The author highly thanks the anonymous reviewer for his/her very helpful and insightful comments that lead to significant improvement of the quality of this manuscript. The author has checked the manuscript carefully and tried the best to address all the comments. Below, the italic is used for quoting the comments from the reviewer and following with the point-by-point responses.

1 General Comments

This paper presents a new technique in estimating model error covariance inflation factor which is widely used in ensemble-based filters. An inflated model error covariance is necessary to arrest the divergence of the filter. This paper relies on estimating the inflation factor from an objective function inspired from the domain of generalized cross validation (GCV) techniques widely used in the field of machine-learning. The author also shows that this method, in comparison to a basic Ensemble Kalman Filter, considerably improves the root-mean-squared error and enhances the influence of observations on the analysis when applied to the Lorenz 96 model.

Response: Thank you for your review and comments.

However, it is well known that a basic Ensemble Kalman Filter (EnKF) falls short on many accounts and a mere improvement with respect to it does not give much credence to this new technique. Even introducing a simple constant multiplicative inflation factor to the basic EnKF considerably improves the analysis. The author should address the following questions:

1) How does this method fare when compared to simple multiplicative inflation techniques like setting a constant inflation factor in the basic ensemble Kalman filter? It will be more interesting to see this method pitted against other sophisticated inflation schemes.

Response: Thank you for your comment. In the simple multiplicative inflation techniques like setting a constant inflation factor, the selection of the constant is often determined by repeated experimentation and prior knowledge (Anderson; Anderson 1999). Hence such experimental tuning is rather empirical and subjective.

The improved EnKF is compared with the constant inflated EnKF in the revised version. The constant is particularly selected as the median of the estimated inflation factor by minimizing the GCV function. Besides small fluctuation, the mean GAI value of the constant inflated EnKF is 27.80%, which is smaller than that of the improved EnKF. The mean spread value of improved EnKF is 3.32, which is slightly larger than that of the constant inflated EnKF (3.25). It illustrates that the underestimation of forecast ensemble spread can be effectively compensated for the two EnKF schemes with forecast error inflation, while the improved EnKF is more effective than the constant inflated EnKF. The analysis RMSE, as well as the values of the GCV functions, decrease sharply no matter which inflation scheme is adopted. However, the GCV function and the RMSE values of the improved EnKF are smaller than those of the constant inflated EnKF, indicating that the on-line estimate method performs better than the simple multiplicative inflation techniques with a constant.

2) How does the time-series of the inflation factor look like?

Response: The time series of estimated inflation factors are shown in Figure 2 in the revised version, which vary between 1 and 6 with greatly majority. The median is 1.88, which is used in the comparison of the improved EnKF and the simple multiplicative inflation techniques like setting a constant inflation factor. This has been added to section 3.2 in the revised version.

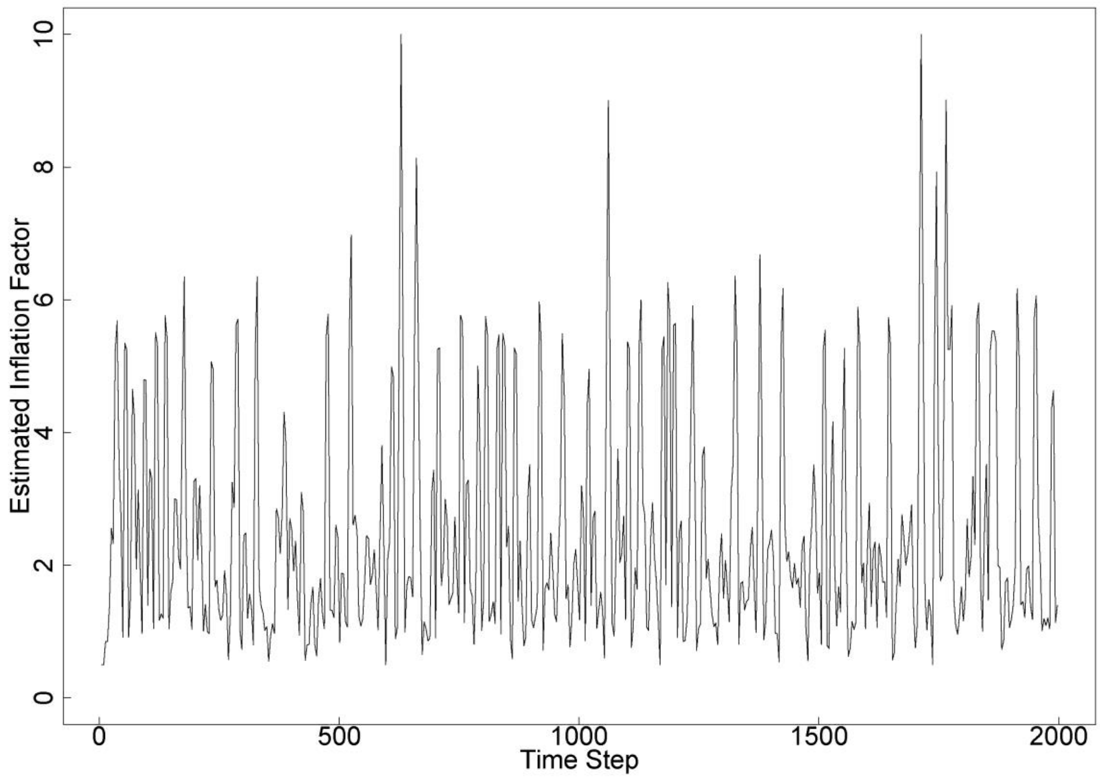

Figure 2. The time series of estimated inflation factors by minimizing GCV function.

3) What are the improvements in other statistical measures like spatial correlation, correlation coefficients, measure of ensemble spread, etc?

Response: The forecast ensemble spread of the conventional EnKF, improved EnKF and constant inflated EnKF are plotted in Figure 3. For the conventional EnKF, since the forecast states usually shrink together, the forecast ensemble spread is quite small with mean value 0.36. The mean spread value of improved EnKF is 3.32, which is slightly larger than that of the constant inflated EnKF (3.25). It illustrates that the underestimation of forecast ensemble spread can be effectively compensated for the two EnKF schemes with forecast error inflation, while the improved EnKF is more effective than the constant inflated EnKF.

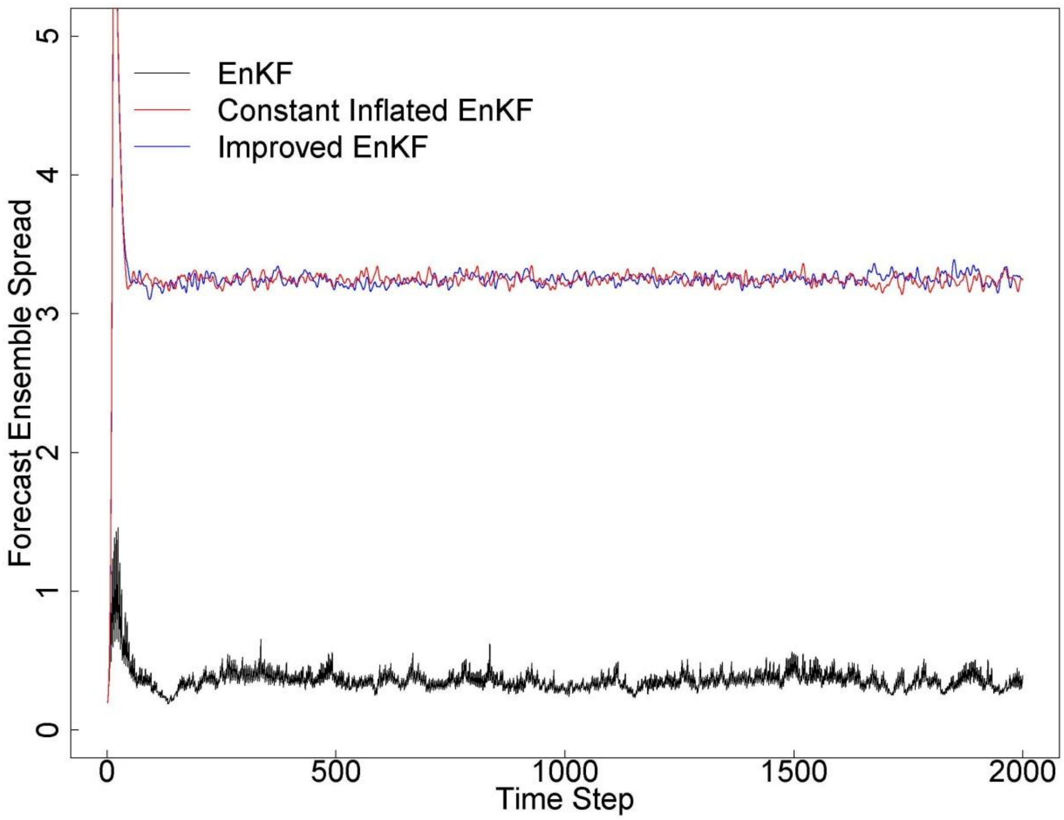

Figure 3. The forecast ensemble spread of the conventional EnKF (black line), the improved EnKF (blue line) and the constant inflated EnKF (red line).

4) What are the computational challenges in estimating the inflation factor?

Response: For the aspect of computational cost in minimizing the GCV function, the most expensive part is in computing the influence matrix $\mathbf{A}_i(\lambda)$. Since the matrix multiplication is commutative for the trace, the GCV function can be easily re-expressed as

$$GCV_i(\lambda) = \frac{p_i \mathbf{d}_i^{\mathrm{T}} \left(\mathbf{H}_i \lambda \mathbf{P}_i \mathbf{H}_i^{\mathrm{T}} + \mathbf{R}_i\right)^{-1} \mathbf{R}_i \left(\mathbf{H}_i \lambda \mathbf{P}_i \mathbf{H}_i^{\mathrm{T}} + \mathbf{R}_i\right)^{-1} \mathbf{d}_i}{\left[\mathrm{Tr}\left(\left(\mathbf{H}_i \lambda \mathbf{P}_i \mathbf{H}_i^{\mathrm{T}} + \mathbf{R}_i\right)^{-1} \mathbf{R}_i\right)\right]^2}. \qquad (1)$$

Since both of the numerator and denominator of the GCV function are scalars, the inverse matrix is only needed in $\left(\mathbf{H}_i \lambda \mathbf{P}_i \mathbf{H}_i^{\mathrm{T}} + \mathbf{R}_i\right)^{-1}$, which can be effectively calculated using the

Sherman–Morrison–Woodbury formula (Golub; Loan 1996). Furthermore, the calculation of the inverse matrix and the multiplication are also indispensable for the conventional EnKF (Eq (6)). There is no additional computation burden for the improved EnKF by minimizing the GCV function essentially. Therefore, the total computation of the improved EnKF is feasible.

5) How does this method fare in presence of sparse observations?

Response: Thank you for your comment. In presence of sparse observations, the state which is not observed can be only improved by the physical mechanism of the forecast model, although this improvement is very limited. Therefore, a multiplicative inflation may not be effective enough to enhance the assimilation accuracy. In this case, the additive inflation and the localization technique can be applied to improve the assimilation quality further presence of sparse observations (Miyoshi; Kunii 2011; Yang et al. 2015).

This discussion has been added to section 4 in the revised version.

It is clear that English is not the native language of the author. One of the suggestions is to seek help and revise the language of this paper. This paper, in its present version, is more of a curious exercise and needs to be considerably revised to make it publishable. If these revisions are made, this paper may be accepted for publication.

Response: Thank you for your comment. The grammars have been checked carefully and the language has been polished in the revised version.

2 Specific Comment

1) "The objective function needs to be minimized to estimate the inflation parameter" is not explicitly mentioned when it is introduced in the main text. It is however casually mentioned in the Discussion section.

Response: The inflation factor is estimated by minimizing the GCV (Eq. (9)) as an objective function. This has been explicitly mentioned in section 2.2 in the revised version.

2) What is the motivation of generating observations at every 4 time-steps?

Response: In realistic problems, the observation cannot be obtained every time. The time step for generating the numerical solution is set at 0.05 non-dimensional units, which is roughly equivalent to 6 hours in real time, assuming that the characteristic time-scale of the dissipation in the atmosphere is 5 days (Lorenz 1996). Therefore the observation obtained every 4 time-steps can be used to mimic the daily observation in practical problems such as satellite data.

This explanation has been added to section 3.1 in the revised version.

3) The simplest observation error covariance matrix is a diagonal R. There are many in-situ observation systems in which R is diagonal. What happens when R is diagonal? Also, what is the motivation behind choosing that particular expression of R? What is the harm in introducing a parameter in the expression of R which may be conveniently tuned to set R diagonal?

Response: In in-situ observation systems, the observation error covariance matrix is diagonal. But for remote sensing and radiances data, it is usually spatially correlated. The correlation coefficient of two observation grids is inversely proportional to their distance. Therefore the particular expression of the observation error covariance matrix is chosen, which may potentially be applied to assimilate remote sensing observations and radiance data. The performances of these assimilation schemes are very similar in the case of diagonal *R*, which is a special form of the general case in the manuscript.

*4) P4 L1: What does the author mean by "**gradually important**"?*

Response: It means researchers realize that, the covariance inflation is becoming more and more important. The following texts have been added in the revised version.

Covariance inflation, as a technique used to mitigate filter divergence by inflating the empirical covariance in EnKF, can increase the weight of the observation in the analysis state (Xu et al. 2013). Actually, it will perturb the subspace spanned by the ensemble vectors and better capture the sub-growing directions that may be missed in the original ensemble (Yang et al. 2015).

5) P5 L9-10: Kindly elaborate on what the "favorable properties" of GCV are?

Response: The Cross Validation is a general procedure that can be applied to estimate tuning parameters in a wide variety of problems, which aims at minimizing the estimated error at the observation grid point. The GCV criterion is a weighted version. Originally proposed to reduce the computational burden, GCV is one of a number of criteria which all involve an adjustment to the average mean-squared-error over the training set (Craven; Wahba 1979).

6) P6 L20-21: What does the author mean by "The EnKF assimilation result is . . . ***sufficiently close*** *to the corresponding true state . . ."?*

Response: The common assumption is the existence of a "true" underlying state of the system for a state variable in the geophysical research fields. Data assimilation is a powerful mechanism to estimate the true trajectory. The difference of the estimated vector to the truth (such as the root-mean-square-error used in the literature) is used to evaluate the accuracy of an assimilation scheme. The text "sufficiently close" means "accurate enough". The sentence has been changed to "The EnKF assimilation result is . . . an accurate estimate of the corresponding true state …"

7) Please elaborate the flowchart in the main text as well.

Response: The flowchart is elaborated exhaustively in section 2.1 and 2.2 in the revised version. The inflation factor is estimated by minimizing the GCV function is specifically stated.

8) What is N in Fig 1?

Response: The scalar N is the total time step in the forecast procedure.

9) P13, L8-9: "The model forecast changes very much along with the change in F . . ..". This is a very general statement. Please be a little more specific. Also cite references that show the model to be chaotic for F>3.

Response: The text has been changed to as follows:

For the Lorenz-96 model, modifying the forcing strength *F* changes the model forecast considerably. For values of *F* larger than 3 the system is chaotic (Lorenz; Emanuel 1998).

3 Technical Comment

1) There are many notable absences of articles, usage of wrong prepositions and a few grammatical corrections to point out. One such instance is in P13 L8-9. "The model forecast . . . and is chaos with integer values of F larger than 3". This should be "The model forecast . . . and is ***chaotic for*** *integer values of F>3".*

Response: Thank you for your comment. The grammars have been checked carefully in the revised version.

2) P13 L22: "The variety of the analysis RMSE . . .". It is not clear what the author wants to convey.

Response: It has been changed to "The variability of the analysis RMSE is very consistent with that of the GCV function …".

Again, thanks for your constructive comments and helpful suggestions.

The references in this reply are listed as follows (Some of them are already in the original manuscript and some are newly added in the revised version).

Anderson, J. L., and S. L. Anderson, 1999: A Monte Carlo implementation of the nonlinear fltering problem to produce ensemble assimilations and forecasts. *Monthly Weather Review*, **127,** 2741-2758.

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Lorenz, E. N., 1996: Predictability a problem partly solved.

Lorenz, E. N., and K. A. Emanuel, 1998: Optimal sites for supplementary weather observations simulation with a small model. *Journal of the Atmospheric Sciences*, **55,** 399-414.

Miyoshi, T., and M. Kunii, 2011: The Local Ensemble Transform Kalman Filter with the Weather Research and Forecasting Model: Experiments with Real Observations. *Pure & Applied Geophysics*, **169,** 321-333.

Xu, T., J. J. Gómez-Hernández, H. Zhou, and L. Li, 2013: The power of transient piezometric head data in inverse modeling: An application of the localized normal-score EnKF with covariance inflation in a heterogenous bimodal hydraulic conductivity field. *Advances in Water Resources*, **54,** 100-118.

Yang, S.-C., E. Kalnay, and T. Enomoto, 2015: Ensemble singular vectors and their use as additive inflation in EnKF. *Tellus A*, **67**.