Nonlin. Processes Geophys. Discuss.,
doi:10.5194/npg-2016-72-RC2, 2017

Nonlinear Processes in Geophysics
Discussions

Interactive comment on "Trajectory encounter number as a diagnostic of mixing potential in fluid flows" *by* Irina I. Rypina and Larry J. Pratt

Anonymous Referee #2

Received and published: 2 January 2017

In this paper the authors introduce a new Lagrangian descriptor to give a measure of the effectiveness of a flow to mix over a finite time. The idea is to start with a finite grid of K initial trajectories, and, for each trajectory compute the number other trajectories that come within a radius R of the given one, thus they compute

$$N(x,R,T,K) = \sum_{k=1}^{K} I\left(\min_{s,t\in[0,T]}(\|\phi_s(x_k) - \phi_t(x)\|) \le R\right)$$

where we define the indicator function I to return 1 if true and 0 if false, and the flow $\phi_t(y) = x(t;y)$ for an initial point y. (The authors never give such a formula, and ignore the dependence on the gird).

While this is an intriguing idea, it is not clear how to make it mathematically well-

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Discussion paper

defined. It seems to have some relation to finite time entropy, as introduced in the reference by Froyland âĂŤthis computes the growth rate of number of distinguishable trajectories. Would it be better to talk about a growth-rate here too? I feel that one should not just compute something that is so specific to choices, but first make a consistent mathematical definition: something that exists in the limit as the grid of initial points becomes infinitely fine, say, and then compute it, showing that the computations are, to some approximation, giving the desired quantity.

1. The authors fix the grid size and do not investigate how the number depends on grid size. The do not even tell the reader what grids are used in the first two examples!

2. It seems like it would be better to define something that (like they mention in the conclusions) represents a "fraction" or "density" of encounters. Mathematically one would probably define something that uses an $\epsilon - \delta$ construction: Given trajectories on an ϵ grid, how many get closer than δ? Then take limits. if possible, of a density or growth-rate?

3. Another possible quantity, though instead of measuring "mixing" would be one that measures "ergodicity": How many grid cells does a given trajectory cover? This might also be an interesting quantity, and much easier to compute. Note that mixing is equivalent to each trajectory visiting every grid cell.

4. The authors do not really compare their results with any of the other many possible descriptors like FTLE, or perhaps more relevantly the finite time entropy.

5. The authors do not discuss the complexity of this computation. It seems to me that it is much more computationally intensive than, e.g the FTLE, which does not involve comparing all distances between all trajectories. It this really a feasible calculation? How does it scale with the number of trajectories and the time?

6. The authors do some basic investigation of how Ndepends upon R and t, but the computations of N for the simple diffusive and shear cases seem wrong to me:

In particular, if we take a planar diffusive process with diffusion coefficient D, and make the assumption (not clear to me) that one can transform to a frame moving with one particle (doesn't this double the diffusivity?), then one should compute the probability of a particle finding itself inside a disk of radius R for any time $0 < t < T$, given it starts at some point (x_0, y_0) in the plane. For example for the process on the line, then at a FIXED time t this means evaluating the integral

$$P(|x(t)| < R | x(0) = x_0) = \frac{1}{2\sqrt{Dt}} \int_{-R}^{R} \exp\left(-\frac{(x - x_0)^2}{2Dt}\right) dx$$

ï£ijwhich can be evaluated in terms of error functions. The authors seem to assume a deterministic motion with the root mean square distance, which seems to me to be wrong. They also ignore particles that start inside the circle of radius R (not so important if they want a large t limit I suppose). Now to compute N you have to sum (or integrate?) this probability over an initial distribution of initial points, say x_0 is uniform on a box, perhaps? And you have to somehow compute the probability over all times $0 < t < T$. This calculation seems very different from the one given in the paper.

The shear flow is easier, but I think not done correctly either. One has to compute the area of the region that sweeps into the circle of radius R, but also include the particles that start inside the disk.

While this paper has an intriguing idea, I think it needs substantial revision and correction before publication.

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