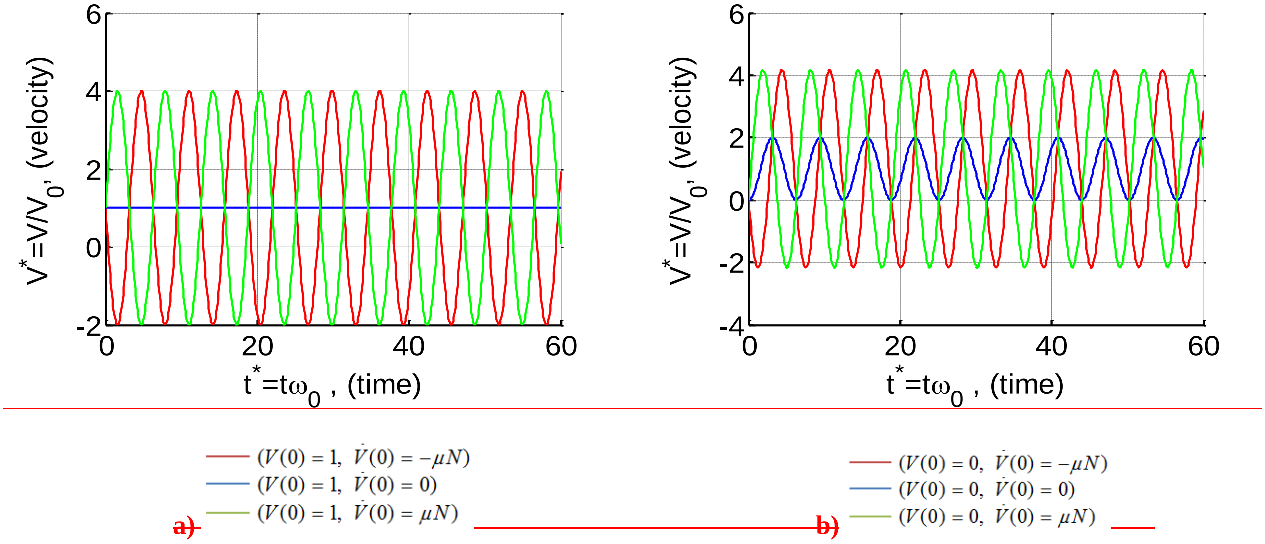
~~under different initial conditions leading to the steady sliding and stick-slip-type regimes we assume velocity V >0 and consider the block sliding under the following two sets of initial conditions:~~

$$V(0)=1,\ \dot V(0)=-\mu N;\quad V(0)=0,\ \dot V(0)=-\mu N$$

~~$V(0)=1,\ \dot V(0)=0;\quad V(0)=0,\ \dot V(0)=0$~~ ~~(5)~~

$$V(0)=1,\ \dot V(0)=\mu N;\quad V(0)=0,\ \dot V(0)=\mu N$$

~~Figure 2 represents the corresponding two types of behaviour of the system (dimensinless velocity vs. dimensionless time).~~



~~**Figure 2: Block sliding under different initial conditions.**~~

It is ~~seen that~~observed that the system exhibits self-excited oscillations even with constant friction coefficient, which somewhat resemble the stick-slip-type sliding. Furthermore, the energy in the system does not change with time, obviously due to the constant energy influx by velocity V_0, where the excess of the V_0 is dissipated by friction.

~~This is a harmonic motion with the frequency is equal to the eigen frequency of the system. The friction coefficient only affects the initial conditions. In more~~A detailed investigation of the behaviour of ~~such~~ a system described in a section 2 was undertaken in our previous works ~~is investigated in our previous works~~ (Karachevtseva et al., 2014; Karachevtseva et al., 2014).). It should also be noted that similar oscillation-type movements were observed in laboratory experiments on sliding of two granite blocks under biaxial compression (Sobolev et al., 2016).

3 Stress wave propagation in frictional sliding (generalisation 1D solid)

In t~~T~~he previous section, we show~~ed~~s the stick-slip-like motion occurring even when the friction coefficient is constant. In this section we ~~Now this understanding will~~ will expand our understanding to incorporate the ~~be generalised to~~ slide over a fault where a stick-slip phenomenon is traditionally flagged as a mechanism of earthquakes. We shall keep assuming the constant friction law, which will permit us to obtain an analytical solution. For this purpose, following Nikitin (1998), we consider the simplest possible 1D model of fault sliding, which takes into account the rock elastic response and the associated dynamic behaviour~~, shown in Figure 3~~. The model is shown in Figure 3. It consists of

~~To this end,~~ an infinite elastic rod of height (thickness) h, and of ~~per~~ unit length in the direction normal to the plane of drawing in Figure 3. ~~and linear~~The linear density is ~~ρ~~ ρ and the rod is assumed to be able ~~sliding~~ slide over a stiff surface ~~is considered~~. The sliding is resisted by friction. The stiff surface can be described as a symmetry line such that instead of the (horizontal) fault, only the upper half of the line is considered.~~though of as a symmetry line, such that instead of the (horizontal) fault only the upper half of it is considered.~~ The rod is connected to a stiff layer moving with a constant velocity V_0. The connection is achieved through a series of elastic shear springs. Both the elastic rod and the elastic springs describe the model of the ~~model the~~ elasticity of the rock around the fault, as shown in Figure 3. We assume that the system is subjected to a uniform compressive load σ_N ~~σ_n~~ such that the friction stress is kept constant, which is ~~; it is~~ assumed equal to ~~$\tau_{fr} = \kappa\mu\sigma_n = const$~~ $\tau_f = \mu\sigma_N = const$.

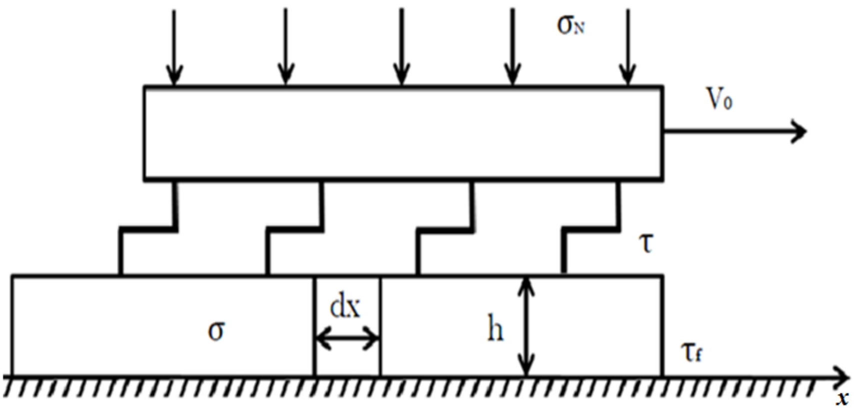


Figure 3: The model of infinitive elastic rod driven by elastic shear spring.

~~Let the longitudinal (normal) stress in the rod be σ and the contact shear stress be τ, friction stress τ_f and the load point velocity V_0.~~ ~~The~~ E~~e~~quation of movement of the rod reads:

$$\frac{\partial\sigma}{\partial x} + \frac{1}{h}(\tau - \tau_f) = \rho\frac{\partial V}{\partial t} \qquad (6)$$

Where σ is the longitudinal (normal) stress in the rod, τ is the contact shear stress, τ_f is the frictional stress, V_0 is the load point velocity and $V(x,t)$ is the velocity of point x of the rod at time t, as shown in Figure 3.

~~$V(x,t)$ is the velocity of point x of the rod at time t, Figure 3.~~

~~If the Young's modulus of the rod is E, then~~ According to the Hooke's law: ~~gives~~

$$\sigma = E\frac{\partial u}{\partial x}$$

(7)~~,~~

where $u(x,t)$ is the displacement and E is the Young's modulus of the rod. After differentiating, we have:

~~. After differentiating the Hooke's law is expressed as:~~

$$\frac{\partial \sigma}{\partial t} = E\frac{\partial V}{\partial x}$$

(~~7~~8)

The elastic reaction of the shear springs is expressed ~~through the following equation~~as:

$$\frac{\partial \tau}{\partial t} = k_2(V - V_0)$$

(~~8~~9)

where k_2 is the spring stiffness relating stress and displacement discontinuity (the difference between the rod displacement and the zero displacement of the base). ~~In the usual way system of equations (6)-(8) produces the wave equation:~~

Defining $\Delta V = V - V_0$ and solving the system of equations (6)-(9), we get the following wave equation:

$$\frac{\partial^2 \Delta V}{\partial t^2} = c^2\frac{\partial^2 \Delta V}{\partial x^2} - \omega^2 \Delta V$$

(~~9~~10)

where $c = \sqrt{Eh/\rho}$ is the velocity of the longitudinal wave (p-wave), $\omega = \sqrt{k_2/(h\rho)}$ is ~~what can be~~ regarded as eigen frequency of the system consisting as a unit length of the rod considered as a lamp mass on the shear springs.

It is ~~seen~~ observed that despite the ~~frictional sliding between the rod and the stiff surface~~ presence of shear springs and friction between the rod and the stiff surface, the waves propagate with the p-wave velocity determined by the Young's modulus and density of the rod. ~~So~~ Therefore~~according~~, according to the terminology described in ~~Introduction~~the introduction, the wave should be named *p-sonic wave*. It should be highlighted ~~emphasizes~~ that while such waves look like the shear waves, they are in fact compressive waves propagation along the rod, hence denoted as the p-wave velocity.

In order to analyse the way the pulse propagates, equation (~~9~~10) is complemented by the initial conditions as:

$$\Delta V(x,t) = f_0(x); \quad \frac{d\Delta V}{dt} = F_0(x) \qquad (11)$$

~~(10)~~

Solution of wave equation (10~~9~~) can be found by using the Riemann method (e.g., Koshlyakov, 1964).

$$\Delta V(x,t) = \frac{1}{2}[f(x-ct) + g(x+ct)] + \frac{1}{2}\int_{x-ct}^{x+ct}\Phi(x,t,z)dz \qquad (12)$$

~~(11)~~

where

$$\Phi(x,t,z) = \frac{1}{\sqrt{c^2t^2 - (z-x)^2}}\varphi(x,t,z)$$

(~~12~~13)

The integral from (12~~1~~) can be found by using the Chebyshev-Gauss method

$$I(x,t)=\int_{x-ct}^{x+ct}\Phi(x,t,z)dz\approx\frac{\pi}{n}\sum_{j=1}^{n}\varphi(x,t,\ x+\zeta_j at),\quad \xi_j=\cos\left(\frac{2j-1}{2n}\pi\right) \qquad (14)$$

~~(13)~~

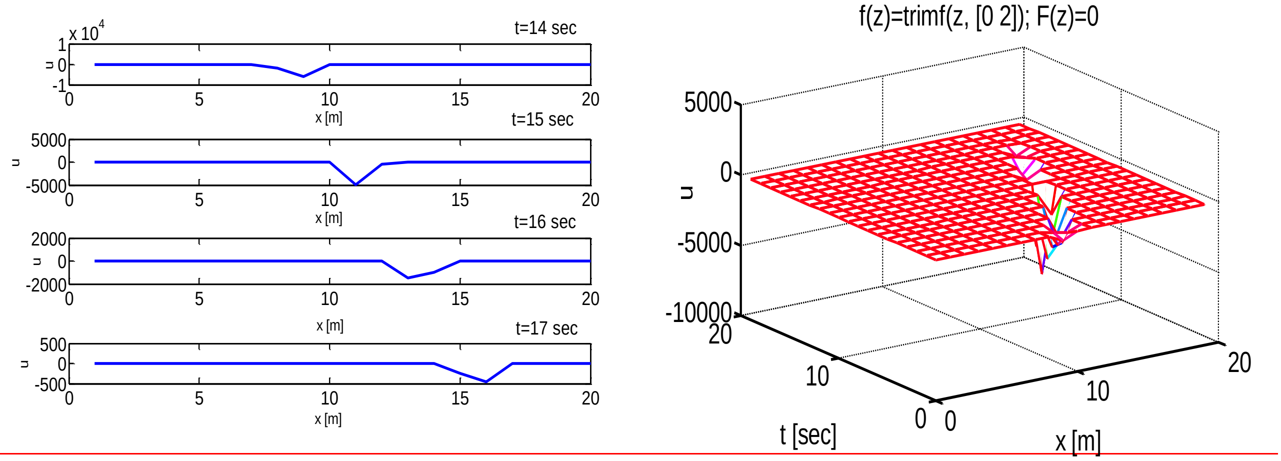
where

$$\varphi(x,t,z)=\frac{1}{c}F(z)J_0\left(\frac{\omega}{c}i\sqrt{c^2t^2-(z-x)^2}\right)\sqrt{c^2t^2-(z-x)^2}+\omega tf(z)\left(\frac{1}{i}\right)J_0'\left(i\frac{\omega}{c}\sqrt{c^2t^2-(z-x)^2}\right)$$

(~~14~~15)

3.1 Propagation of an initial ~~disturbance~~ sliding

Figures 3-~~5~~ 4 represent the propagation of initial ~~disturbance~~ sliding under the different initial conditions. Particularly, a triangular ~~displacement~~ velocity impulse, equation (16) and zero ~~velocity~~ acceleration were used as initial conditions for Figure 3. As shown ~~F~~in~~or~~ Figure 4, linear and harmonic functions ~~were~~ are used for ~~displacement~~ velocity and ~~velocities~~ acceleration as initial conditions.



$$f(x;a,b,c)=\max\left(\min\left(\frac{x-a}{b-a},\frac{c-x}{c-b}\right),0\right) \qquad (16)$$

where x is the vector, a, b, c are scalar parameters.

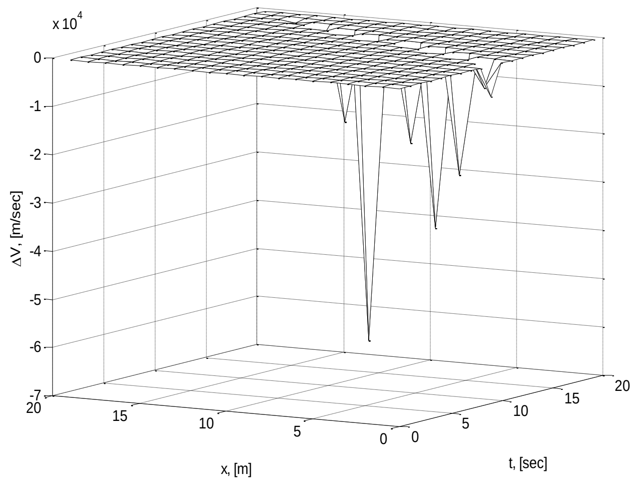
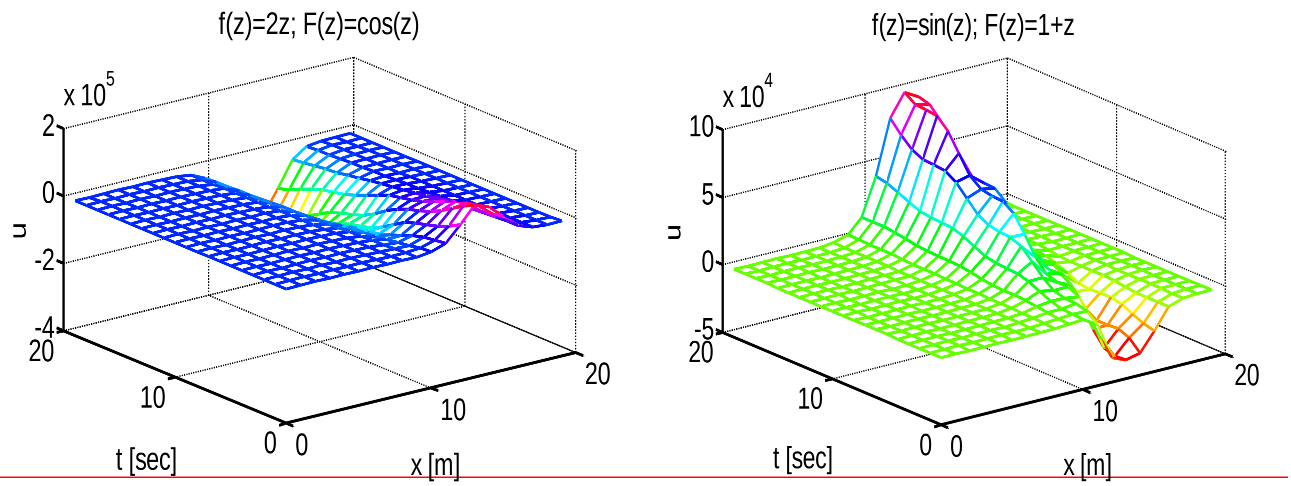


Figure 3: Propagation of initial sliding in the form of a triangular function *f*(z) of zero area. ~~disturbance (f (z) = trimf (z, [0 2]); F(z)=0).~~



~~**a) f(z)=2z; F(z)=cos(z); b) f(z)=sin(z); F(z)=1+z;**~~

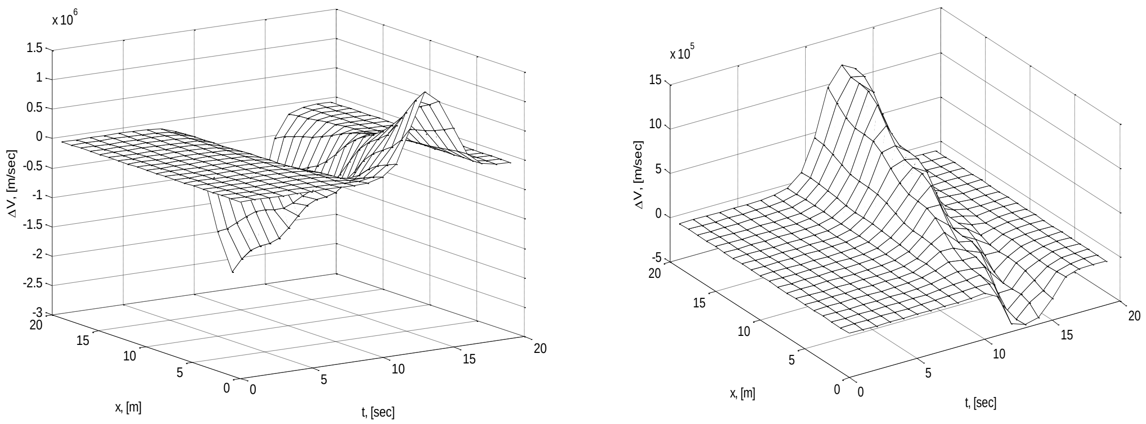
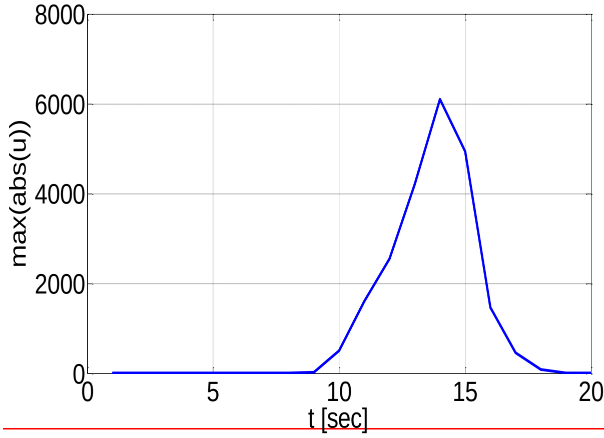


Figure 4: Propagation of initial ~~disturbances~~sliding with different initial conditions.

It is seen that the initial ~~disturbance~~ sliding (impulse) propagating with p-wave velocity keeps its width but the amplitude reduces with time. It is also observed that as ~~Obviously as~~ the impulse propagates, it ~~looses~~loses energy which goes to increas~~eing~~ the energy of shear springs.

~~Figure 5 shows the peak of initial disturbance changing with time (here the triangular displacement and zero velocity were set as initial conditions).~~



~~Figure 5: Maxima of initial disturbance.~~

4 Discussion

This paper introduced the notion that the frictional movement resembling the stick-slip sliding, which are often observed and usually attributed to the rate dependence of friction, can be obtained with constant friction by taking into account the elasticity of the surrounding and its self-oscillations. This understanding is applied to propagation of slip over infinitely long fault leads to a simple model that predicts that the slip will propagate with p-wave velocity. This conclusion is made under the assumption of constant (rate-independent) friction. Relaxing this assumption, that is taking into account that

$\tau_f = \tau_f(\frac{\partial \Delta V}{\partial t})$ leads to the following equation replacing equation (10):

$$\left(1+\frac{1}{\rho h}\frac{d\tau_f}{d\Delta V_t'}\right)\frac{\partial^2 \Delta V}{\partial t^2} = c^2\frac{\partial^2 \Delta V}{\partial x^2} - \omega^2 \Delta V, \quad \Delta V' = \frac{\partial \Delta V}{\partial t} \qquad (17)$$

It is seen that when the sliding rate changes slowly, the propagation speed of rupture c_1 can be approximated as:

$$c_1^2 \approx c^2\left(1+\frac{1}{\rho h}\frac{d\tau_f}{d\Delta V_t'}\right)^{-1} \qquad (18)$$

Furthermore, it is observed that when the friction increases with the sliding rate, c_1 becomes smaller than p-wave velocity. If the rate dependence of friction is lowered further, the slip propagation can become intersonic.

~~4~~ 5 Conclusions

In this paper, it is shown that the~~The~~ accumulation of elastic energy in the sliding plates on both sides of the fault can produce oscillations in the velocity of sliding even when the friction is constant. These oscillations resemble stick-slip movement, but they manifest themselves in terms of sliding velocity rather than displacement. The sliding exhibits wave-like propagation over long faults. Furthermore, ~~an infinite elastic rod~~ the 1D model shows that the zones of ~~disturbances~~ sliding propagate along the fault with the velocity of p-wave (the propagation speed can however be lower if the rate dependence of friction is taken into account). The mechanism of such fast wave propagation is the normal (tensile/compressive) stresses in the neighbouring elements (normal stresses on the planes normal to the fault surface) causing a p-wave propagating along the fault rather than the shear stress controlling the sliding. This manifests itself as a p-sonic propagation of an apparent shear rupture.

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