Nonlin. Processes Geophys. Discuss.,
doi:10.5194/npg-2016-57-RC1, 2016

Nonlinear Processes in Geophysics
Discussions
Open Access

Interactive comment on “Controllability, not chaos, key criterion for ocean state estimation” *by* Geoffrey Gebbie and Tsung-Lin Hsieh

Anonymous Referee #1

Received and published: 26 October 2016

This manuscript contains a description of the results of a series of experiments in which 4DVAR was used to assimilate simulated data from a nonlinear system corresponding to the damped driven pendulum in a chaotic parameter regime. This example differs from most other examples in the literature of data assimilation in strongly nonlinear systems in that it is a non-autonomous system, unlike, say, Lorenz (63). With a few reservations, the example is fairly well worked out. The application of the χ^2 test is particularly noteworthy. The basic results are worth publishing in some form.

The authors never state their model system explicitly. It is not (1). The system with which they are actually working differs from (1) in that it has a white noise term with variance S_f (see (5)) added to the right hand side. The distinction is not trivial. I assume that the reference solution in their twin experiments is the stochastic system with the stochastic term set to zero. The effect of adding the unknown stochastic term

Printer-friendly version

Discussion paper

is to increase the number of degrees of freedom in the control space from two, i.e., the initial conditions in the purely deterministic problem, to the number of time steps taken by the numerical method, which is potentially infinite.

The general level of discussion in this manuscript might have been marginally acceptable twenty years ago, when implications of applications of techniques from the engineering world were still being explored, but most of the manuscript is far below the current state of the art.

Studies of chaotic systems forced by white noise have appeared in a number of places in the literature. One example can be found in a paper by Tziperman from the early 90s.

There is nothing novel about writing the 4DVAR cost function in terms of a Lagrange multiplier. The use of Lagrange multipliers in variational formulations of estimation and control problems has been in the engineering textbooks since the 70s, and appeared in the early work of Thacker in the ocean modeling literature. In the present context, in which the task is to estimate an unknown stochastic forcing function, the Lagrange multiplier formulation is valid, but the same Euler-Lagrange equations result from equivalent cost function formulations without Lagrange multipliers, see, e.g., the text by Kalnay or either of the books by Bennett, as well as many of the reviews in the literature.

The authors should note that the estimation problem is the dual of the control problem. General questions of linear controllability and observability are dealt with in engineering textbooks. This topic has been well worked out in the context of models of the ocean and atmosphere in the work of S. E. Cohn in the late 80s and early 90s. The question of nonlinear observability is very complex. There was a book by Casti on the subject published some time ago.

The question of dealing with underdetermination has been discussed extensively in the literature. Solutions to underdetermined problems are not, in general, unique. The problem, in practice, is the fact that minimizing the cost function (5) involves searching a space of corrections that is potentially infinite. The highly irregular reconstructed forcing shown in the bottom panel of figure 4 is most likely one of an enormous number of minimizers of (5). There are almost certainly many others that will minimize the cost function, some smoother, many even more irregular.

Bennett showed that, in the linear problem, one solution can be found by choosing a correction to the first guess that lies in an N_y dimensional space spanned by representer functions, where N_y is the number of observations. This solution corresponds to the Moore-Penrose inverse. Arguments as to why that solution should be preferred over others are the stuff of textbooks.

Similar practical results can be had without explicit calculation of representers. In practical problems in modeling the ocean and atmosphere, the correction to the forcing function lies in a space of enormous dimension, so it is common to precondition the search for a cost function minimizer. This effectively reduces the dimension of the control space by choosing corrections to be a linear combination of singular vectors of the error covariance matrix. This approach is documented in the work of A. Lorenc and O. Talagrand. In the present problem, it might be reasonable to impose nontrivial temporal correlation on the forcing correction, which might have the effect of limiting the spectrum of the correction and thus ruling out irregular forcing corrections like that shown in figure 4.

The authors have a choice. They can simply report on the results of their twin experiments on their nonautonomous system and eliminate nearly all of the discussion, or they can go back over twenty or twenty five years of literature and rewrite the discussion to make it a meaningful contribution to the current state of the art.

Interactive comment on Nonlin. Processes Geophys. Discuss., doi:10.5194/npg-2016-57, 2016.