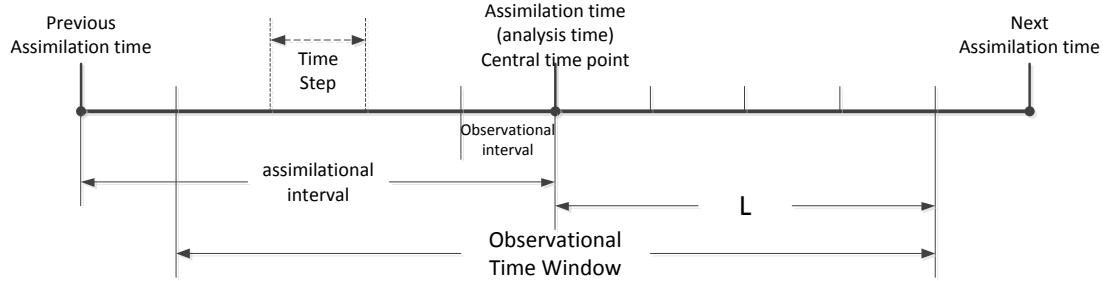


**Figure 1**



**Figure 1:** The schematic for the assimilation interval, the length of observational time window (OTW) as well as observational interval in terms of the model integration time step. Here  $L$  represents the time steps at one side of OTW. For example, OCN-OTW ( $L$ ) in the content stands for an ocean observational time window with total observations of  $2L+1$ .

Figure 2

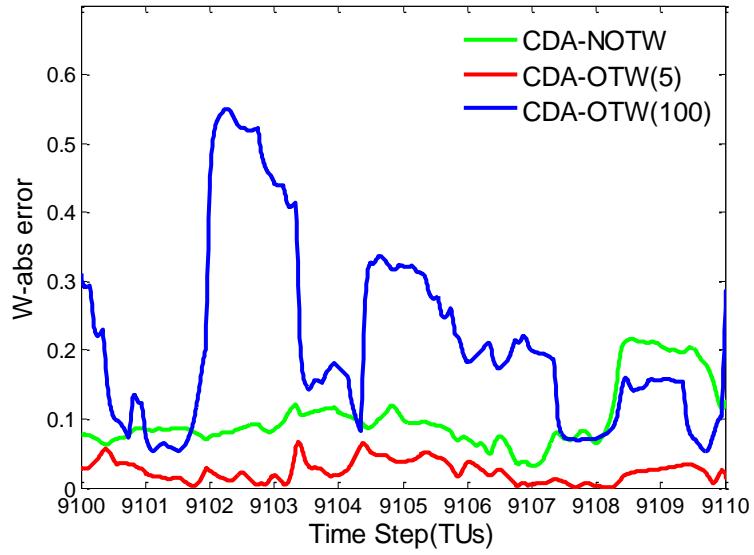
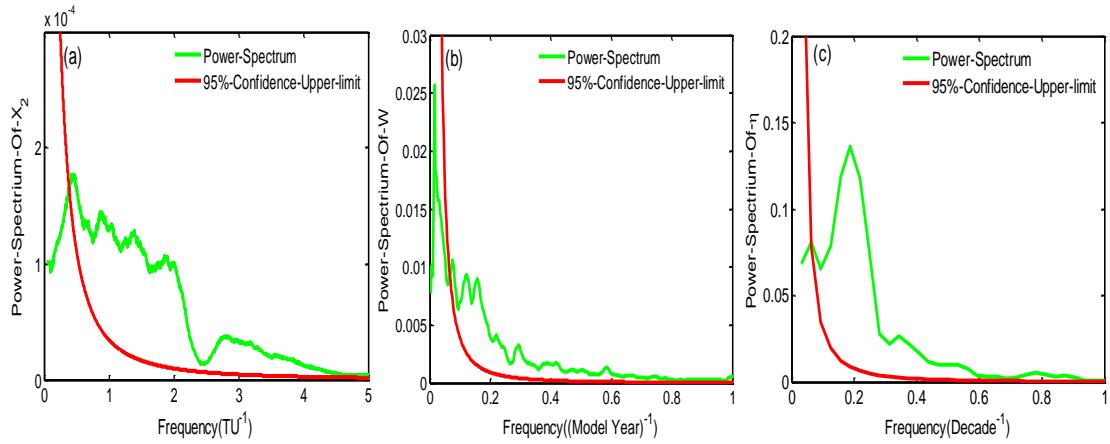


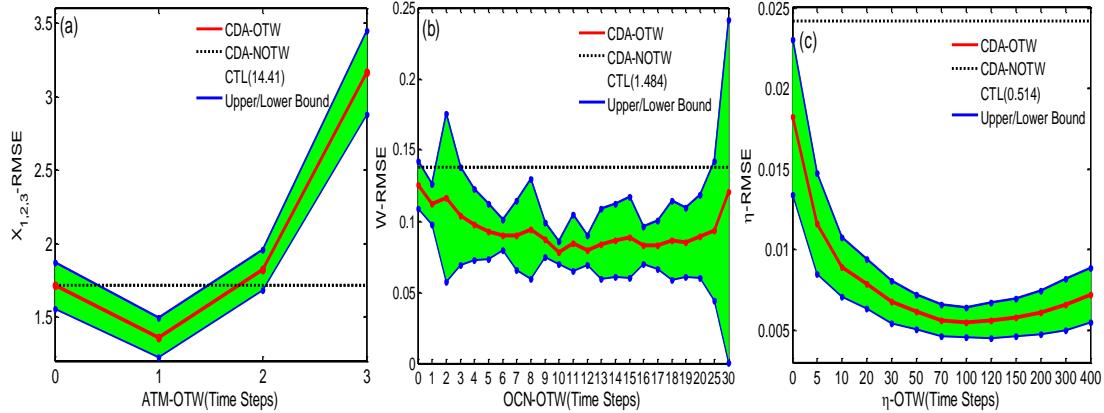
Figure 2: Time series of the absolute errors of the slab ocean variable ( $\omega$ ) in 3 assimilation experiments based on the model states between 9100 TUs and 9110 TUs assimilation results in the perfect model experiment framework with the uni-variate adjustment scheme. Green – CDA control with the standard update intervals of 0.05 TU for  $X_{1,2,3}$  and 0.2 TU for  $\omega$ ; Red – CDA with an ocean observational time window (OCN-OTW) of 5 time steps [OCN-OTW(5)]; Blue – CDA with OCN-OTW(100).

**Figure 3**



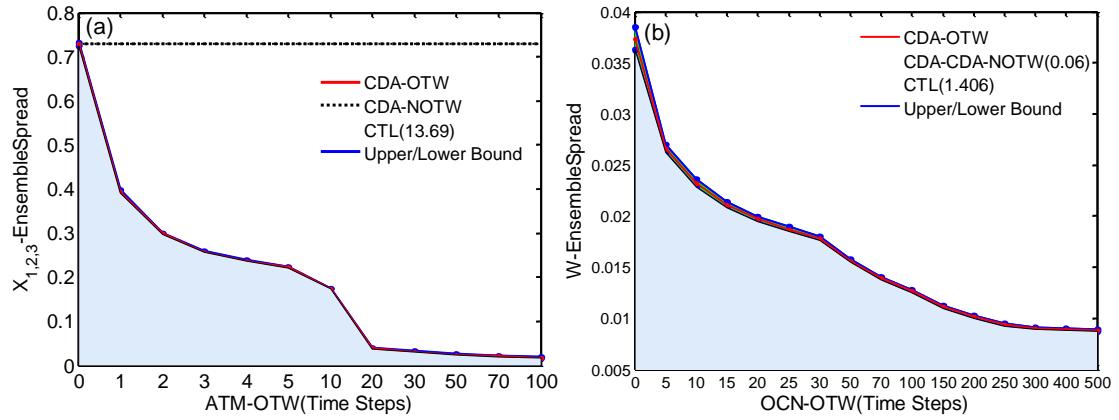
**Fig. 3 The power spectrum (green) of a)  $X_2$  b)  $\omega$ , c)  $\eta$  based on the model states between 5000TUs and 9800TUs integrations after the spin-up which integrates for 10000TUs from the initial condition (0,1,0,0,0) with respect to the frequency, with 95% statistics significance (red).**

Figure 4



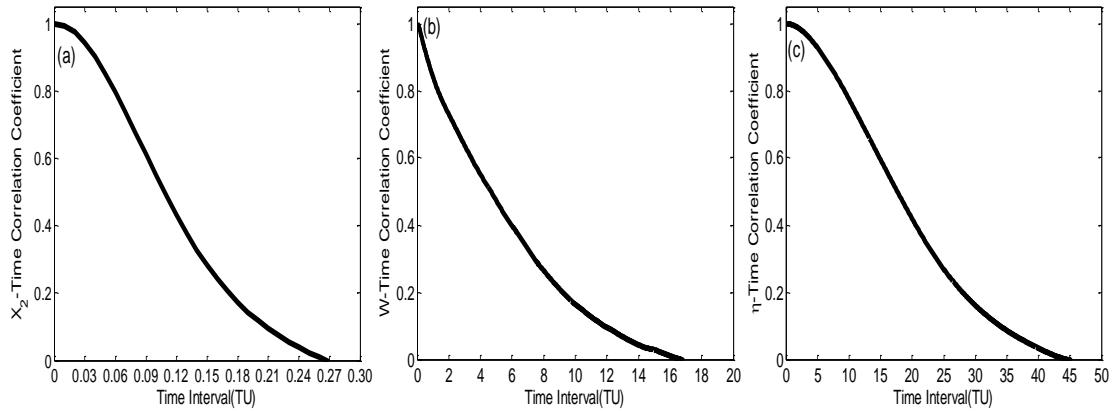
**Fig. 4** Variations of root mean square errors (RMSEs) of a “atmospheric” states  $X_{1,2,3}$  (namely the average of  $X_1, X_2$  and  $X_3$  RMSEs) in the space of ATM-OTW length when the “oceanic” state ( $\omega$ ) only uses a single observation at the assimilation time; b) “upper ocean” state ( $\omega$ ) in the space of OCN-OTW length when the ATM-OTW is fixed 1 as shown in panel a (1 for the ATM-OTW, i.e. 3 observations in each window, see the caption of Fig. 1) but the OCN-OTW (for  $\omega$ ) is varying and c) “deep ocean” state ( $\eta$ ) in the space of  $\eta$ -OTW length when the “deep ocean” observations are assumed to be valid and the ATM-OTW and OCN-OTW are fixed as 1 and 10, respectively. The experiments are conducted in a perfect model setting with a simple uni-variate adjustment scheme. The red lines are the 20-case mean, each using different initial conditions taken from different periods in the control integration (see description in section 2.2), and the blue lines represent the upper/lower bounds (mean  $\pm$  standard deviations) of the RMSEs. An OTW with the length of 0 represents only assimilating the observation at the assimilation time (i.e. with no OTW, dashed-black lines). The RMSE values of the control case (no observational constraint, called CTL) are marked in the parenthesis.

**Figure 5**



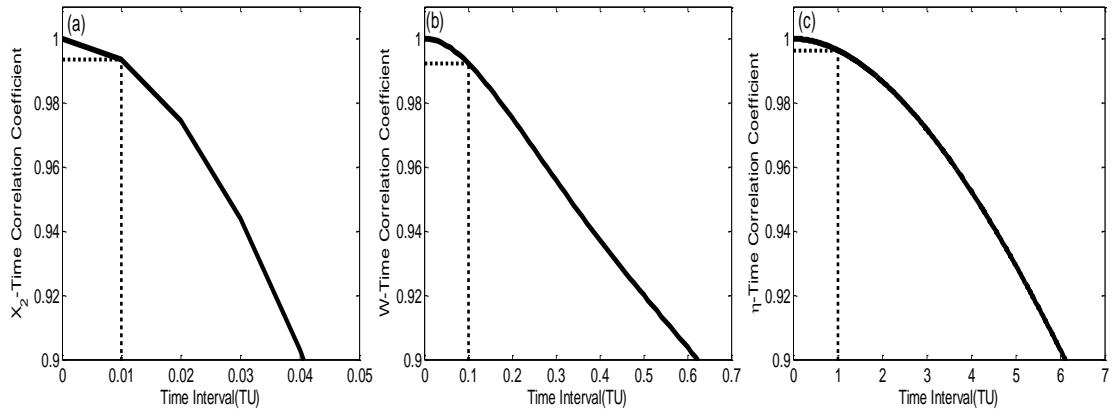
**Fig. 5** Same as the panels a and b in Fig. 4 but for the variation of ensemble spreads of the model states. In panel b the optimal ATM-OTW is also set as 1. The area between the lower and upper bounds (blue) represents the range evaluated from the 20 cases.

**Figure 6**



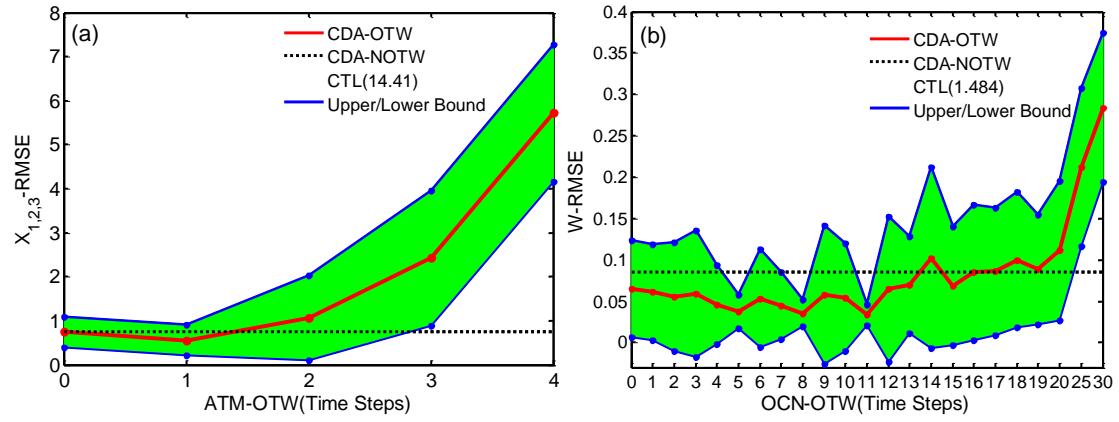
**Fig. 6** The auto-correlation coefficient of a)  $X_2$  b)  $\omega$ , c)  $\eta$  in the space of lag times. What are shown is the mean of 20 cases. In each case, an independent section (each has 10000 data of the state – 100 TUs with the interval of 0.01 TU) is used to evaluate the lag correlation coefficient. The 20 independent sections are taken from the model states apart each 200TUs between 5000TUs and 9000TUs integrations after the spin-up of 10000TUs from the initial condition (0, 1, 0, 0, 0).

**Figure 7**



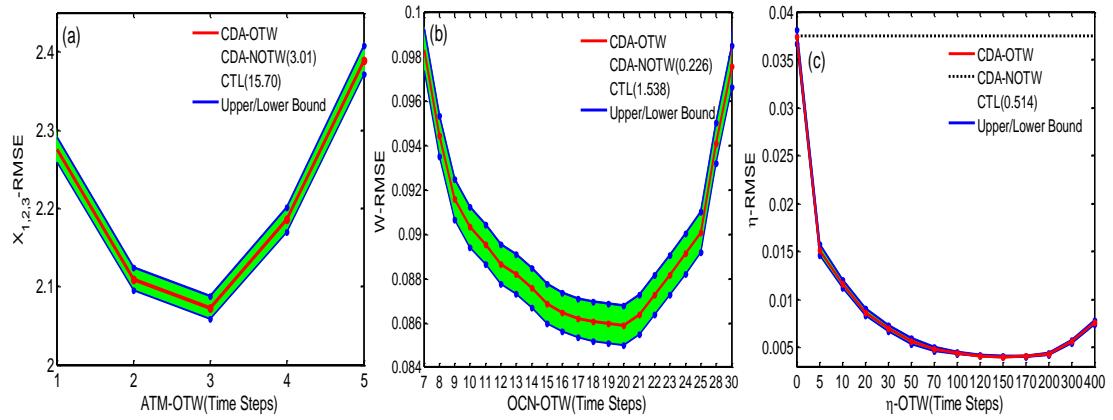
**Fig. 7** Same as Fig.6 but for the zoomed-out version to show relationship between the optimal OTW and de-correlation. The black dashed lines mark the corresponding time correlation coefficients at the time scale (L) of optimal OTWs as detected by Fig. 4 for different media.

**Figure 8**



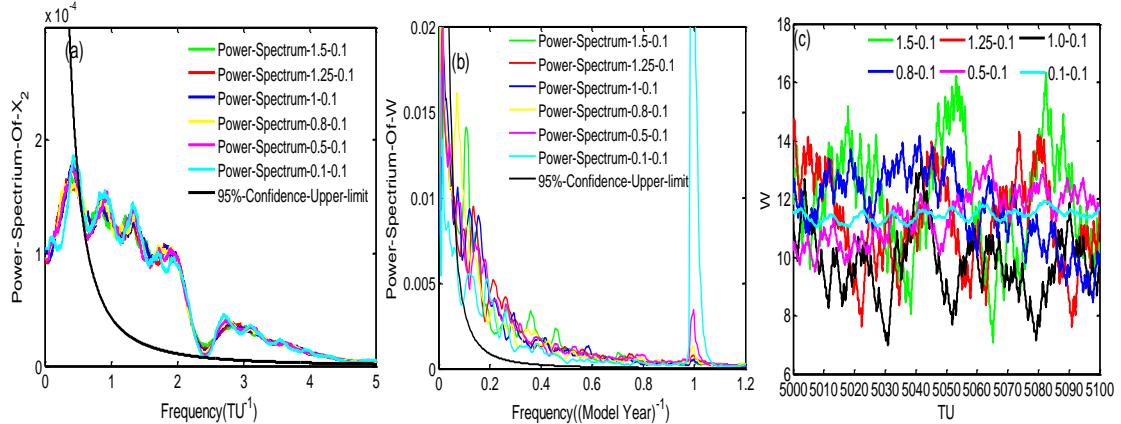
**Fig. 8** Same as Fig. 4 but using multi-variate adjustment scheme. In panels b the optimal ATM-OTW is also set as 1.

**Figure 9**



**Fig. 9** Same as Fig. 4 but using the biased model setting. In panels b) the optimal ATM-OTW is set as 3. And in panels c) the optimal ATM-OTW and OCN-OTW are kept as 3 and 20, when the “deep ocean” observations are assumed to be valid.

**Figure 10**



**Fig. 10** The power spectrum of a)  $X_2$  and b)  $\omega$  based on the model states between 5000 TUs and 9800 TUs integrations after the spin-up which integrates for 10000TUs from the initial condition (0,1,0,0,0) with different coupling strength (  $C_2$  is set as 1.5, 1.25, 1.0, 0.8, 0.5 and 0.1). Panel (c) shows the time series of the model state  $\omega$  between 5000 TUs and 5100 TUs integrations corresponding to the six cases.

Figure 11

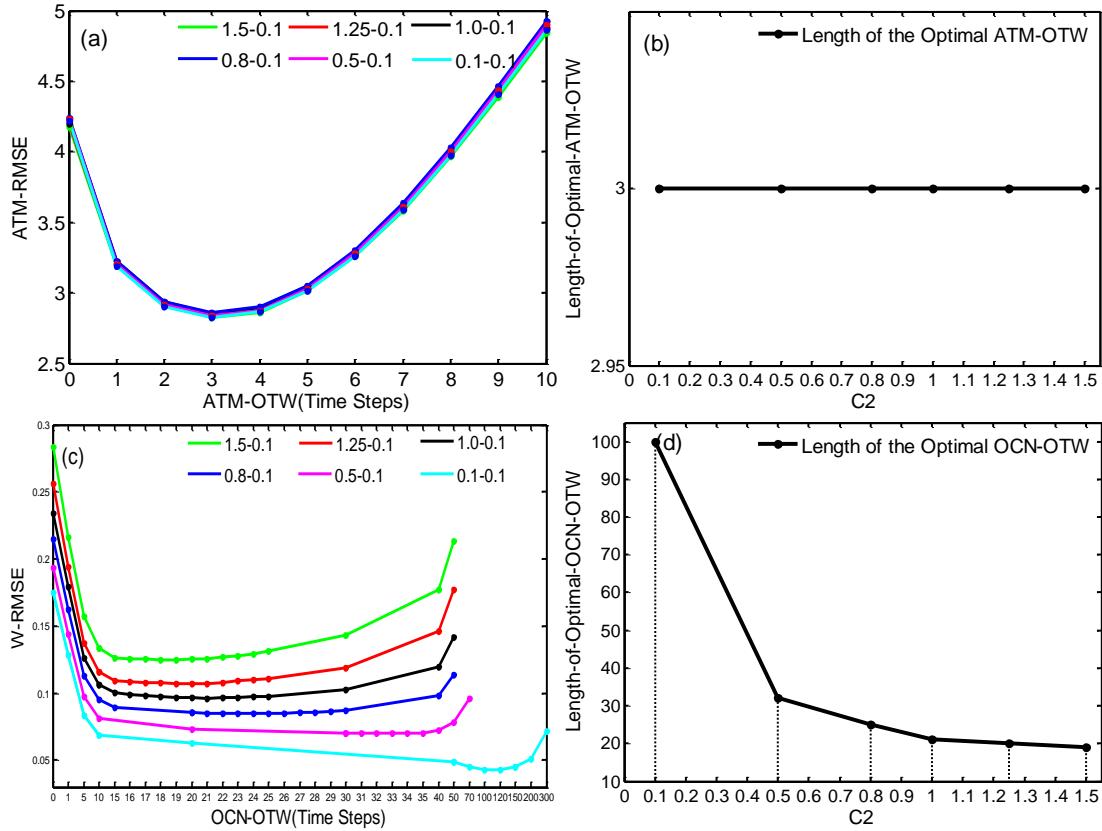


Fig. 11 Panels a) and c) are the same as panels a) and c) in Fig. 7 but for cases using the biased model setting and six different coupling strength cases (with  $C_2$  values as 1.5, 1.25, 1.0, 0.8, 0.5, 0.1). Panels b) and d) are the variation of the length of the optimal ATM-OTW and OCN-OTW with respect to the values of  $C_2$ .