Dynamics of the Hadley circulation in an axisymmetric model undergoing stratification periodic forcing

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Reply to reviewers

I wish to thank both reviewers for their time and observations they did. My answers are in *italics.*

Reply to reviewer 1

The forcing from MJO is not stationary because MJO is an eastward moving ’pulse’ of cloud and rainfall near the equator that typically lasts for a time period of from 30 to 60 days. I am not convinced that t the forcing from MJO can be represented by the axisymmetric model in this paper (pag 10 line 4)

The reviewer is right and I wrote a similar sentence in the first version (page 10, line 4)

“We have used an dry axisymmetric model to simulate the Hadley circulation and to investigate the role of a changing stratification of the thermal forcing, as if there were moist convection that alters the static stability of the model atmosphere. The bi-dimensionality of the model prevents the generation of any eastward traveling wave. “

Although observations suggest that intra-annual oscillations are better described when it being driven by large-scale disturbance, there are many studies developed conceptual model by considering coupled oscillations in a single column suggesting the oscillation can be self-contained in a single region. Adopting simple model allow to understand some aspects that may be particularly relevant to understanding the role of certain features. Further explanations are given in the answer to the reviewer 2. I hope that they clarify the my position..

Reply to reviewer 2

I wish to thank the reviewer for time and attention he put to read through my paper. He caught the philosophy behind the paper.

1) Perhaps a little pedantic, but I would disagree with the classification of the response at moderate forcing periods as “quasi-periodic”. The latter term is normally understood to represent a system exhibiting 2 or more incommensurate frequencies - otherwise the attractor is a “simply periodic” limit cycle (or perhaps it really is a torus?). In the cases shown in Fig. 1, the responses show spectral peaks predominantly at the forcing period and integer fractions thereof. These additional peaks in the spectrum look very much as though they are simple harmonics of the forcing frequency omega - i.e. at 2*omega, 3*omega etc. Such components are not incommensurate with the forcing and are therefore not independent. This can be easily tested by constructing phase portraits e.g. using standard delay embedding of the time series shown in Fig. 1 (i.e. plot (t) vs (t+q)), where q is a time interval around 1/4 of the forcing period). If the additional frequencies are indeed harmonics, the phase portrait will resemble a simple closed
loop, whereas for a genuinely quasi-periodic evolution a topologically more complex object (such as a torus) will result. Examples could perhaps be presented to the readerin an additional figure?

The reviewer is not pedantic, the reviewer is right, the forcing is not truly quasi-periodic and I was undecided until the end on which term to use. I noticed that when the forcing period is a prime number the response might be classified ‘formally’ as quasi-periodic, in fact I added the periodogram for the case t0=23 days, how it may be seen the dominant frequencies are 22.5 and 11.625, although close to 23 and 11.5 they are not commensurate. At this point even this case could be classified periodic. I added an additional figure (new Fig. 2) as suggested by the reviewer. Description of new Fig. 2 is on page 7 line 18 of the marked-up manuscript. I was very careful to classify any motion as quasi-periodic.

2.) For periods longer than 60 days, an apparent dynamic bifurcation to an intermittently chaotic behavior appears. The chaotic bursting behavior seems to appear around the phase in the forcing where static stability is becoming very weak? Are you sure the chaotic behavior that is seen is due to a physical instability and not a numerical artifact?

I would have like to have seen some kind of evaluation of this (e.g. by varying the spatial resolution and/or an exploration of what kind of motion is occurring when the chaotic bursts appear). Is this due to some kind of symmetric baroclinic instability or some other process? This should be explored more carefully to make sure we are not simply seeing the result of a defect in the model numerics.

As the reviewer can see the formulation of the forcing lead a weaker static stability periodically for any period considered, however, a weaker static stability alone is not the only condition to trigger a burst in the energy transfer to higher latitudes, the other condition is that has to persist for a while.

In order to satisfy the reviewer request, a sensitivity study was performed. Four experiments were designed and run:

1) Halving time step but with unchanged horizontal and vertical resolution.

2) Halving the horizontal resolution

3) Halving the vertical resolution

4) Halving the horizontal and vertical resolution
See new Fig. 6

I included a description of what happens in my opinion, even in the light of Cessi (1998) results. (see new Fig. 7)
See pages 11, 12 and 13 of the marked-up manuscript for the discussion about these results.

3) On a related point, the model configuration chosen looks to have a number of singular symmetries, both geometrical (symmetric about the equator) and dynamical (static stability being forced precisely towards neutrality). Is the response and bifurcation sequence dependent on satisfying these symmetries or is the observed behavior generic? This would be important to check, since non-generic responses are unlikely to be observed in a real atmosphere.

I am not sure what the reviewer means. What I can say is that even when the model shows a periodic oscillation, this is only in the strength of the stream-function but the pattern remains almost the same. I have studied this model extensively and I can say that it almost impossible that its output diverges from the classical Hadley cell configuration. Hence, the model maintains the symmetry and the features we see are independent of specific details of the model and the forcing.
However, the atmosphere, even though restricted to the tropical circulation is more complex than the model adopted here and consequently we cannot transfer all the results to the real atmosphere tout court.

4) On a similar theme, are these bifurcations to intermittent chaos likely to be observed in a fully 3D atmosphere? This ought at least to be considered and the means to test this discussed in the closing sections of the paper. Are there plans to extend the study to a fully 3D model with similar forcing? This would be the logical next step, but I would be frankly somewhat skeptical that some of these phenomena are robust to interactions with non-axisymmetric flow components.

I followed the suggestion of the reviewer, adding a paragraph in the conclusion. See page 16 of the marked-up manuscript.

Last but not least a professional editor corrected the paper.

Dynamics of the Hadley circulation in an axisymmetric model undergoing stratification periodic forcing

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Abstract. The time-dependent response of the Hadley circulation to a periodic forcing is explored ~~by using~~ via a simplified non-linear axisymmetric model. Thermal forcing towards a given equilibrium potential temperature drives the model atmosphere. The vertical stratification of this temperature is forced to become periodically neutral with a period t_0. ~~Some simulations were performed with different~~ Simulations performed with values of t_0 ~~,~~ ranging from 10 to 90 days ~~, they exhibit a stronger circulation when comparing with~~ exhibit stronger circulation compared to the results of a constant thermal forcing experiment. As the period increases, a transition ~~takes place~~ occurs first from a stationary regime, obtained when forcing is constant, to a periodic (and possibly quasi-periodic) regime, and then to an intermittent regime, albeit one with a strong periodic component. The stream-function response to periodic forcing is ~~a quasi-periodic~~ generally a periodic oscillation, with two main frequencies dominating, one with a period equal or close to ~~that of forcing~~ the forcing period and another with a period that is half of the forcing period. The former is dominant for values of t_0 larger than 30 days, whereas the latter is prevalent for t_0 smaller than 30 days. The ~~quasi-periodic~~ periodic oscillations obtained in this model ~~might~~ may be associated with the ~~quasi-periodic~~ periodic oscillations observed in the tropical regions. In this case the periodic charge and discharge of moisture in the tropical atmosphere, with consequent change of stratification, may be linked to those oscillations. In the model, ~~with forcing periods~~ at forcing periods of over 63 days the response of the stream-function periodically enters ~~in a sort of intermittent regime, with chaos appearing with high frequency oscillations, which~~ into a quasi-intermittent regime, exhibiting high frequency chaotic oscillations that are modulated by the slow timescale of forcing. The vertical viscosity plays a role in determining ~~even~~ the evolution of the Hadley circulation under the conditions established by the forcing.

1 Introduction

The Hadley cell is a specific and observable feature of tropical circulation (Dima and Wallace, 2003).The main driving process ~~undergoing~~ of this feature can be seen as a combination of low-latitude convective processes and interactions of tropical circulation with ~~higher latitude~~ higher-latitude eddies (Charney, 1969). Although tropical circulation follows a seasonal cycle due to the asymmetry of solar forcing during a solar year (Fang and Tung, 1999), it is also subjected to ~~alternated intra–seasonal~~ alternating intra-seasonal strong and weak periods (Goswami and Shukla, 1984). The cycles modulate ~~the precipitation activities~~ precipitation activity in the equatorial regions.

It is well known that the rainfall distribution over Indian tropical regions varies considerably from day to day with an ~~alternate~~ alternating behavior associated with a few periodic or quasi-periodic cycles. Cycles of 3–7, 10–20 and 30–60 days are present in the atmospheric dynamics of tropics and subtropics. The 3–7 ~~days~~ day cycle is related to synoptic-scale convective systems generated over the Bay of Bengal, while the others have been ~~related~~ associated with the cycles of monsoon rainfall over the Indian region ~~how~~ as discussed by Kripalani et al. (2003) who showed a prevalence of the ~~cycle~~ 10–20 ~~days~~ day cycle during a normal monsoon year and ~~one of~~ of the 30–60 ~~days~~ day cycle during a drought year~~and the dominance~~.

The latter cycle is known as the ~~Madden–Julian~~ Madden-Julian oscillation (Madden and Julian, 1972). Observational analyses and modeling studies ~~have~~ revealed that there are dominant periods in the tropics and subtropics. ~~Tropical~~ Since the publication of that paper, tropical oscillations have been extensively studied ~~in following years~~ with observations (Yasunari, 1979, 1980, 1981; Sikka and Gadgil, 1990, Yoneyama et al., 2013) as well as by means of models (Goswami and Shukla, 1984, Zhu and Hendon, 2015, Wang et al., 2016). The Madden-Julian oscillation has a significant impact on the Indian (Murakami 1976; Yasunari 1979) and Australian monsoons (Hendon and Liebmann 1990). Kessler and McPhaden (1995) for instance suggested that it could play an important role in the onset and development of El Niño events. The relationship between this oscillation and tropical cyclogenesis has also been ~~linked~~ posited in some works (Maloney and Hartmann, 2000; Mo 2000; Maloney and Hartmann, 2001). Moreover, He et al. (2011) showed that Hadley circulation variability is closely related to Madden-Julian oscillation convection. Goswami and Shukla (1984) suggested that this oscillation in the Hadley circulation is due to the interaction between the internal dynamics of tropical circulation with moist convection; in fact, with constant latent heat included in their model, the quasi-periodic oscillation vanished. This ~~lead~~ led them to consider that latent heat released during the moist processes can play ~~an~~ a fundamental role in the dynamics of this cycle.

~~Knowledge of tropical~~ Tropical atmosphere dynamics can be ~~obtained by using~~ explored via complex general circulation models as well as rather simpler models like the axisymmetric ones. Axisymmetric models focus on the main processes occurring in the tropospheric region, capturing the central process of the Hadley circulation. In such ~~a kind of~~ models, for example, eddies are not allowed, and all the processes involved in the higher latitudes are not considered. ~~An~~ In this paper, an axisymmetric model (Cessi, 1988) is used ~~in this paper~~ to perform an analysis of Hadley cell behavior when periodic forcing is applied. The model spin up takes less than 100 days. When the imposed forcing is not variable, the developing circulation results in a steady state with the Hadley cell representing a fixed point solution within the phase space. In this kind of configuration, other atmospheric processes, acting on longer or shorter spatial and temporal scales, ~~such as~~ like those mentioned earlier, are essentially excluded.

The Hadley circulation is the meridional overturn that develops in response to a temperature that is in radiative-convective equilibrium, and so it is worth analyzing how the model atmosphere behaves when the vertical stratification of equilibrium temperature becomes neutral. Although the equilibrium temperature towards which the model atmosphere ~~is brought~~ evolves is essentially stable, ~~we can think that~~ a less stable temperature stratification ~~occurs, for~~ may occur, example, due to tropospheric heating caused by condensation of water vapor, ~~and that the atmosphere is not always stable~~ resulting in a less stable atmosphere. However, it is reasonable to ~~think that the system is not always forced to become less stable, but~~ assume that this condition occurs only periodically.

In Section 2 we ~~will~~ describe the model we used and how we set up the stratification by modulating the vertical distribution of the temperature ~~in order~~ so as to periodically reach neutral static stability. ~~When~~ In Section 3, we describe how modulation of the vertical stratification ~~is modulated~~ by a periodic function ~~we shall see, in Section 3, that this have~~ has an impact on the time evolution ~~with a quasi-periodic response , with~~ of the stream-function, which exhibits a periodic response of two main frequencies, ~~of the stream function~~ suggesting that a ~~quasi-periodic~~ periodic response is intrinsic to this model when stratification changes periodically. ~~The~~ In this section, we also investigate the role of the vertical viscosity~~will be investigated too~~. In Section 4 we ~~will draw the~~ present our conclusions.

2 Model equation and ~~forcing~~ applied forcing

~~The model used in this paper is~~ This paper employs the full model described in Cessi (1998), who studied its analytic and numerical solutions ~~deriving the expansion into power series~~ via a power series expansion in the Rossby number ~~R~~ R of the prognostic variables, the zonal momentum M, the potential temperature θ, and the ~~stream function~~ stream-function ψ. The model follows the same hypothesis of the classical axisymmetric models such as those used by Held and Hou (1980) ~~or by~~ and Fang and Tung (1999).

The horizontal coordinate is defined as $y = a\,\sin\,\phi$,from which we have

$$c(y) = \cos\phi = \sqrt{(1 - y^2/a^2)}, \tag{1}$$

where a is the radius of a planet having a rotation rate Ω, with an atmosphere height H. The model is similar to the Held and Hou model~~(HH80)~~, but it prescribes a horizontal and vertical viscosity, respectively equal to ν_{H} ν_{V}. The prognostic variables are the angular momentum $M = \Omega\,a\,c^2 + u\,c$, where u represents the zonal velocity; the zonal vorticity ψ_{zz} ~~with the meridional stream function~~ where the meridional stream-function ψ is defined by

$$\begin{aligned}\partial_y\psi &\equiv w,\\ \partial_z\psi &\equiv -cv,\end{aligned} \tag{2}$$

and the potential temperature θ that is forced towards a radiative–convective equilibrium temperature θ_{E}.

Starting from the dimensional equations of the angular momentum, zonal vorticity and potential temperature, we ~~will~~ obtain a set of dimensionless equations. The new equations are non-dimensionalized using a scaling that follows Schneider and Lindzen (1977), but the zonal velocity u is scaled with $\Omega\,a$. A detailed description can be found in Cessi (1998).

The non-dimensional model equations are

$$M_t = \frac{1}{R}\left\{M_{zz} + \mu\left[c^4\left(c^{-2}M\right)_y\right]_y\right\} - J(\psi, M), \tag{3a}$$

$$\psi_{zzt} = \frac{1}{(R^2E^2)} y c^{-2}\left(M^2\right)_z - \frac{1}{c^{-2}} J\left(\psi, c^{-2}\psi_{zz}\right) + \frac{1}{(RE^2c^{-2})}\theta_y + \frac{1}{(Rc^{-2})}\left[c^{-2}\psi_{zzzz} + \mu\psi_{zzyy}\right], \tag{3b}$$

$$\theta_t = \frac{1}{R}\left\{\theta_{zz} + \mu\left[c^2\theta_y\right]_y + \alpha\left[\theta_{\mathrm{E}}(y,z) - \theta\right]\right\} - J(\psi, \theta). \tag{3c}$$

~~The~~ where the term $J(A, B) = A_y B_z - A_z B_y$ is the Jacobian.

The thermal Rossby number R, the Ekman number E, the ratio of the horizontal to the vertical viscosity μ and the parameter α are defined as

$$R \equiv gH\Delta_{\mathrm{H}}/\left(\Omega^2 a^2\right);\ E \equiv \nu_{\mathrm{V}}/\left(\Omega H^2\right);$$
$$\mu \equiv \left(H^2/a^2\right)\nu_{\mathrm{H}}/\nu_{\mathrm{V}};\ \alpha \equiv H^2/(\tau\nu_{\mathrm{V}}). \tag{4}$$

The term α is the ratio of the viscous timescale across the depth of the model atmosphere to the relaxation time τ toward ~~the radiative–convective~~ radiative-convective equilibrium.

The boundary conditions for the set of Eq. (3) are

$$M_z = \gamma\left(M - c^2\right),\ \psi_{zz} = \gamma\psi_z,$$
$$\psi = \theta_z = 0 \text{ at } z = 0,$$
$$M_z = \psi_{zz} = \psi = \theta_z = 0 \text{ at } z = 1, \tag{5}$$

where $\gamma = \frac{CH}{\nu_{\mathrm{V}}}$ is the ratio of the spin-down time due to ~~the~~ drag to the viscous timescale; the bottom drag relaxes the angular momentum M to the local planetary value $\Omega a c^2$ through a drag coefficient C. The model forcing is represented by an equilibrium potential temperature θ_{E} defined by

$$\theta_{\mathrm{E}} = \frac{4}{3} - y^2 + \frac{\Delta_{\mathrm{V}}}{\Delta_{\mathrm{H}}}\left(z - \frac{1}{2}\right). \tag{6}$$

A simulation with the forcing ~~(~~given by Eq. 6 ~~)~~ leads to a stationary situation where the Hadley cell is a ~~point fix~~ fixed point for the system of Eq. (3). ~~This simulation will be referred~~ We refer to this simulation as the control experiment. The parameters used in the simulations ~~described~~ we describe in this paper are shown in Table 1.

In order to change vertical stratification we can define ~~$\theta_textrmE$~~ θ_{E} as an exponential function of z:

$$\theta_{\mathrm{E}} = \frac{4}{3} - |y|^n + \frac{\Delta_{\mathrm{V}}}{\Delta_{\mathrm{H}}}\left(z^k - \frac{1}{2}\right).$$

This approach was used by Tartaglione (2015), who used constant ~~k~~ k values, to investigate the role of vertical stratification on the strength and position of the Hadley circulation. The response of the model atmosphere to vertical stratification will be

Table 1. Values of the parameters used in this work

Parameter	Value	Formula
H	$8 \cdot 10^3\, m$	
Δ_{H}	$1/3$	
Δ_{V}	$1/8$	
C	$5 \cdot 10^{-3}\, m^2\, s^{-1}$	
ν_{H}	$5\, m^2\, s^{-1}$	
ν_{V}	$1.86\, m^2\, s^{-1}$	
τ	$1.728 \cdot 10^6\, s$	
a	$6.4 \cdot 10^6\, m$	
g	$5 \cdot 10^{-3}\, m\, s^{-2}$	
Ω	$2\pi/(8.64 \cdot 10^4)\, s^{-1}$	
R	0.121	$gH\Delta_H/(\Omega^2 a^2)$
E	$1.07 \cdot 10^{-3}$	$\nu_{\mathrm{V}}/(\Omega H^2)$
α	19.9	$H/(\tau \nu_{\mathrm{V}})$
Δ	$3/8$	$\Delta_{\mathrm{V}}/\Delta_{\mathrm{H}}$
γ	25500	CH/ν_{V}
μ	$4.2 \cdot 10^{-6}$	$(H^2/a^2) \cdot \nu_{\mathrm{H}}/\nu_{\mathrm{V}}$

defined by variations of the exponent ~~k~~ k in the forcing equation as a function of time. In general, the vertical stratification can change the intensity of the Hadley cell only when the forcing is concentrated on the equator, whereas it loses importance when the forcing is represented by a weak meridional gradient in the equilibrium temperature (Tartaglione, 2015).

Thus, we ~~will~~ analyze the response of our model to an equilibrium temperature that moves periodically from a stable stratification to a neutral one~~. In other words, we will have the exponent k of the equation~~ ; i.e. we allow exponent k of Eq. (6) as a function of time and we ~~will~~ assume that $n = 2$. Thus Eq. (6) ~~considered in this work will be~~takes the form:

$$\theta_{\mathrm{E}} = \frac{4}{3} - |y|^n + \frac{\Delta_{\mathrm{V}}}{\Delta_{\mathrm{H}}} \left(z^{1+sin\,\frac{2\pi t}{t_0}} - \frac{1}{2} \right). \tag{7}$$

When the exponent ~~will be nil~~ is zero the thermal forcing ~~will have~~ has an adiabatic stratification with no variation of the potential temperature along the vertical direction. The parameter t_0 ~~will have a value smaller~~ takes a value less than or equal

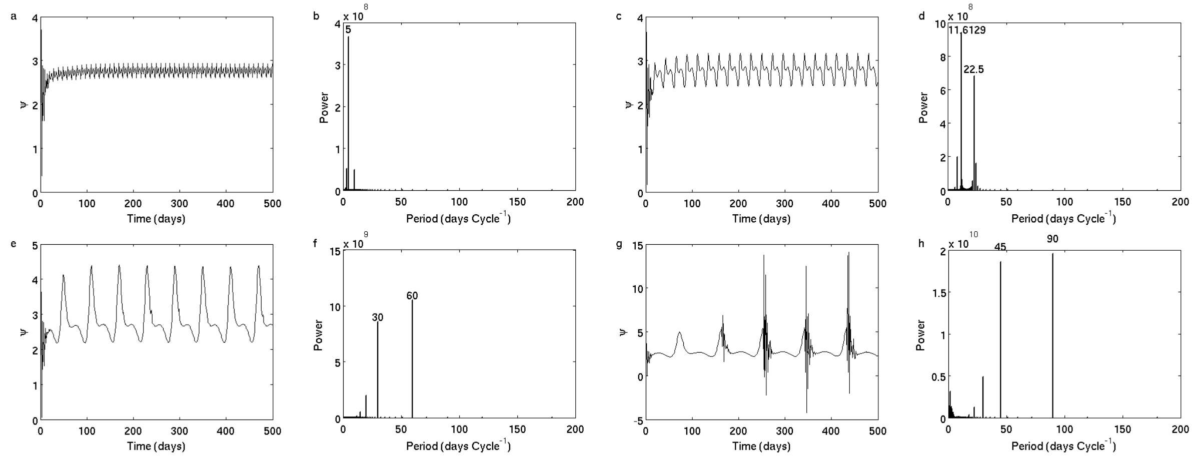

Figure 1. Temporal evolution of ~~stream function~~ the stream-function at 3° North and 2400 m (a,c,e) and its power spectral density (b,d,f) when $t_0 = 10$ (a,b), 23 (c,d) 60 days (e,f) and 90 days(g,h). The figures on the bars are the cycle periods in days.

to 90 days, and ~~the parameter t_0 will define~~ defines the time in which the forcing ~~will become~~ becomes adiabatic, forcing the model atmosphere to become neutral. ~~We can think that when~~ When t_0 is small, the main mechanisms involved in the change of the static stability are associated ~~to~~ with short time convective events, whereas large values of t_0 can be ~~thought related to meteorological events lasting longer~~ related to longer-lasting meteorological events like monsoons. Whatever ~~the process~~ process is involved in changing the vertical stratification, we ~~are assuming~~ assume in our model that ~~such a process~~ it is periodic.

A cautionary note is necessary. The main role of convection is to bring the atmosphere into a state of neutral vertical stratification, deleting the effects of the unstable layer created by ~~the only-radiative~~ radiative only processes. Thus the dry unstable condition is almost never met in the real atmosphere as there is always an overturn that leads the atmosphere to be statically stable and the most ~~present~~ prevalent instability is the conditional stability due to the presence of water vapor. However, the neutral condition of θ_0 represents for this model a less stable condition compared with the θ_0 prescribed in the control simulation (Eq. 6). We parameterize the ~~contribute~~ contribution of the water cycle in the model by means of a time function of θ_0 in such a way that the model atmosphere becomes ~~alternatively~~ alternately more or less stable.

3 Results

3.1 Model response to periodic forcing

Using Eq. (7) as forcing, we ~~will have~~ obtain a vertical stratification of the equilibrium temperature that becomes periodically neutral. The values of t_0 are ~~set in order~~ chosen so as to explore the rate of vertical stratification change, and consequently, the effects on the static stability. The value of t_0 ~~will be set into the range from~~chose lies in the range of 10 to 90 days. The aim of these simulations is to relax the atmosphere to neutral stability, but ~~leaving~~ leave the time average of vertical lapse rate equal to that used in the literature, i.e. equal to one (see Eq. 6).

~~Let us have~~ We now take a closer look at the details in a specific point of the domain, that corresponding to 3° N and 2400 m of altitude. The choice of this specific point is ~~due to~~ based on the fact that the dynamics of this model ~~is~~ are essentially equatorial, and that point is in the ascending branch of the cell and its signal in term of spectrum power is higher than that obtained where ~~there is the maximum~~ the value of the stream-function is a maximum, at about 9° N and 1600 m of altitude. ~~When $t_0 = 10$ days,~~

The time evolution of the stream-function, at 3° N and 2400 m, and its power spectrum for $t_0 = 10$ days are shown in Figs. 1a,1b. It is easy to see that the evolution resembles a ~~quasi-periodic~~ periodic solution with more peaks in the spectrum; however, the dominant frequency is ~~that~~ one corresponding to the period of 5 days, i.e., half of the forcing period. As we will see, as the time period increases, the second component related to the time forcing becomes more important. Figures 1c and 1d show the temporal evolution of the ~~stream function~~ stream-function at a point in the model domain, and the spectral density, ~~when~~ for $t_0 = 23$ days. When the forcing period is a prime number, the quasi-periodicity is ~~more marked, with more peaks at high frequencies close one another~~formally present, as for this value of t_0 the two main frequencies are incommensurate. When $t_0 = 30$ days the solution is still ~~quasi-periodic~~ periodic with a main period of 30 days, although there is a signal even at 15 days~~(not shown)~~. This behavior is the same at least when $t_0 = 60$ days (Figs. 1e,1f). The peak in the spectrum ~~of~~ corresponding to a period of 60 days is higher than that corresponding to the half forcing period.

If we look at the phase space evolution and the plot of couples(ψ(t);ψ(t+t_0)), we can see that for both $t_0 = 23$ days and $t_0 = 60$ days (Fig. 2), the solution does not lie on the same curve, but rather assumes a behavior that appears quasi-periodic. As the solution is essentially numeric, this behavior could be related to rounding errors rather than a real quasi-periodic solution.

Time evolution of the solution, ~~when~~ for t_0 = 90 days, is shown in Figs. 1h ~~,~~and 1g. ~~Let us have a closer look at the details of the~~ Looking at the time evolution of the ~~stream function when t_0 = 90 days. The~~ stream-function, the motion is chaotic only when the time is close to multiples of t_0 = 90 days, whereas it returns to the steady solution when the stratification θ_E is far away from the condition of neutral stability. Thus, the slow variation ~~of~~ with time, because of the high value of t_0, triggers a fast response in the model that allows instabilities to grow faster than those obtained by using small values of t_0. Therefore, we ~~can~~ observe that there is an unusual sort of slow-fast dynamic. This phenomenon can be ~~view~~ viewed as a sort of intermittency with the slow component, which modulates the fast process.

Thus, if the adiabatic forcing is reached with a relatively fast change of the stratification, the ~~solutions follow the adiabatic forcing~~solution follows adiabatic forcing, increasing the strength of the circulation (Fig. ~~2a)and this~~ 3a), which leads to strong subtropical jet streams and stronger easterly winds in the equatorial region (Fig. ~~2b~~3b). As in Fang and Tung (1999)~~who found a~~, who found stronger circulation when they replaced a fixed sun (equinoctial Hadley cell) with a moving sun, here it is ~~a~~ the time-varying stratification stability of θ_0 that causes a stronger circulation with respect to a fixed lapse rate.

~~The quasi-periodicity~~ Periodicity with two dominant cycles in the model response ~~appears to be~~ is interesting in light of ~~quasi-periodic~~ the oscillations observed in the tropical atmosphere. Madden and Julian (1971,1972) were the first to show the existence of an oscillation in pressure and winds with a predominant peak in the spectrum ~~with~~ at a period of 40-50 days. They also showed that the amplitude of this peak was greater in the tropical station and was weaker in the sub-tropical stations. Yasunari (1979) ~~showed~~ demonstrated by means of spectral analysis that cloudiness fluctuations have two dominant periodicities: one of about 15 days and another of 40 days. Other studies documented ~~15 day~~ 15–day oscillations within the tropical regions related to monsoons (e.g. Krishnamurti and Bhalme, 1976; Krishnamurti and Ardanuy, 1980; Krishnamurti and Subrahmanyam, 1982). Yasunari (1981) showed that even the 40 day oscillation has some relation to the Asian summer monsoon. Anderson and Rosen (1983) found similar results by using ~~the~~ zonally averaged zonal winds. Thus, the features of these oscillations suggest that it may be possible to understand them with a zonally averaged model. Goswami and Shukla (1984) used a symmetric model with hydrology to study the Hadley circulation and found that it has well-defined strong and weak episodes. These oscillations of the Hadley circulation occurred in their model in two dominant ranges of periodicities: one with a period of between 10 and 15 days and another with a period of between 20 and 40 days. Since our model does not include hydrology, this double period has to be related to the internal dynamics of the system. In fact, if periodicity ~~was expected ,~~ is expected with a time period equal to the forcing ~~one, the quasi–periodic behavior~~ time period t_0, the response with cycles with more frequencies has to come from the interaction between ~~the~~ changing static stability and the internal dynamics of the model. As the changing stratification stability implies a way to simulate the moist convection, our result seems to be in agreement with the findings of Goswami and Shukla (1984). They found that quasi-periodic oscillations of the Hadley circulation were seen only when the moist convective heating ~~is active changing dynamics of~~ was turned on in their model.

The time series of the ~~vertically averaged stream function~~ vertically-averaged stream-function shows that the ~~quasi-periodic~~ observed periodic response involves mainly low-level processes. Figure ~~3a~~ 4a shows the time series of the ~~stream function~~ stream-function averaged over the lower 3200 m, while ~~in Fig. 3b it is~~ Fig. 4b shows it averaged over all the domain height. In both cases strong and weak patterns in the stream-function are present, and the strong episodes of the ~~stream function~~ stream-function at lower levels do not dump suddenly, but ~~last~~ persist for a while. However, when averaged over the entire domain time series shows strengthening and weakening of the ~~stream function~~ stream-function that seem to be periodic. This periodic behavior tends to become intermittent ~~when~~ whit increasing t_0. Higher values of t_0 ~~means~~ mean that the model atmosphere takes more time to become neutral, ~~but~~ while at the same time ~~it remains~~ remaining close to the neutral stability~~more time inducing~~ , which induces a perturbation that produces ~~dramatic increasing~~ a dramatic increase in the stream-function values.

The interaction of ~~the~~ slow parameter variation with the fast rate of motions in the phase space ~~is the cause of phenomena~~ causes a phenomenon known as "dynamic ~~bifurcations~~bifurcation" (Guckenheimer and Holmes, 2002). Figure ~~4~~ 5 shows a one-dimensional bifurcation diagram, i.e., the differences between two ~~stream functions~~ stream-functions at the same point of the domain (3° N and 2400 m of altitude) having a time lag of t_0 days, plotted for each value of t_0. The outliers, indicating ~~a~~ chaotic behavior, start to appear at $t_0 = 64$ days, indicating the presence of spikes in the ~~stream function~~ stream-function values associated with a chaotic solution. Thus, the periodic behavior starts losing force by allowing ~~the~~ chaos to emerge when the z exponent approaches zero.

Periodicity is still present, in the sense that chaotic behavior of the model appears periodically. This occurs, for instance, for $t_0 = 90$ days (Figs. 1g,1h). In some respects we can say that the low variability is still governed by ~~quasi-periodic~~ periodic

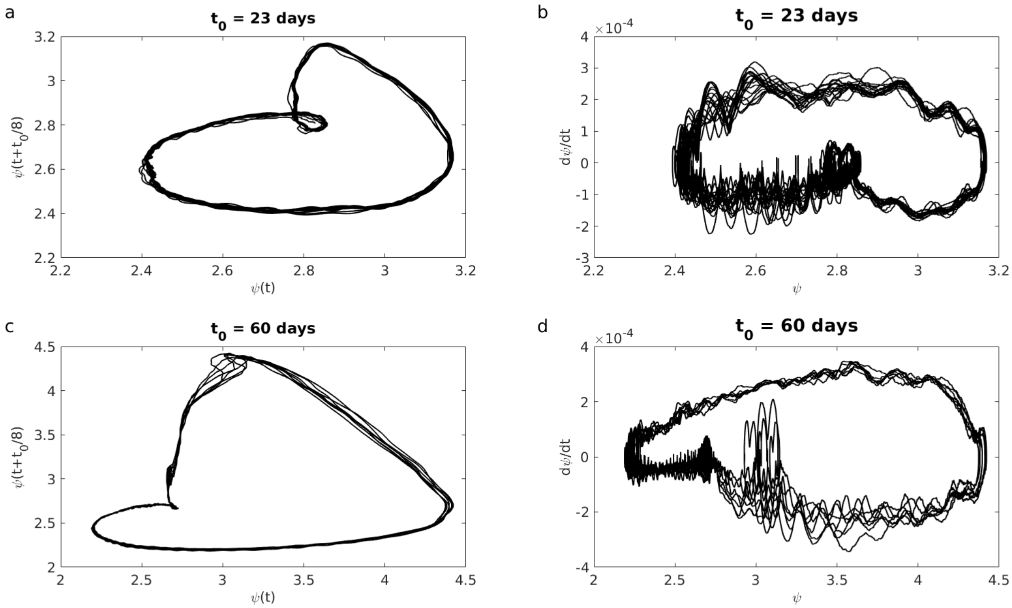

Figure 2. Difference Representation of stream-function (in dimensionless unit) the evolution of the experiment with $t_0 = 50$ days stream-function at 3° North and the control experiment 2400 m ψ(at), the zonal wind U -ψ(in ms^{-1}t+t_0/8) averaged on the height as a function of latitude y (bleft panels) , for the experiment $t_0 = 50$ 23 days (in redupper panels) and the control experiment $t_0 = 60$ days experiments (in blacklower panels)after 500 simulation days.

oscillation, with the presence of chaotic behavior associated with the neutral static stability of the forcing. If the results found in this work are really linked to Madden-Julian oscillation, since Since the transition occurs at $t_0 = 64$ days, which is very close to in the 50-60 daysday range of the Madden and Julian oscillation, the question arises whether results of this work imply that the Madden-Julian oscillation also has chaotic components. Unfortunately, with real data this is not easy to determine with real data as high frequency components are usually removed in order to study the Madden-Julian oscillations, and because of high non-linearity of the atmosphere where chaotic features are normalin such a system.

It seems that a bifurcation delay might be active when t_0 is longer than 63 days. The system fails to notice the onset of instability. This is quite evident in FigFigs. 1h ,and 1g. We can observe that, where it is clear that, most of the time, the low frequency (with period of 90 days) component is the dominating one. Its modulating action is recognizable only when it triggers ; however, a fast component is observed when the chaotic behavior is triggered, a result obtained even with simpler models (e.g. Zaks et al., 2002). Having reached a chaotic state, the system wanders along this state for a time, until it finds itself with the exponent of zk, which defines the slow component of the system, having a value "far enough" from zero that the system goes back to steady solution. At this time, self-modulation is temporarily switched off until the cycle is repeated.

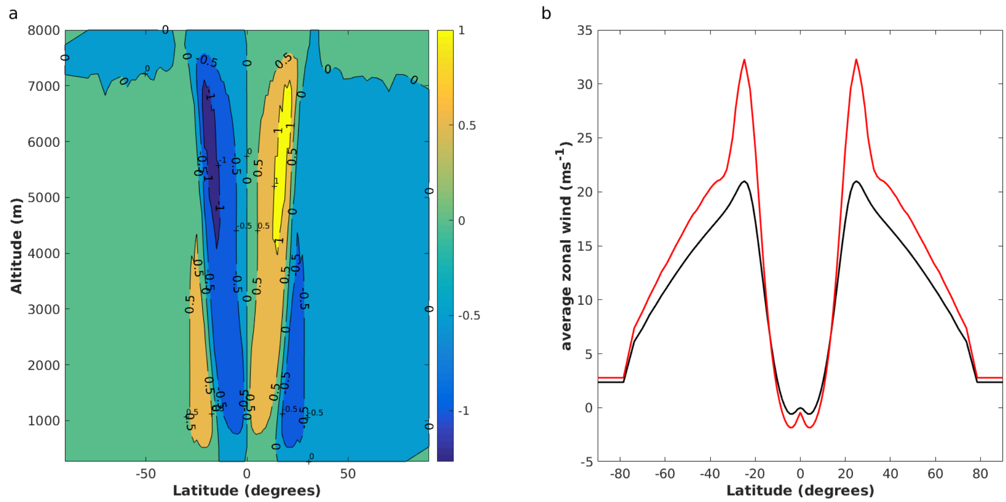

Figure 3. Difference between the stream-function (in dimensionless units) of the experiment with $t_0 = 50$ days, and the control experiment (a), the zonal wind U (in ms^{-1}) averaged on the height as a function of latitude y (b), for the experiment $t_0 = 50$ days (in red) and the control experiment (in black) after 500 simulation days.

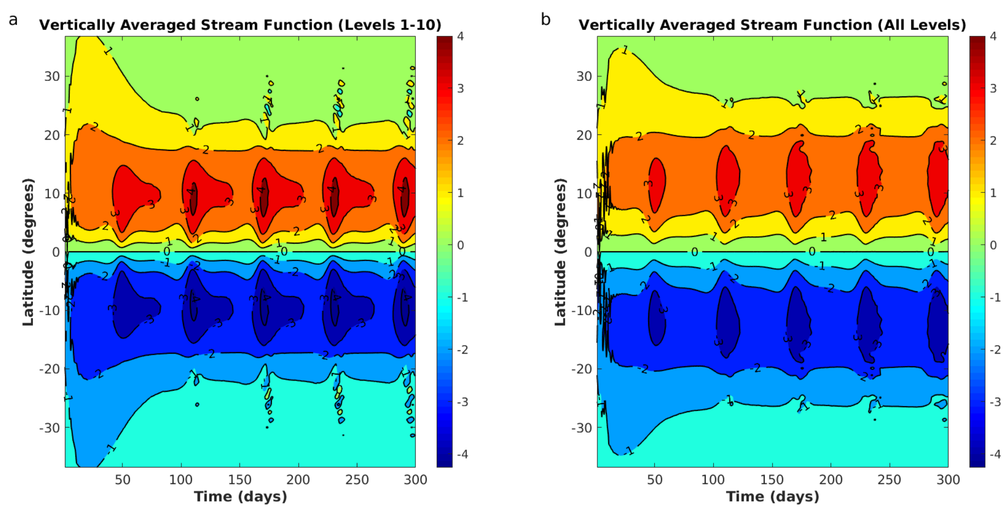

Figure 4. Time evolution of the vertically averaged ~~stream function~~ stream-function (non-dimensional unit) over lower levels, up to 3200 m (a) and over ~~all~~ the entire height of the domain (b), for the experiment with $t_0 = 60$ days.

We can say that identification of dynamic bifurcations caused by slow variation can, in general, be a problematic task~~for the dynamic bifurcation~~, because of transitions. During these transitions two situations could occur: bifurcations with abrupt change of the attractor size (and in such a case the dynamic bifurcation could be visible), or transitions occurring with changes

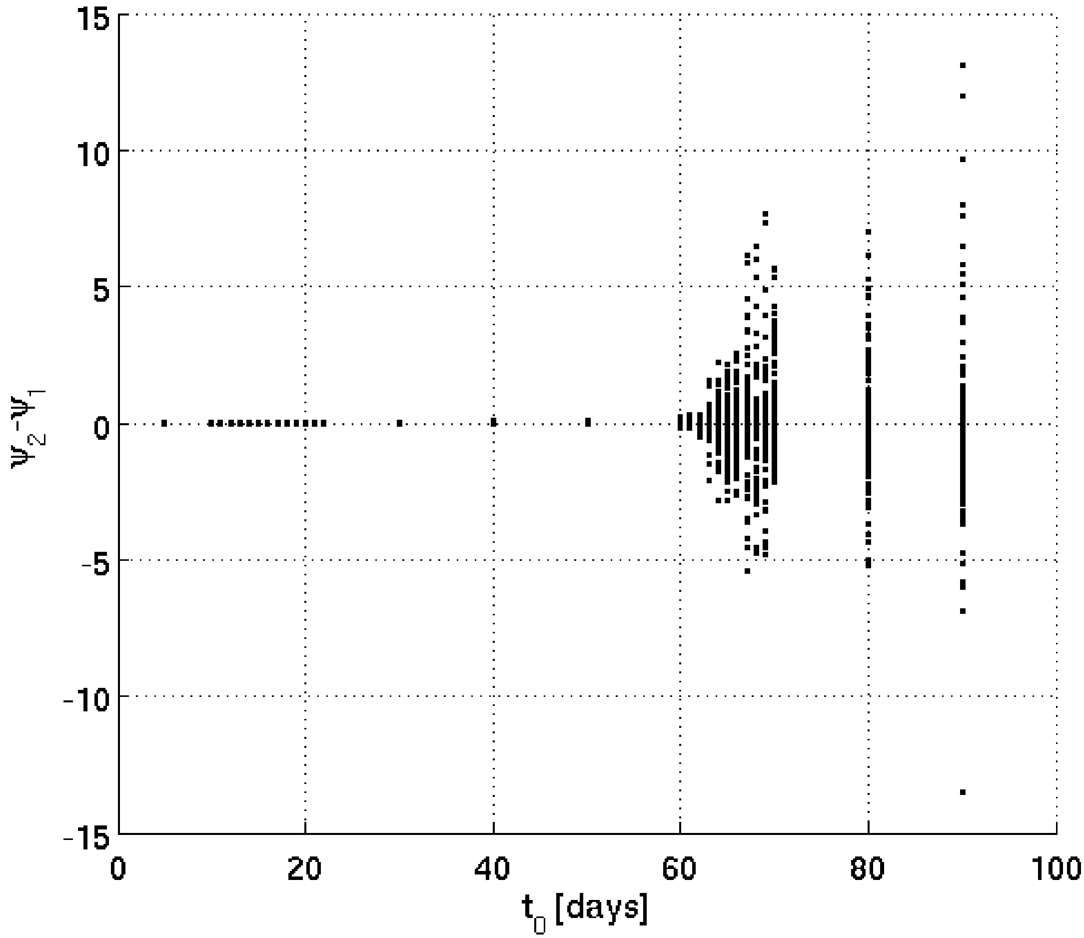

Figure 5. The 1-D bifurcation diagram as a function of t_0. ~~With~~ At a period of 63 days, deviations from periodic behavior start to appear. The plot shows the differences of two ~~stream functions~~ stream-functions (in dimensionless ~~unit~~units) at the same point of the domain, but with a time lag of t_0 days. The value of the vertical viscosity is set to 5 m^2s^{-1}.

~~of~~ in the geometry of the chaotic attractor. In the latter case it ~~might~~ may be difficult to observe these variations in the short time that the system is in the chaotic state.

~~Does history affect~~

3.2 Model Sensitivity to time and grid steps

One can argue that the amplification seen in the model solution when t_0 equals 90 days may be an artifact of the model. In order to eliminate this possibility, some simulations were performed in order to assess the model sensitivity to time and grid

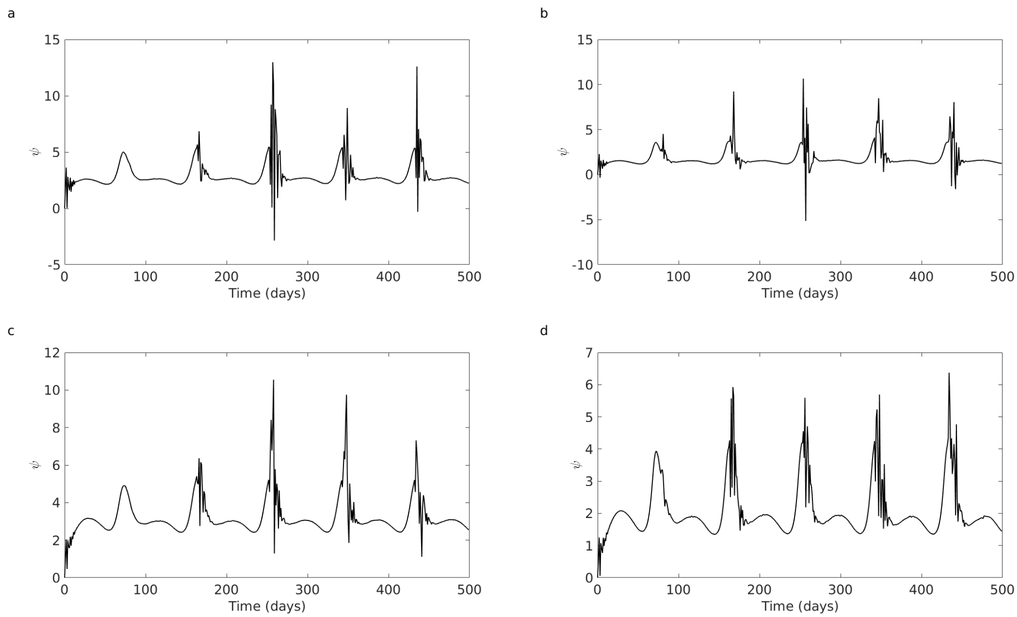

Figure 6. As in Fig. 1g, but halving the time step, and increasing the spatial resolution, i.e. halving the horizontal (b), vertical (c) and both (d) grid steps..

steps. Figure 6 shows the temporal evolution of the stream-function at 3° North and 2400 m (as in Fig. 1), obtained by halving the time step (Fig. 6a), doubling the horizontal (Fig. 6b) and the vertical (Fig. 6c) resolutions and halving all the spatial grid steps (Fig. 6d). In all these cases, albeit with some slight differences, there is an explosive increase of the stream-function value at some point, with the most remarkable difference seen when the vertical resolution is increased. There is still periodic strengthening of the Hadley circulation, but its variability is essentially reduced with the negative spikes vanishing. Although not shown here, these results hold when the time step and the spatial grids are even further reduced.

The behavior of the numerical solution for large values of t_0 has some similarities with the results found by Cessi (1998), who discussed the analytical solution in the case of no stratification. The surface angular momentum departs from the planetary value and is homogenized in the tropical region. The vertically averaged potential temperature is homogenized as well. This homogenization leads to easterly surface winds that are a maximum at the equatorial region, which is dominated by a barotropic core of easterlies that extends upward (see Fig. 7). Westerly winds have higher speed than those obtained with constant stratification. This is already visible in Fig. 2b, where t_0 is 50 days. Although there are similarities between the analytic solution found by Cessi (1998) and our numerical solution, they depart from one other, in the stream-function especially, when $t_0 = 90$ days, and even for the case with no stratification. The stream-function behavior starts to appear chaotic, and is actually chaotic when no stratification is imposed (not shown). In fact, the intense chaotic solution with $t_0 = 90$ and no stratification does not blow up, and circulation strength remains limited. With no imposed stratification due to radiative equilibrium, on the

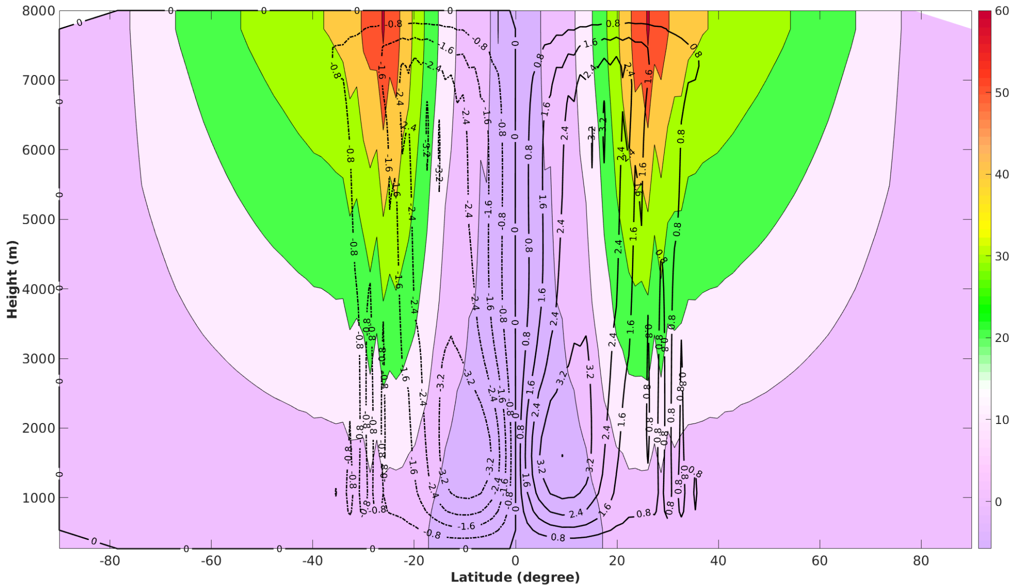

Figure 7. Mean stream-function in dimensionless units (contour) and zonal wind in $m^2 s^{-1}$ (color) for the case t_0 = 90 days.

one hand the circulation follows potential instability, which tends to lead the isotherms to a vertical position, while on the other hand isotherms are tilted horizontally by the meridional velocity producing a dynamically driven stratification.

Moreover, a second circulation is produced at the poleward edge of the Hadley circulation in the case where t_0 equals 90 days (Fig. 7). While this second circulation is unrealistic (we are dealing with a simplified model of tropical atmosphere), it is interesting from a mathematical point of view, as it shows how a numerical solution can differ from the analytic solution, though it was already clear in Cessi (1998) that in absence of stratification second order equations showed a singularity.

To investigate whether history affects the value of the internal state of our model ~~? In this respect~~ we performed a simulation ~~changing~~ where we changed the value of t_0 during the run from 10 ~~days~~ to 80 days~~and returning back at~~, and back again to 10 days (Fig. ~~5~~8). The model adjusted its response, which is the same of that obtained starting from an initial condition at rest, almost immediately, even though the change of t_0 was sudden. This result shows that the model response does not depend on history and on the initial conditions. Moreover the quick adjustment of the model suggests that the fluctuations observed in the simulation are an intrinsic characteristic of ~~this model~~ it and not an artifact~~, which is valid with the vertical viscosity set to 5 $m^2 s^{-1}$~~.

3.3 Role of the vertical viscosity

The periodic chaotic behavior is related to vertical viscosity. It appears even for time periods less than 63 days, when the vertical viscosity is close to zero. When the vertical viscosity is 0.5 $m^2 s^{-1}$, i.e. one tenth of that used previously, the appearance of

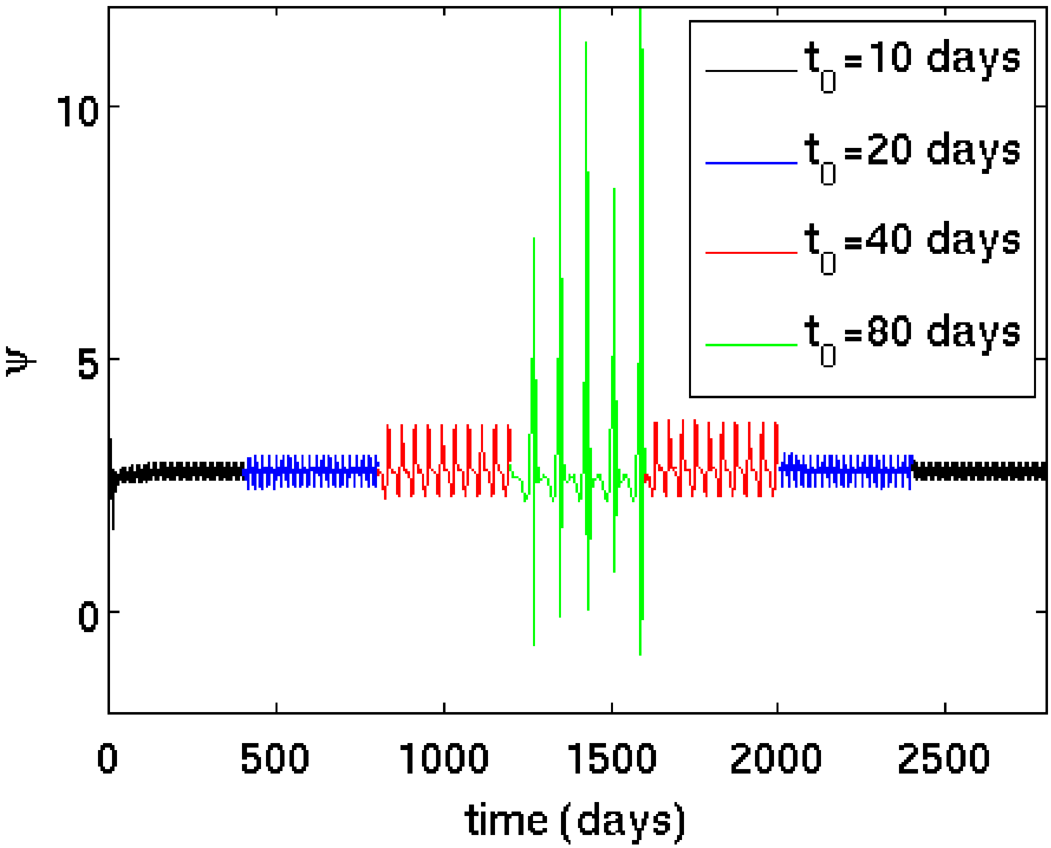

Figure 8. Temporal evolution of ~~stream function~~ the stream-function (in dimensionless ~~unit~~units) at 3° North and 2400 m of altitude for a simulation where ~~t0~~ t_0 changes with time.

the chaotic behavior is immediate for small values of t_0 , as we can see when $t_0 = 10$ days (Fig. ~~6a~~9a). When $t_0 = 30$ days, the solution ~~has a chaotic solution~~ is chaotic for almost each of the 30 days (Fig. ~~6b~~9b), and the same occurs when $t_0 = 60$ days (Fig. ~~6c~~9c). In such a case, the evolution of the ~~stream function~~ stream-function will have a steady solution with intermittent chaotic behavior when the static stability gets close to the neutral condition, especially for higher t_0. Figure ~~7d~~ 9d illustrates the situation for $t_0 = 90$ days. This representation is only intended to show the effect of the vertical viscosity ~~that has to~~ which must be taken into account when we consider the development and evolution of the ~~stream function~~stream-function. The high values of the non-dimensional ~~stream function~~ stream-function appear to be unrealistic, ~~ν_V~~ but we cannot exclude ~~that the~~ the possibility transport of momentum and heat from the equator could occur by means of bursts (Majda and Stechmann, 2008). If we ~~want to estimate at which level of ν_V~~ wish to estimate for a specific value of t_0 ~~we can observe Fig. 7 give us an insight of~~ at which level of ν_V this occurs Fig. 10 gives us insight as to what happens for different values of vertical viscosity, for example when $t_0 = 30$ days. For values less than 2 $m^2 s^{-1}$ the systems ~~start to showing~~ begin to exhibit some spikes in the solutions

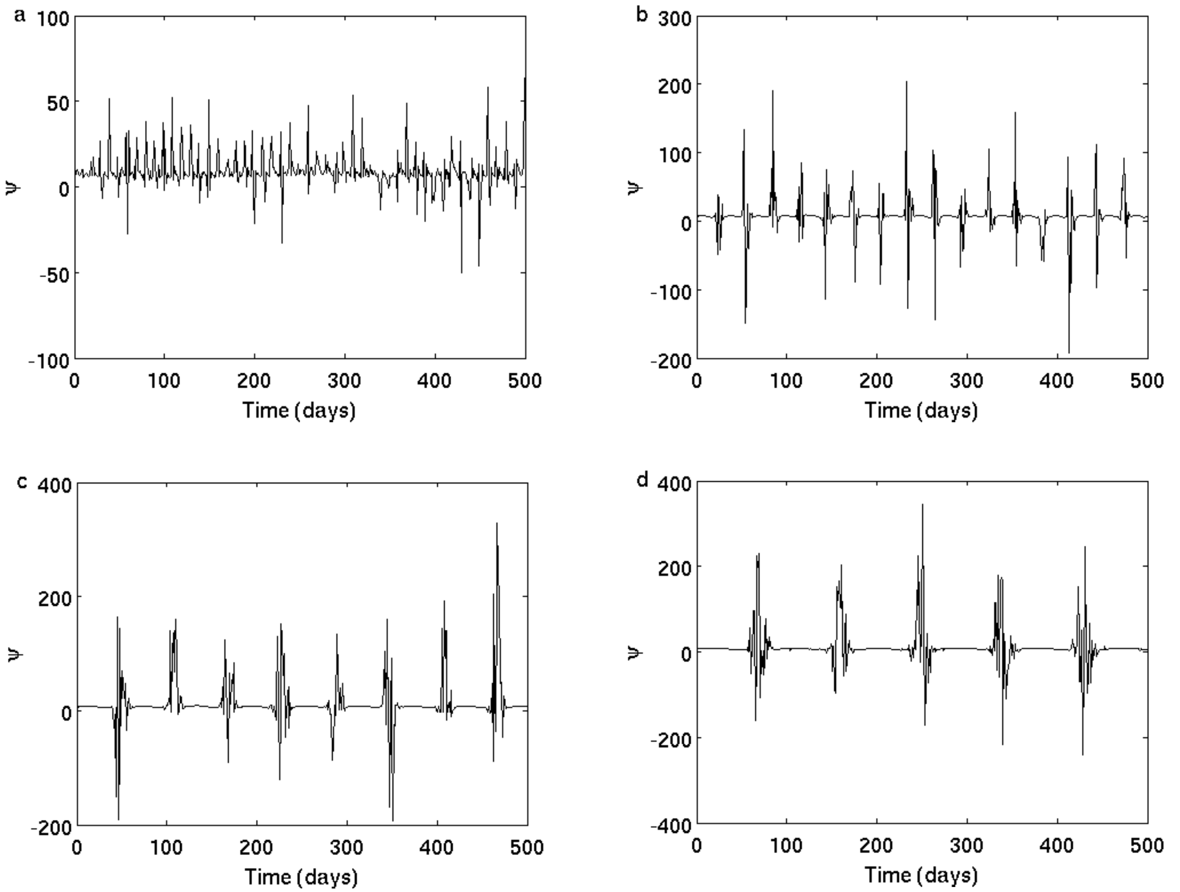

Figure 9. The time evolution of ~~stream function~~ stream-function (in dimensionless unit) at 3° North and 2400 m for t0 = 10 (a), 30 (b), 60 (c) and 90 (d) when $\nu_{\mathrm{V}} = 0.5$.

associated with a chaotic behavior of the system. For higher values of t_0, the amplification of the model response occurs for higher values of ~~$nu_textrmV$~~ ν_{V}. On the ~~contrary~~other hand, high values of vertical viscosity kill the chaotic behavior, making the evolution ~~quasi-periodic with the trajectories of the stream function still lying on a torus~~ periodic (not shown).

4 Conclusions

We have used ~~an~~ a dry axisymmetric model to simulate the Hadley circulation and to investigate the role of a changing stratification of the thermal forcing, ~~as if there were~~ which simulates moist convection that alters the static stability of the model atmosphere. The bi-dimensionality of the model prevents the generation of any eastward traveling wave. Hence, in our discussion, the influence of eddies' momentum fluxes is not taken into account. We have shown that the stream-function representing the Hadley circulation in an axisymmetric model can exhibit a ~~quasi-periodic~~ periodic behavior when the vertical

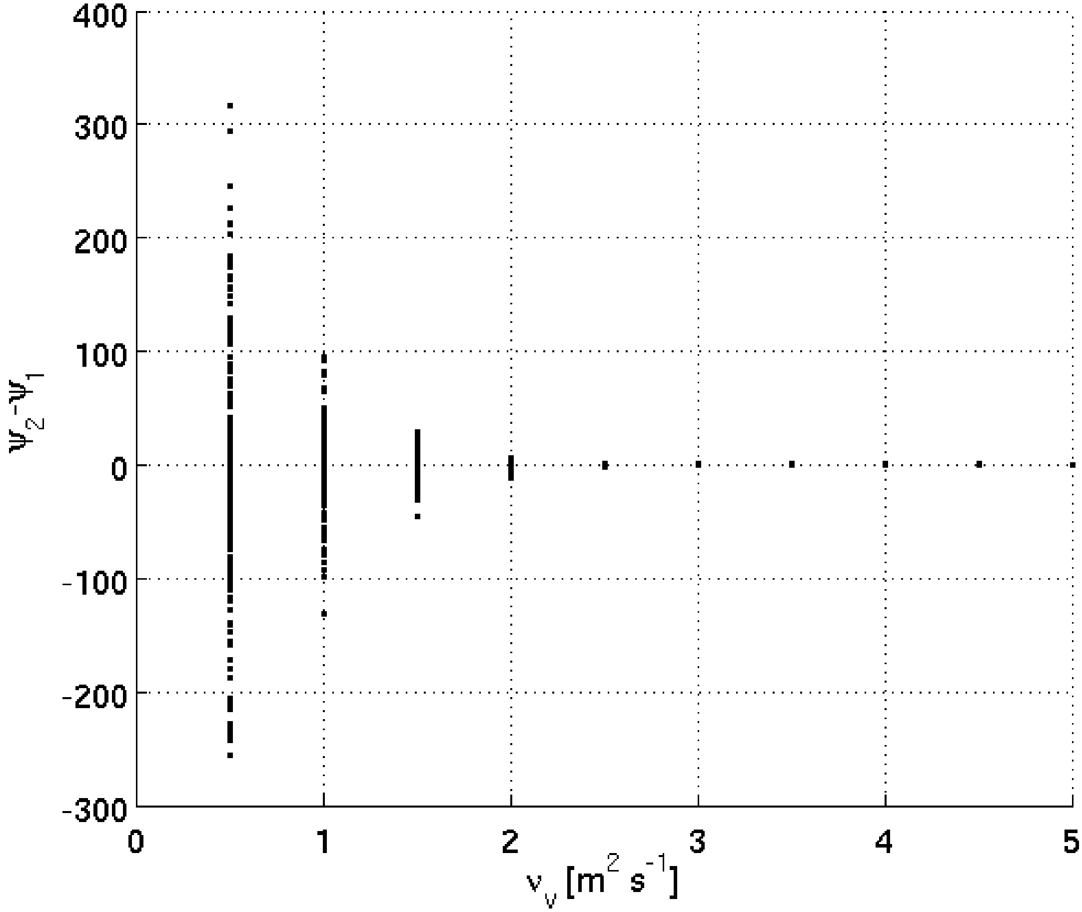

Figure 10. The 1-D bifurcation diagram as a function of ν_V for simulations with for $t_0 = 30$ days.

stratification of the thermal forcing is periodically forced to become neutral. The question that arises is whether this oscillation can be linked with the observed atmospheric fluctuations within the tropical region. Although Madden-Julian oscillation is a three dimensional phenomenon with the development of a Kelvin-Rossby wave (Gill, 1980), it is certainly associated with the evolution of convective anomalies (Hendon and Salby, 1994). It has already suggested that the been suggested that quasi-periodic oscillation seems to be an intrinsic characteristic of the tropical atmospherein according , in accordance with the results of Goswami and Shukla (1984). Bi-dimensional models, while they can give us a framework of basic physics underlying atmospheric processes are quite limited as the real atmosphere is naturally three-dimensional. The findings of this work suggest that if a cyclic process perturbs tropical stratification, the Hadley circulation strength may be periodic with possible bursts when the perturbation time is larger than 60 days. These results must necessarily be compared with observations of the real atmosphere or to the results of three-dimensional models.

If the forcing period is up to 63 days the ~~stream function~~ stream-function evolution shows a ~~classic quasi-periodic~~ periodic behavior. For period forcing longer than 63 days, the slow frequency associated with the forcing period modulate a fast response in the system, generating a chaotic motion that ~~lasts for a while before going back to a steady solutionwhen the chaotic phase finishes~~persists for a period of time before returning to a non-chaotic solution. It is not clear whether these high frequency characteristics are actually present in the meridional circulation of our planet. In fact, when we look for periodicities of the order of tens ~~days into~~ of days in observations, the higher frequency signals are usually removed. Moreover, even though we can detect such signals, it is not easy to associate them with the oscillations caused by a change of static stability~~rather than~~ , as opposed to other processes. The chaotic dynamics observed in the model could exist in other planets where the vertical stratification takes longer to become neutral. The change of vertical stability that in our model simulates the cycle of large-scale convection might be equivalent to the recharge and discharge of moisture that supports the Madden-Julian oscillation (Zhu and Hendon, 2015). This ~~can be thought~~ change of stability can be imagined to control the aggregation process of convection, which allows a bistable equilibrium between moist ~~or~~ and dry situations (Raymond and Zeng, 2000, Zhang et al., 2003, Arnold and Randall, 2015). An important aspect is the rate at which this process occurs. As we have shown in the model we consider, when this rate ~~in the considered model~~ is over 63 days ~~we can have~~ it results in the short term in a chaotic impact on the Hadley circulation strength. The role of friction in the symmetric circulation, driven by a meridional thermal gradient of a fast rotating planet like the Earth, is contradictory. On the one hand it is an essential ingredient to allow a meridional overturn, instead of a strong zonal wind in cyclostrophic balance only~~, on~~ . On the other hand, the value must be close enough to zero ~~in order to allow the~~ to allow angular momentum conservation. Although these conditions are met in the model we ~~have considered~~consider, the presence of a time-varying stratification alters the classic view of a stably stratified vertical temperature gradient. Other than the meridional thermal gradient, this imposed time-varying stratification represents another nonlinear forcing, which amplifies the model response when the vertical viscosity is small, itself representing a source of amplification of the model response in the inviscid case.

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