Thanks for your comments very much.

To facilitate discussions, the following quick responses are provided. More detailed discussions will be given in the final responses after the discussion period ends.

We are aware that a further simplified 3D-NLM (e.g., with a particular set of initial conditions) and the Duffing equation (as discussed in the Appendix) (or double-well potential system) may be dynamically equivalent. However, while the former (the 3D-NLM) has three ordinary differential equations (ODEs) and the latter (with or without an external forcing) is a second-order ODE. Therefore, without providing a proof regarding a homeomorphism, we avoided a detailed comparison between the two systems. While our focus of this study is on the role of the nonlinear feedback loop in producing oscillatory solutions (and the homoclinic orbit), our new paper is to examine the impact of the extended nonlinear feedback loop on the periodicity (or quasi-periodicity) of solutions in a five-dimensional non-dissipative Lorenz model, which is simplified from the five-dimensional Lorenz model (Shen, 2014). Here, we would like to present how the nonlinear feedback loop (i.e., -XZ and XY) may lead to the nonlinear restoring forcing term (i.e., X**3), which plays a role in producing oscillatory solutions.

The Lorenz model (1963) has been studied extensively and been used to illustrate the sensitive dependence of solutions on initial conditions (i.e., the butterfly effect of the first kind.). Three types of physical processes in the Lorenz model are: heating, dissipation and nonlinear interactions. The nonlinearity is from the horizontal advection of temperature term, which appears in all of climate and weather models (e.g., Shen et al., 2006, 2012, 2013). Therefore, improving the understanding of the nonlinear term and the associated (thermodynamic) feedback may help improve the representation of the thermodynamic feedback in numerical models, which remains big uncertainties in climate model simulations.

In this study, we focus on the role of nonlinear processes and heating term (i.e., without the inclusion of dissipation). We present several closed-form solutions to the simplified Lorenz model. In addition to the closed-form solution using trigonometric functions, we also present a closed-form solution using elliptic functions in Appendix B as verifications. It is our believe the simple form of the solutions (to X"+X**3/2=0) using trigonometric functions can effectively help illustrate the role of the nonlinear term (X**3) in producing oscillatory solutions. The solutions can improve our understanding of the nonlinear processes and thus help examine the competing impact between the heating and nonlinearity. The relationship between the nonlinear feedback loop (-XZ and XY) and the nonlinear term X **3 is given on page 3 of the response file. The physical processes (i.e., downscaling and upscaling processes) associated with -XZ and XY were first discussed in Shen (2014).

Additionally, we discuss how the collective impact of the nonlinear feedback loop and the heating may produce three types of solutions, including nonlinear periodic solutions with a small or large cycle and the homoclinic orbit solution, and discuss the energy cycle. Note that X**2, Y**2+Z**2 and Z are associated with kinetic, available potential and potential energy, respectively. (This should be different from the energy cycle in the Duffing equation). As mentioned, we currently examine the impact of the extended nonlinear feedback loop on the periodicity (or quasi-periodicity) of solutions using a five-dimensional non-dissipative Lorenz model. More detailed responses will be given soon.

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The non-dissipative Lorenz model (3D-NLM) with no heating term is written as:

$$\frac{\mathrm{d}X}{\mathrm{d}\tau} = \sigma Y, \tag{1}$$

$$\frac{\mathrm{d}Y}{\mathrm{d}\tau} = -XZ, \tag{2}$$

$$\frac{\mathrm{d}Z}{\mathrm{d}\tau} = XY. \tag{3}$$

Equations (1-2) lead to

$$\frac{d^2X}{d\tau^2} = \sigma \frac{dY}{d\tau} = -\sigma XZ. \tag{4}$$

From Eqs. (1) and (3) (i.e., $X * Eq.(1) - \sigma * Eq.(3)$), we have

$$\frac{d}{d\tau}\left(\frac{X^2}{2} - \sigma Z\right) = 0, \tag{5}$$

and

$$\frac{X^2}{2} - \sigma Z = C, \tag{6}$$

where C is a constant. The initial condition of $(X,Y,Z) = (0,1,0)$ leads to $C = 0$. Therefore, Eqs. (4) and (5) with the above initial condition gives us

$$\frac{d^2X}{d\tau^2} = -\frac{X^3}{2}.$$

The above derivations illustrate the relationship between the X^3 and the nonlinear feedback loop (i.e., $-XZ$ and XY), which are associated a pair of upscaling and downscaling processes as discussed in Shen (2014).