

# ***Interactive comment on “Characterization of HILDCAA events using Recurrence Quantification Analysis” by Odim Mendes et al.***

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## **Characterisation of HILDCAA events using Recurrence Quantification Analysis**

by  
Mendes et al.

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Referee = **R** and Authors' answer = **A**

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**General comments of the authors:**

Initially, we thank the Reviewer for the suggestions. A PDF of the paper revised is attached and presents the complete information (In it, the color blue indicates parts of the text revised or even sections completely revised).

**R:** This paper has many weaknesses: the methodology is not explained in sufficient detail, the conclusions are simply not understandable, the English is very poor.

**A:** We rewrote the text taking into account all the remarks. We added details in the RP and RQA explanations in the methodology and some new references to clarify some points. We also revised the content according to the suggestions taking care of language issues.

**R:** Section 2 should be devoted to explain the mathematical method. Instead, it is merely a list of nomenclature and definitions. How is the Shannon entropy used in the paper? How are the four parameters defined? What do we learn from them?

**A:** We rewrote this section to clarify the measure definitions. The patterns presented in the RP can give a qualitative interpretation of the complexity of data being analysed. For that, with some practice, we can observe the patterns described in the vertical/-diagonal/horizontal lines, structures and clusters, isolated points, and the corner of the matrix. While a resource to a visual inspection, the RP technique is still an important tool in the complexity analysis. However, the RQA measurements provide an objective quantification of RP based on the structures presented on it. RQA becomes a power-

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ful tool to characterise complex non-linear data, helping us to analyse the dynamical system under investigation on the data.

Entropy (ENT) refers to ideas presented in the Shannon entropy according to Shannon(1964). This measure reflects the complexity of the recurrence plot with respect to the diagonal lines. It represents the probability to find exactly a diagonal line of length  $\ell$  in the RP. Nevertheless, the interpretation of the values of this measure is the opposite of traditional Shannon entropy, *i.e.*, larger ENT values are related to low entropy, as presented in Letellier(2006). In the general sense, the concept of entropy is the basis for the quantitative aspects of the Information Theory, translated by the researchers in this area in a mathematical formalism to build non-linear analysis tools for applying, for instance, to physics dataset.

**R:** Section 4 presents the result in a very hurried and superficial way. On line 29, page 5 the Authors argue that the behaviour in Figures 1 and 2 are very similar. They look very different to me. Why should they be similar? One is storm time, the other is quite time!

**A:** The Section 4 was rewritten to improve the presentation of the results and their interpretations. On the mentioned figures the focus was the variability feature, not to the signal intensity. Now, it is presented taking into account both the variability and the amplitude.

**R:** I do not understand how are Figures 3 and 4 generated and what they represent.

**A:** They represent a space phase plot (portrait), that is a geometric representation of the trajectories of the dynamics of the system in the phase plane. The fundamental starting point of many approaches in non-linear data analysis is the construction of a phase space portrait of the considered system. The state of a system can be described by its state variables  $x_1(t), x_2(t), \dots, x_d(t)$ , for example the both state variables temper-

ature and pressure for a thermodynamic system. The " $d$ " state variables at time  $t$  form a vector in a  $d$ -dimensional space which is called phase space. The state of a system typically changes in time, and, hence, the vector in the phase space describes a trajectory representing the time evolution, the dynamics, of the system. The shape of the trajectory gives hints about the system; periodic or chaotic systems have characteristic phase space portraits.

The observation of a real process usually does not yield all possible state variables. Either not all state variables are known or not all of them can be measured. However, due to the couplings between the system's components, we can reconstruct a phase space trajectory from a single observation  $u$  by a time delay embedding, as described in Takens' embedding Theorem, 1981. Takens proved that instead of  $2m + 1$  generic signals, the time-delayed versions

$$u(t), u(t - \tau), u(t - 2\tau), \dots, u(t - 2m\tau),$$

of one generic signal would suffice to embed the  $m$ -dimensional manifold. There are some technical assumptions that must be satisfied, restricting the number of low-period orbits with respect to the time-delay  $\tau$  and repeated eigenvalues of the periodic orbits. The phase space reconstruction is not exactly the same to the original phase space; nevertheless, its topological properties are preserved, if the embedding dimension is large enough (the embedding dimension has to be larger than twice the phase space dimension). Details can be found in N. Marwan webpage <http://www.agnld.uni-potsdam.de/~marwan/matlab-tutorials/html/phasespace.html>.

**R:** Figures 5 and 6 show the RP matrix, but what do we learn from them?

**A:** Quantification measurements from RP come basically from the recurrence patterns it presents, such as point density, diagonal structures, and vertical structures in the recurrence plot. By visual inspection, we can see the signatures (from typology and texture) of the processes under consideration. We compare the dynamical behaviour

of processes to identify features and, maybe, similarities. We rewrote the text of Section Results to improve the content and the flux of ideas.

**R:** On line 16-17, page 6 we learn the RQA parameters calculated for the two cases, but again I have no idea what they mean, without reading their definition and physical interpretation.

**A:** We rewrote the text to improve the understanding in this part.

**R:** Finally, I simply cannot make any sense of the three conclusions on page 7. The first one seems wrong: it cannot be that auroral activity is responsible for energy transfer from the solar wind to the magnetosphere-ionosphere. The causality relationship is obviously in the opposite direction!

**A:** We improved the way we present the ideas in this section.

**R:** Regarding the second and third conclusions I do not argue that they are wrong. I just do not understand what they are supposed to mean.

**A:** We rewrote the text (with the major points) to improve the understanding in this part.

**R:** Despite all my criticism, I support the idea of using methods from dynamical systems and chaos theory to analyse geomagnetic events. It should be done, however, in a much more clear and accessible way. As it stands, this paper would not be understood/ appreciated by the largest majority of the community.

**A:** We improved the readability of the text to clarify the points the Reviewer presented.

## References

- C. E. Shannon. *The Mathematical Theory of Communication*. University of Illinois, Urbana, IL, University of Illinois, Urbana, IL, 1964.
- C. Letellier. Estimating the Shannon Entropy: Recurrence plots versus symbolic dynamics. *Physical Review Letters*, 96(254102), 2006.

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