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Nonlinear Processes in Geophysics
Discussions

# Dynamics of the Hadley circulation in an axisymmetric model undergoing periodic change in forcing stratification

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**Abstract.** The time-dependent response of the Hadley circulation to a periodic forcing is explored by using a simplified non-linear axisymmetric model. Thermal forcing towards a given equilibrium potential temperature drives the model atmosphere. The vertical stratification of this temperature is forced to become periodically neutral with a period $t_0$. Some simulations were performed with different values of $t_0$, from 10 to 90 days, they exhibit a stronger circulation when comparing with constant thermal forcing experiment. As the period increases, a transition takes place from a stationary regime, obtained when forcing is constant, to a quasi-periodic regime, and to an intermittent regime. The stream-function response to periodic forcing is a quasi-periodic oscillation, with two main frequencies dominating, one with a period equal or close to that of forcing and another with a period that is half of the forcing period. The former is dominant for values of $t_0$ larger than 30 days, whereas the latter is prevalent for $t_0$ smaller than 30 days. The quasi-periodic oscillations obtained in this model might be associated with the quasi-periodic oscillations observed in the tropical regions. In this case the periodic charge and discharge of moisture in the tropical atmosphere may be linked to those oscillations. In the model, with forcing periods over 63 days the response of stream-function periodically enters in a sort of intermittent regime, with chaos appearing with high frequency oscillations, which are modulated by the slow timescale of forcing. The vertical viscosity plays a role in determining even the evolution of the Hadley circulation under the conditions established by the forcing.

## 1 Introduction

The Hadley cell is a specific and observable feature of tropical circulation (Dima and Wallace, 2003). The main driving process undergoing this feature can be seen as a combination of low-latitude convective processes and interactions of tropical circulation with higher latitude eddies (Charney, 1969). Although tropical circulation follows a seasonal cycle due to the asymmetry of solar forcing during a solar year (Fang and Tung, 1999), it is also subjected to alternated intra–seasonal strong and weak periods (Goswami and Shukla, 1984). The cycles modulate the precipitation activities in the equatorial regions.

It is well known that the rainfall distribution over Indian tropical regions varies considerably from day to day with an alternate behavior associated with a few quasi-periodic cycles. Cycles of 3–7, 10–20 and 30–60 days are present in the atmospheric dynamics of tropics and subtropics. The 3–7 days cycle is related to synoptic-scale convective systems generated over the Bay of Bengal, the others have been linked to the cycles of monsoon rainfall over the Indian region how discussed by Kripalani et al. (2003) who showed a prevalence of the cycle 10–20 days during a normal monsoon year and one of 30–60 days in a

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Nonlinear Processes in Geophysics
Discussions

drought year. The latter is known as the Madden–Julian oscillation (Madden and Julian, 1972). Observational analyses and modeling studies have revealed that there are dominant periods in the tropics and subtropics. These tropical oscillations have been extensively analyzed with observations (Yasunari, 1979, 1980, 1981; Sikka and Gadgil, 1990, Yoneyama et al., 2013) as well as by means of models (Goswami and Shukla, 1984, Zhu and Hendon, 2015, Wang et al., 2016). The Madden-Julian oscillation has a significant impact on the Indian (Murakami 1976; Yasunari 1979) and Australian monsoons (Hendon and Liebmann 1990). Kessler and McPhaden (1995) for instance suggested that it could play an important role in the onset and development of El Niño events. The relationship between this oscillation and tropical cyclogenesis has also been linked in some works (Maloney and Hartmann, 2000; Mo 2000; Maloney and Hartmann, 2001). Moreover, He et al. (2011) showed that Hadley circulation variability is closely related to Madden-Julian oscillation convection. Goswami and Shukla (1984) suggested that this oscillation in the Hadley circulation is due to the interaction between the internal dynamics of tropical circulation with moist convection; in fact, with constant latent heat in their model the quasi-periodic oscillation vanished. This lead to consider that latent heat released during the moist processes can play an fundamental role in the dynamics of this cycle.

Knowledge of tropical atmosphere dynamics can be obtained by using complex general circulation as well as rather simpler models like the axisymmetric ones, which focus on the main processes occurring in the tropospheric region, capturing the central process of the Hadley circulation. In such a kind of models, for example, eddies are not allowed, and all the processes involved in the higher latitudes are not considered. An axisymmetric model (Cessi, 1988) is used in this paper to perform an analysis of Hadley cell behavior when periodic forcing is applied. The model spin up takes less than 100 days. When the imposed forcing is not variable, the developing circulation results in a steady state with the Hadley cell representing a fixed point solution within the phase space. In this kind of configuration other atmospheric processes, acting on longer or shorter spatial and temporal scales, such as those mentioned earlier, are essentially excluded.

The Hadley circulation is the meridional overturn that develops in response to temperature difference between equator and poles that is in radiative-convective equilibrium, it is worth analyzing how the model atmosphere behaves when the vertical stratification of equilibrium temperature becomes neutral. Although the equilibrium temperature towards which the model atmosphere is brought is essentially stable, we can think that a less stable temperature stratification occurs,for example, due to tropospheric heating caused by condensation of water vapor. However, it is reasonable to think that the system is not always forced to become less stable, but that this condition occurs periodically.

In Section 2 we will describe the model used and how we set up the stratification by modulating the vertical distribution of the temperature in order to periodically reach neutral static stability. When the vertical stratification is modulated by a periodic function we shall see, in Section 3, that this have an impact on the time evolution with a quasi-periodic response, with two main frequencies, of the stream function suggesting that a quasi-periodic response is intrinsic to this model when stratification changes periodically. The role of the vertical viscosity will be investigated too. In Section 4 we will draw the conclusions.

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Published: 8 November 2016

Nonlinear Processes in Geophysics
Open Access
Discussions

## 2 Model equation and forcing applied

The model used in this paper is the full model described in Cessi (1998), who studied its analytic and numerical solutions deriving the expansion into power series in the Rossby number R of the prognostic variables, the zonal momentum M, the potential temperature $\theta$, and the stream function $\psi$. The model follows the same hypothesis of the classical axisymmetric models such as those used by Held and Hou (1980) or by Fang and Tung (1999).

The horizontal coordinate is defined as $y = a\sin\phi$,from which we have

$$c(y) = \cos\phi = \sqrt{(1 - y^2/a^2)}, \tag{1}$$

where $a$ is the radius of a planet having a rotation rate $\Omega$, with an atmosphere height $H$. The model is similar to the Held and Hou model, but it prescribes a horizontal and vertical viscosity, respectively equal to $\nu_{\mathrm{H}}$ and $\nu_{\mathrm{V}}$. The prognostic variables are the angular momentum $M = \Omega\, a\, c^2 + u\, c$, where $u$ represents the zonal velocity; the zonal vorticity $\psi_{zz}$ with the meridional stream function $\psi$ is defined by

$$\begin{aligned} \partial_y \psi &\equiv w, \\ \partial_z \psi &\equiv -cv, \end{aligned} \tag{2}$$

and the potential temperature $\theta$ that is forced towards a radiative–convective equilibrium temperature $\theta_{\mathrm{E}}$.

Starting from the dimensional equations of the angular momentum, zonal vorticity and potential temperature, we will obtain a set of dimensionless equations by using a scaling that follows Schneider and Lindzen (1977), except for the zonal velocity $u$ that is scaled with $\Omega\, a$. A detailed description can be found in Cessi (1998).

The non-dimensional model equations are

$$M_t = \frac{1}{R}\left\{M_{zz} + \mu\left[c^4\left(c^{-2}M\right)_y\right]_y\right\} - J(\psi, M), \tag{3a}$$

$$\begin{aligned} \psi_{zzt} = & \frac{1}{(R^2E^2)} y c^{-2}\left(M^2\right)_z - \frac{1}{c^{-2}} J\left(\psi, c^{-2}\psi_{zz}\right) \\ & + \frac{1}{(RE^2c^{-2})}\theta_y + \frac{1}{(Rc^{-2})}\left[c^{-2}\psi_{zzzz} + \mu\psi_{zzyy}\right], \end{aligned} \tag{3b}$$

$$\theta_t = \frac{1}{R}\left\{\theta_{zz} + \mu\left[c^2\theta_y\right]_y + \alpha\left[\theta_{\mathrm{E}}(y,z) - \theta\right]\right\} - J(\psi, \theta). \tag{3c}$$

The term $J(A, B) = A_y B_z - A_z B_y$ is the Jacobian.

The thermal Rossby number $R$, the Ekman number $E$, the ratio of the horizontal to the vertical viscosity $\mu$ and the parameter $\alpha$ are defined as

$$\begin{aligned} & R \equiv gH\Delta_{\mathrm{H}}/\left(\Omega^2 a^2\right);\; E \equiv \nu_{\mathrm{V}}/\left(\Omega H^2\right); \\ & \mu \equiv \left(H^2/a^2\right)\nu_{\mathrm{H}}/\nu_{\mathrm{V}};\; \alpha \equiv H^2/\left(\tau\nu_{\mathrm{V}}\right). \end{aligned} \tag{4}$$

The term $\alpha$ is the ratio of the viscous timescale across the depth of the model atmosphere to the relaxation time $\tau$ toward the radiative–convective equilibrium.

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Published: 8 November 2016


Nonlinear Processes in Geophysics
Open Access
EGU
Discussions

The boundary conditions for the set of Eq. (3) are

$$M_z = \gamma\left(M - c^2\right),\ \psi_{zz} = \gamma\psi_z,$$
$$\psi = \theta_z = 0 \text{ at } z = 0,$$
$$M_z = \psi_{zz} = \psi = \theta_z = 0 \text{ at } z = 1, \quad (5)$$

where $\gamma = \frac{C\,H}{\nu_{\mathrm{V}}}$ is the ratio of the spin-down time due to the drag to the viscous timescale; the bottom drag relaxes the angular momentum $M$ to the local planetary value $\Omega\, a\, c^2$ through a drag coefficient $C$. The model forcing is represented by an equilibrium potential temperature $\theta_{\mathrm{E}}$ defined by

$$\theta_{\mathrm{E}} = \frac{4}{3} - y^2 + \frac{\Delta_{\mathrm{V}}}{\Delta_{\mathrm{H}}}\left(z - \frac{1}{2}\right). \quad (6)$$

A simulation with the forcing (Eq. 6) leads to a stationary situation where the Hadley cell is a point fix for the system of Eq. (3). This simulation will be referred as the control experiment. The parameters used in the simulations described in this paper are shown in Table 1.

In order to change vertical stratification we can define $\theta_{\mathrm{E}}$ as an exponential function of z:

$$\theta_{\mathrm{E}} = \frac{4}{3} - |y|^n + \frac{\Delta_{\mathrm{V}}}{\Delta_{\mathrm{H}}}\left(z^k - \frac{1}{2}\right).$$

This approach was used by Tartaglione (2015), who used constant $k$ values, to investigate the role of vertical stratification on the strength and position of the Hadley circulation. The response of the model atmosphere to vertical stratification will be defined by variations of the exponent $k$ in the forcing equation as a function of time. In general, the vertical stratification can change the intensity of the Hadley cell only when the forcing is concentrated on the equator, whereas it loses importance when the forcing is represented by a weak meridional gradient in the equilibrium temperature (Tartaglione, 2015).

Thus, we will analyze the response of our model to an equilibrium temperature that moves periodically from a stable stratification to a neutral one. In other words, we will have the exponent $k$ of the equation as a function of time and we will assume that $n = 2$. Thus Eq. (6) considered in this work will be:

$$\theta_{\mathrm{E}} = \frac{4}{3} - |y|^n + \frac{\Delta_{\mathrm{V}}}{\Delta_{\mathrm{H}}}\left(z^{1+sin\frac{2\pi t}{t_0}} - \frac{1}{2}\right). \quad (7)$$

When the exponent will be nil the thermal forcing will have an adiabatic stratification with no variation of the potential temperature along the vertical direction. The parameter $t_0$ will have a value smaller or equal to 90 days, and the parameter $t_0$ will define the time in which $\theta_{\mathrm{E}}$ will become adiabatic, forcing the model atmosphere to become neutral. We can think that when $t_0$ is small, the main mechanisms involved in the change of the static stability are associated to short time convective events, whereas large values of $t_0$ can be thought related to meteorological events lasting longer like monsoons. Whatever the process involved in changing the vertical stratification, we are assuming in our model that such a process is periodic.

A cautionary note is necessary. The main role of convection is to bring the atmosphere into a state of neutral vertical stratification, deleting the effects of an unstable layer created by the only-radiative processes. Thus the dry unstable condition

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Published: 8 November 2016


Nonlinear Processes in Geophysics
Discussions
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**Table 1.** Values of the parameters used in this work

| Parameter | Value | Formula |
|---|---|---|
| H | $8 \cdot 10^3\, m$ | |
| $\Delta_\mathrm{H}$ | $1/3$ | |
| $\Delta_\mathrm{V}$ | $1/8$ | |
| C | $5 \cdot 10^{-3}\, m^2\, s^{-1}$ | |
| $\nu_\mathrm{H}$ | $5\, m^2\, s^{-1}$ | |
| $\nu_\mathrm{V}$ | $1.86\, m^2\, s^{-1}$ | |
| $\tau$ | $1.728 \cdot 10^6\, s$ | |
| a | $6.4 \cdot 10^6\, m$ | |
| g | $5 \cdot 10^{-3}\, m\, s^{-2}$ | |
| $\Omega$ | $2\pi/(8.64 \cdot 10^4)\, s^{-1}$ | |
| R | 0.121 | $gH\Delta_H/(\Omega^2 a^2)$ |
| E | $1.07 \cdot 10^{-3}$ | $\nu_\mathrm{V}/(\Omega H^2)$ |
| $\alpha$ | 19.9 | $H/(\tau \nu_\mathrm{V})$ |
| $\Delta$ | $3/8$ | $\Delta_\mathrm{V}/\Delta_\mathrm{H}$ |
| $\gamma$ | 25500 | $CH/\nu_\mathrm{V}$ |
| $\mu$ | $4.2 \cdot 10^{-6}$ | $(H^2/a^2) \cdot \nu_\mathrm{H}/\nu_\mathrm{V}$ |

is almost never met in the real atmosphere as there is always an overturn that leads the atmosphere to be statically stable and the most present instability is the conditional stability due to the presence of water vapor. However, the neutral condition of $\theta_\mathrm{E}$ represents for this model a less stable condition compared with the $\theta_\mathrm{E}$ prescribed in the control simulation (Eq. 6). We parameterize the contribute of the water cycle in the model by means of a time function of $\theta_\mathrm{E}$ in such a way the model atmosphere becomes alternatively more or less stable.

## 3 Results

Using Eq. (7) as forcing, we will have a vertical stratification of the equilibrium temperature that becomes periodically neutral. The values of $t_0$ are set in order to explore the rate of vertical stratification change, and consequently, the effects on the

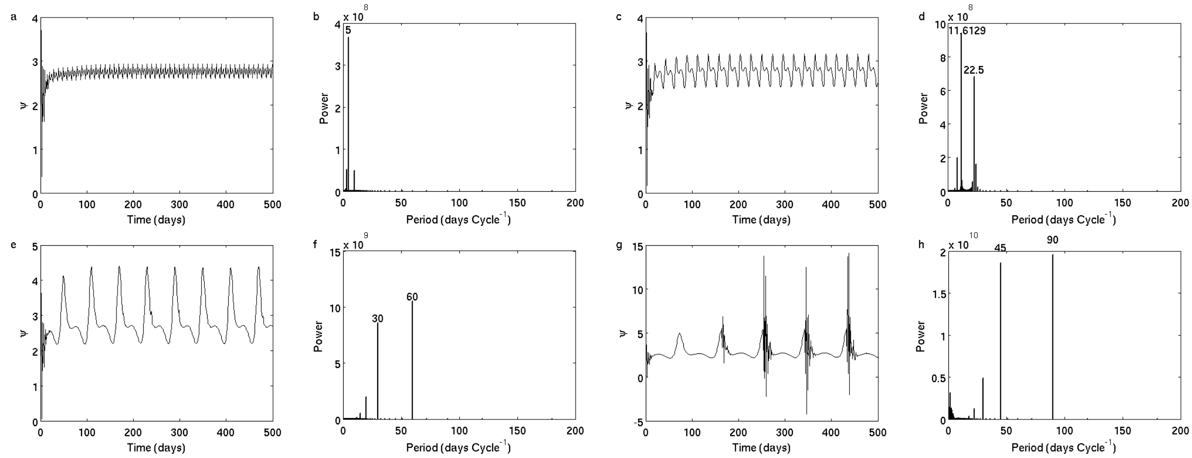


**Figure 1.** Temporal evolution of stream function at 3° North and 2400 m (a,c,e) and its power spectral density (b,d,f) when $t_0 = 10$ (a,b), 23 (c,d) 60 days (e,f) and 90 days(g,h)

static stability. The value of $t_0$ will be set into the range from 10 to 90 days. The aim of these simulations is to relax the atmosphere to neutral stability, but leaving the time average of vertical lapse rate equal to that used in the literature, i.e. equal to one (see Eq. 6).

Let us have a closer look at the details in a specific point of the domain, corresponding to 3° N and 2400 m of altitude. The choice of this specific point is due to the fact that the dynamics of this model is essentially equatorial, that point is in the ascending branch of the cell and its signal in term of spectrum power is higher than that obtained where there is the maximum value of the stream-function, at about 9° N and 1600 m of altitude. When $t_0 = 10$ days, time evolution of the stream-function, at 3° N and 2400 m, and its power spectrum are shown in Figs. 1a,1b . It is easy to see that the evolution resembles a quasi-periodic solution with more peaks in the spectrum; however, the dominant frequency is that corresponding to the period of 5 days, i.e. half of the forcing period. As we will see, as the time period increases, the second component related to the time forcing becomes more important. Figures 1c and 1d show the temporal evolution of the stream function at a point in the model domain and the spectral density, when $t_0 = 23$ days. When the forcing period is a prime number, the quasi-periodicity is more marked, with more peaks at high frequencies close one another. When $t_0 = 30$ days the solution is still quasi-periodic with a main period of 30 days, although there is a signal even at 15 days (not shown). This behavior is the same at least when $t_0 = 60$ days (Figs. 1e,1f). The peak in the spectrum corresponding to a period of 60 days is higher than that corresponds to the half forcing period. Time evolution of the solution, when $t_0$ = 90 days, is shown in Figs. 1h,1g. Let us have a closer look at the details of the time evolution of the stream function when $t_0$ = 90 days. The motion is chaotic only when the time is close to multiples of $t_0$ = 90 days, whereas it returns to the steady solution when the stratification $\theta_E$ is far away from the condition of neutral stability. Thus, the slow variation of time, because of high value of $t_0$, triggers a fast response in the model that allows instabilities to grow faster than those obtained by using small values of $t_0$. Therefore, we can observe that there is an unusual

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Published: 8 November 2016

Nonlinear Processes in Geophysics
Discussions

sort of slow-fast dynamic. This phenomenon can be view as a sort of intermittency with the slow component which modulates the fast process.

Consequently, if the adiabatic forcing is reached with a relatively fast change of the stratification, solutions follow the adiabatic forcing increasing the circulation strength(Fig. 2a) and this leads to strong subtropical jet streams and stronger easterly winds in the equatorial region (Fig. 2b). As in Fang and Tung (1999) who found a stronger circulation when replaced a fixed sun (equinoctial Hadley cell) with a moving sun, here it is a time-varying stratification stability of $\theta_0$ that causes a stronger circulation with respect to a fixed lapse rate.

The quasi-periodicity with two dominant cycles in the model response appears to be interesting in light of quasi-periodic oscillations observed in the tropical atmosphere. Madden and Julian (1971,1972) were the first to show the existence of an oscillation in pressure and winds with a predominant peak in the spectrum with a period of 40-50 days. They also showed that the amplitude of this peak in the tropical station and was weaker in the sub-tropical stations. Yasunari (1979) showed by means of spectral analysis that cloudiness fluctuations have two dominant periodicities: one of about 15 days and another of 40 days. Other studies documented 15 day oscillations within the tropical regions related to monsoons (e.g. Krishnamurti and Bhalme, 1976; Krishnamurti and Ardanuy, 1980; Krishnamurti and Subrahmanyam, 1982). Yasunari (1981) showed that even the 40 day oscillation has some relation to the Asian summer monsoon. Anderson and Rosen (1983) found similar results by using the zonally averaged zonal winds. Thus the features of these oscillations suggest that it may be possible to understand them with a zonally averaged model. Goswami and Shukla (1984) used a symmetric model with hydrology to study the Hadley circulation and found that it has well-defined strong and weak episodes. These oscillations of the Hadley circulation occurred in their model in two dominant ranges of periodicities: one with a period of 10 and 15 days and another with a period between 20 and 40 days. Since our model does not include hydrology, this double period has to be related to the internal dynamics of the system. In fact, if periodicity was expected, with a time period equal to the forcing one, the quasi–periodic behavior has to come from the interaction between the changing static stability and the internal dynamics of the model. As the changing stratification stability implies a way to simulate the moist convection, our result seems to be in agreement with the findings of Goswami and Shukla (1984). They found that quasi-periodic oscillations of the Hadley circulation were seen only when the moist convective heating is active changing dynamics of their model.

The time series of the vertically averaged stream function shows that the quasi-periodic response involves mainly low-level processes. Figure 3a shows the time series of the stream function averaged over the lower 3200 m, while in Fig. 3b it is averaged over all the domain height. In both cases strong and weak patterns in the stream-function are present, the strong episodes of the stream function at lower levels do not dump suddenly, but last for a while. However, when averaged over the entire domain time series shows strengthening and weakening of the stream function that seem to be periodic. This periodic behavior tends to become intermittent when increasing $t_0$. Higher values of $t_0$ means that the model atmosphere takes more time to become neutral, but at the same time it remains close to the neutral stability more time inducing a perturbation that produces dramatic increasing in the stream-function values.

The interaction of slow parameter variation with the fast rate of motions in the phase space is the cause of phenomena known as "dynamic bifurcations" (Guckenheimer and Holmes, 2002). Figure 4 shows a one-dimensional bifurcation diagram,

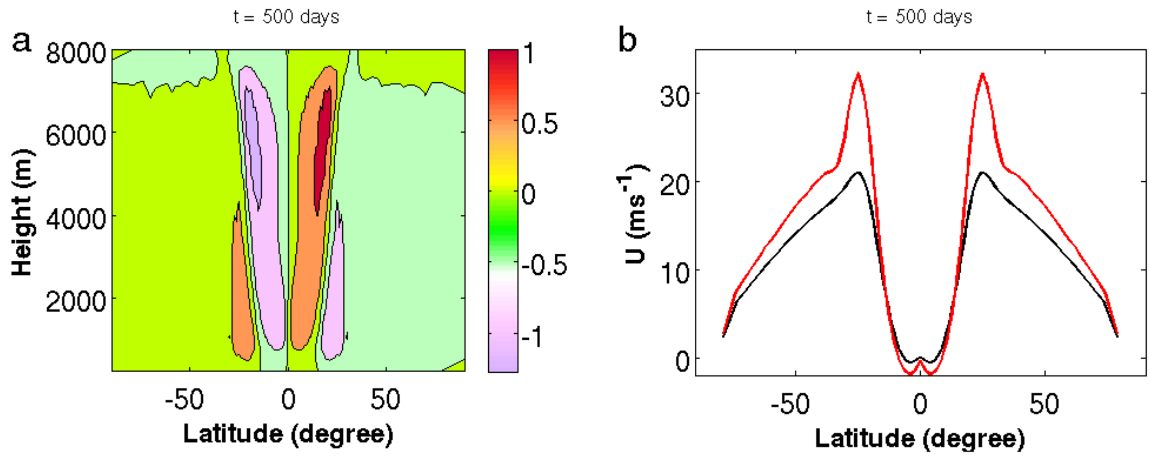


**Figure 2.** Difference of stream-function (in dimensionless unit) of the experiment with $t_0 = 50$ days and the control experiment (a), the zonal wind U ( in $ms^{-1}$) averaged on the height as a function of latitude y (b), for the experiment $t_0 = 50$ days (in red) and the control experiment (in black) after 500 simulation days.

i.e. the differences between two stream functions at the same point of the domain (3° N and 2400 m of altitude) having a time lag of $t_0$ days plotted for each value of $t_0$. The outliers, indicating a chaotic behavior, start to appear at $t_0 = 64$ days indicating the presence of spikes in the stream function values associated with a chaotic solution. Thus, the periodic behavior starts losing force by allowing the chaos to emerge when the $z$ exponent approaches zero. Periodicity is still present, in the sense that chaotic behavior of the model appears periodically. This occurs, for instance, for $t_0 = 90$ days (Figs. 1g,1h). In some respects we can say that the low variability is still governed by quasi-periodic oscillation, with the presence of chaotic behavior associated with the neutral static stability of the forcing. If results found in this work are really linked to Madden-Julian oscillation, since the transition occurs at $t_0 = 64$ days, which is very close to 50-60 days, the question arises whether the Madden-Julian oscillation also has chaotic components. Unfortunately, with real data this is not easy to determine as high frequency components are usually removed in order to study the Madden-Julian oscillations and because of chaotic features due to atmosphere non-linearity.

It seems that a bifurcation delay might be active when $t_0$ is longer than 63 days. The system fails to notice the onset of instability. This is quite evident in Fig. 1h,1g. We can observe that most of time the low frequency (with period of 90 days) component is the dominating one. Its modulating action is recognizable only when it triggers the chaotic behavior, a result obtained even with simpler models (e.g. Zaks et al., 2002). Having reached a chaotic state the system wanders along this state for a time, until it finds itself with the exponent of $z$ in Eq. 7, which defines the slow component of the system, having a value "far enough" from zero the system goes back to steady solution. At this time self-modulation is temporarily switched off until the cycle is repeated.

We can say that identification of dynamic bifurcations caused by slow variation can, in general, be a problematic task for the dynamic bifurcation because of transitions. During these transitions two situations could occur: bifurcations with abrupt

Nonlin. Processes Geophys. Discuss., doi:10.5194/npg-2016-59, 2016
Manuscript under review for journal Nonlin. Processes Geophys.
Published: 8 November 2016

Nonlinear Processes in Geophysics
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Discussions

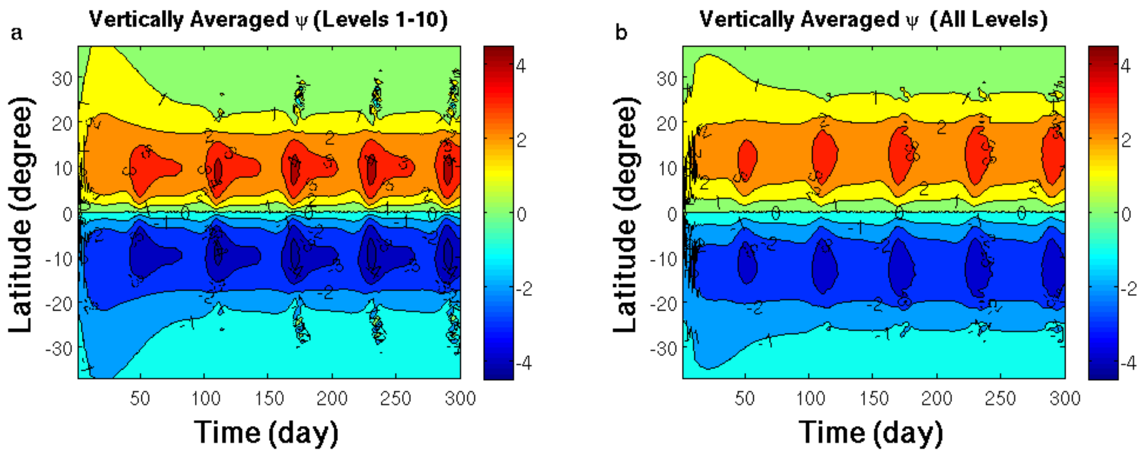


**Figure 3.** Time evolution of the vertically averaged stream function (non-dimensional unit) over lower levels, up to 3200 m (a) and over all the entire height of the domain (b), for the experiment with $t_0 = 60$ days.

change of the attractor size (and in such a case the dynamic bifurcation could be visible), or transitions occurring with changes of the chaotic attractor geometry. In the latter case it might be difficult to observe these variations in the short time in which the system is in a chaotic state.

Does history affect the internal state of our model? In this respect we performed a simulation changing the value of $t_0$ during the run from 10 to 80 days, doubling it after 400 days and returning back at 10 days (Fig. 5). The model adjusted its response, which is the same of that obtained starting from an initial condition at rest, almost immediately even though the change of $t_0$ was sudden. This shows that the model response does not depend on history and the initial conditions. Moreover, the quick adjustment of the model suggests that the fluctuations observed in the simulation are an intrinsic characteristic of this model and not an artifact, which is valid with the vertical viscosity set to 5 $m^2s^{-1}$.

The periodic chaotic behavior is related to vertical viscosity. It appears even for time periods less than 63 days when the vertical viscosity is close to zero. When the vertical viscosity is 0.5 $m^2s^{-1}$, i.e. one tenth of that used previously, the appearance of the chaotic behavior is immediate for small values of $t_0$ , as we can see when $t_0 = 10$ days (Fig. 6a). When $t_0 = 30$ days, the solution has a chaotic solution for almost each of the 30 days (Fig. 6b), and the same occurs when $t_0 = 60$ days (Fig. 6c). In such a case the evolution of the stream function will have a steady solution with intermittent chaotic behavior when the static stability gets close to the neutral condition, especially for higher $t_0$. Figure 6d illustrates the situation for $t_0 = 90$ days. This representation is only to show the effect of vertical viscosity that has to be taken into account when we consider the development and evolution of the stream function. High values of the non-dimensional stream–function appear to be unrealistic, $\nu_V$, although we cannot exclude that the transport of momentum and heat from the equator could occur by means of bursts. If we want to estimate at which level of $\nu_V$ for a specific value of $t_0$ we can observe Fig. 7 give us an insight of what happens for different values of vertical viscosity, for example when $t_0 = 30$ days. For less than 2 $m^2s^{-1}$ the systems start to showing some spikes in the solutions associated with a chaotic behavior of the system. For higher values of $t_0$ the amplification of the model

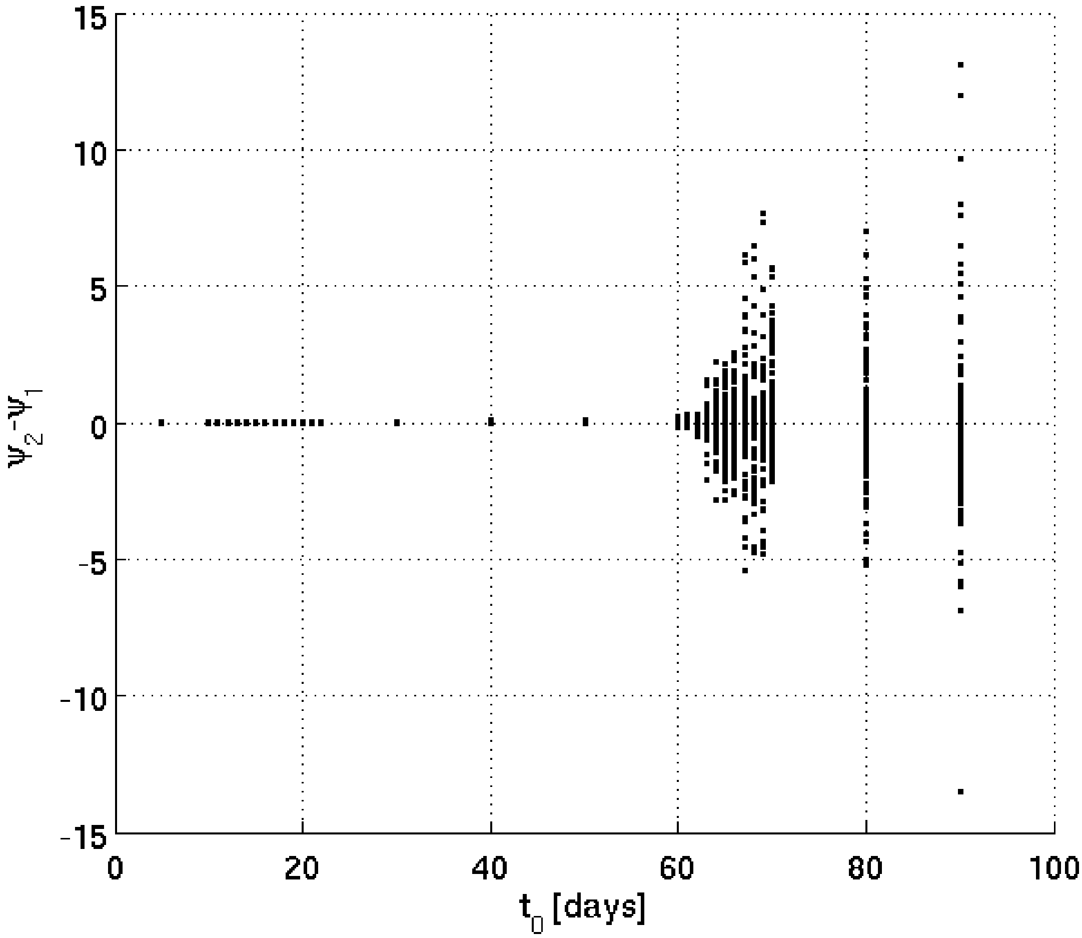


**Figure 4.** The 1-D bifurcation diagram as a function of $t_0$. With a period of 63 days, deviations from periodic behavior start to appear. The plot shows the differences of two stream functions (in dimensionless unit) at the same point of the domain, but with a time lag of $t_0$ days. The value of the vertical viscosity is set to 5 $m^2 s^{-1.}$

response occurs for higher values of $\nu_V$. On the contrary, high values of vertical viscosity kill the chaotic behavior, making the evolution quasi-periodic with the trajectories of the stream function still lying on a torus (not shown).

## 4 Conclusions

We have used an dry axisymmetric model to simulate the Hadley circulation and to investigate the role of a changing stratification of the thermal forcing, as if there were moist convection that alters the static stability of the model atmosphere. The bi-dimensionality of the model prevents the generation of any eastward traveling wave. Hence in our discussion the influence

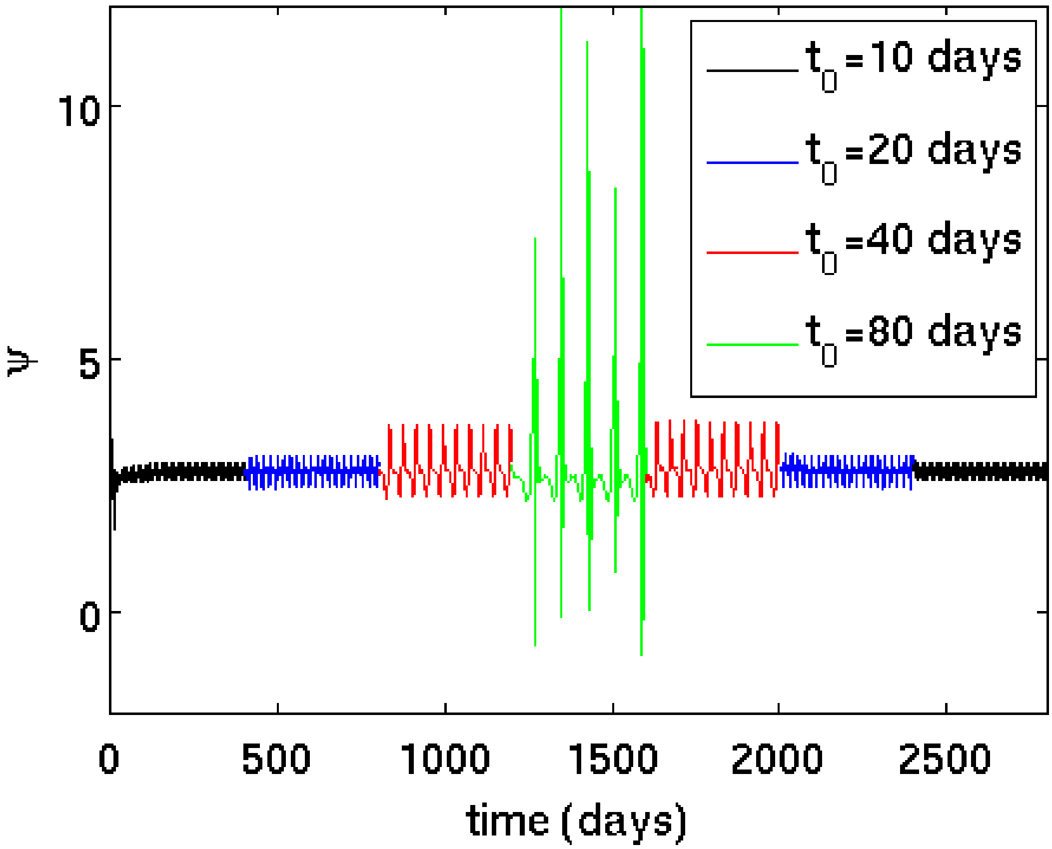


**Figure 5.** Temporal evolution of stream function (in dimensionless unit) at 3° North and 2400 m for a simulation where t0 changes with time.

of eddies momentum fluxes is not taken into account. We have shown that the stream-function representing the Hadley circulation in an axisymmetric model can exhibit a quasi-periodic behavior when the vertical stratification of the thermal forcing is periodically forced to become neutral. The question that arises is whether this oscillation can be linked with the observed atmospheric fluctuations within the tropical region. Although Madden-Julian oscillation is a three dimensional phenomenon with the development of a Kelvin-Rossby wave (Gill, 1980), it is certainly associated with the evolution of convective anomalies (Hendon and Salby, 1994). It has already suggested that the quasi-periodic oscillation seems to be an intrinsic characteristic of the tropical atmosphere in according with the results of Goswami and Shukla (1984).

If the forcing period is up to 63 days the stream function evolution shows a classic quasi-periodic behavior. For period forcing longer than 63 days, the slow frequency associated with the forcing period modulate a fast response in the system, generating a chaotic motion that lasts for a while before going back to a steady solution when the chaotic phase finishes. It is not clear whether these high frequency characteristics are actually present in the meridional circulation of our planet.

Nonlin. Processes Geophys. Discuss., doi:10.5194/npg-2016-59, 2016
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Nonlinear Processes in Geophysics
Discussions
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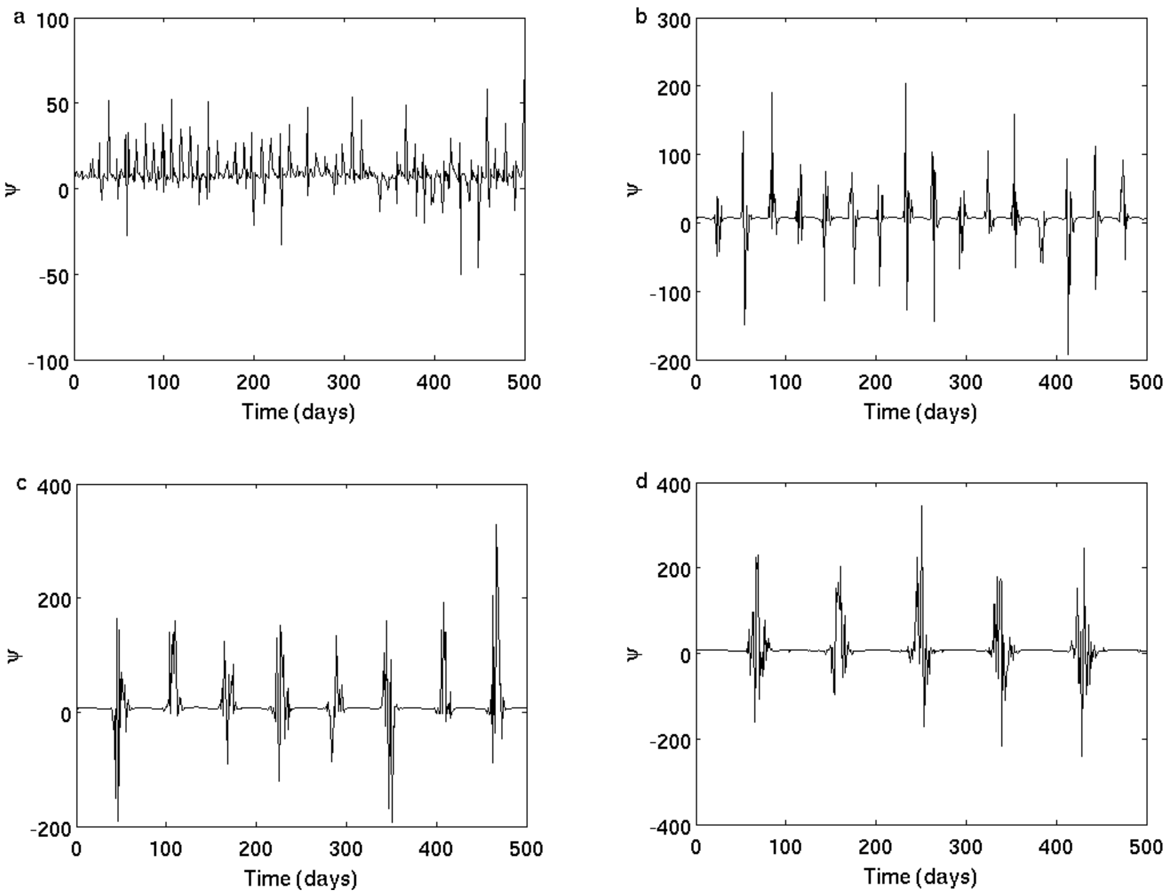


**Figure 6.** The time evolution of stream function (in dimensionless unit) at 3° North and 2400 m for t0 = 10 (a), 30 (b), 60 (c) and 90 (d) when $\nu_V = 0.5$

In fact, when we look for periodicities of the order of tens days into observations, the higher frequency signals are usually removed. Moreover, even though we can detect such signals, it is not easy to associate them with the oscillations caused by change of static stability rather than other processes. The chaotic dynamics observed in the model could exist in other planets where the vertical stratification takes longer to become neutral. The change of vertical stability that in our model simulates the cycle of large-scale convection might be equivalent to recharge and discharge of moisture that supports the Madden-Julian oscillation (Zhu and Hendon, 2015). This can be thought to control the aggregation process of convection, which allows a bistable equilibrium between moist or dry situations (Raymond and Zeng, 2000, Zhang et al., 2003, Arnold and Randall, 2015). An important aspect is the rate at which this occurs. As we have shown, when this rate in the considered model is over 63 days we can have in the short term a chaotic impact on the Hadley circulation strength. The role of friction in the symmetric circulation, driven by a meridional thermal gradient of a fast rotating planet like the Earth is contradictory. On one hand it is an essential ingredient to allow a meridional overturn, instead of a strong zonal wind in cyclostrophic balance only,

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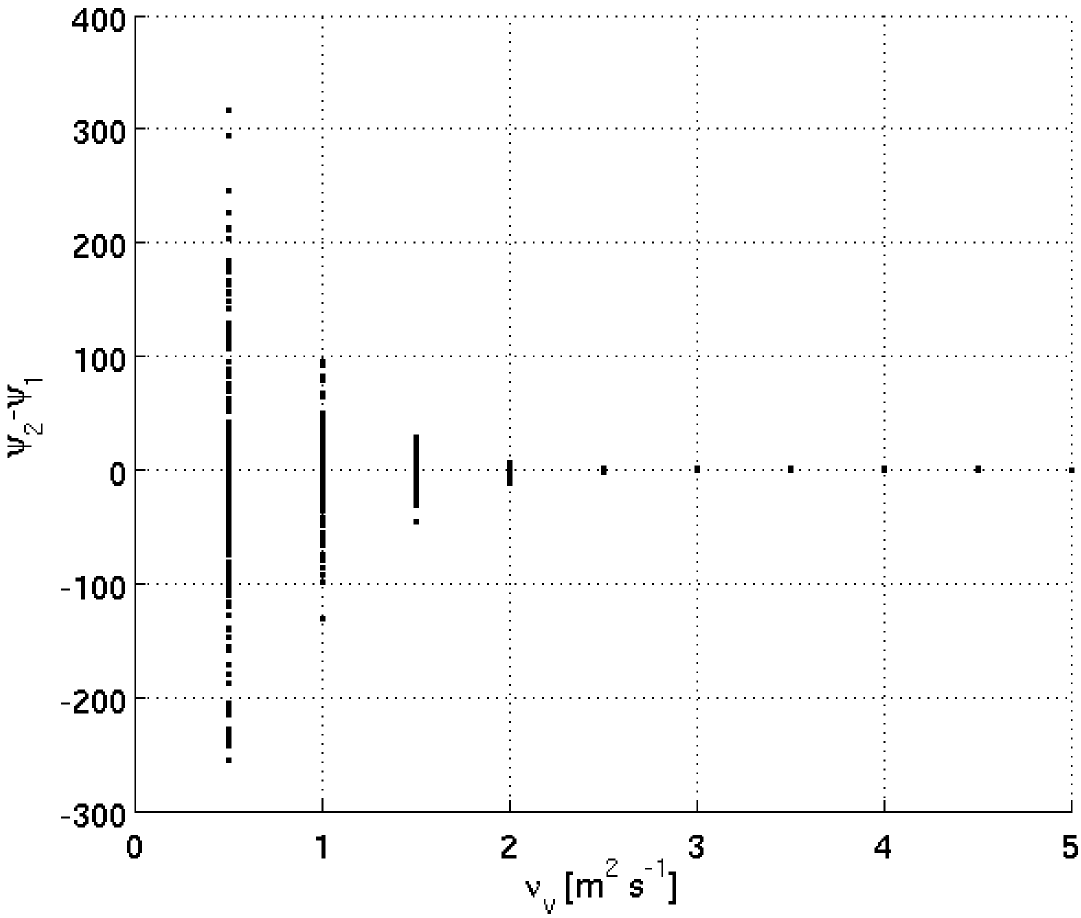


**Figure 7.** The 1-D bifurcation diagram as a function of $\nu_V$ for simulations with for $t_0 = 30$ days.

on the other hand the viscosity value must be close enough to zero in order to allow the angular momentum conservation. Although these conditions are met in the model we have considered, the presence of a time-varying stratification alters the classic view of a stably stratified vertical temperature gradient. Other than the meridional thermal gradient, this imposed time-varying stratification represents another nonlinear forcing, which amplifies the model response when the vertical viscosity is small, which itself representing a source of amplification of the model response in the inviscid case.

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