Interactive comment on "Parametrization of stochastic multiscale triads" *by* Jeroen Wouters et al.

Anonymous Referee #1

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I have reviewed the manuscript entitled "Parametrization of stochastic multiscale triads" (npg-2016-37). I enjoyed this paper and thought the exploration of the weak coupling approach as compared to homogenization was prudent, especially if the goal is to use this tool in future stochastic averaging problems. The paper is laid out in a straight-forward way with a strong emphasis that it is showcasing the application of a particular method (rather than discussion of the method itself) and the appropriate references are provided regarding the method. Overall, I think this paper does a good job of demonstrating the applicability of the weak coupling method and its virtues compared to homogenization. The metrics of comparison are appropriate and I think constitute sufficient evidence for the conclusions the authors have drawn (i.e. that for simple stochastic triad models, weak coupling is a more consistent reduction method than homogenization). I also appreciate that the authors discuss the limitations of the weak

coupling method for the multiplicative triad, giving this paper a proper grounding. Based on the manuscript and the research scope of the journal, I believe this work falls within the desired scope of NPG and meets the review criteria.

I have a few minor suggestions that I think would make this work more complete and informative to the reader. They are mostly intended for those approaching this topic for the first time. I have listed most of them below in a point form.

My advance thanks to the authors for considering my comments. Their responses to my comments are appreciated. My compliments to the authors for their work.

Scientific / Content –

1. p2, line 11 - There is at least one work out there that shows the lack of spectral gap explicitly in climate dynamics. Readers may appreciate a reference to this fact. The work of J. M. Mitchell (1976), although it is not new, offers a discussion of this.

2. p2, line 33 - Is there a good reference for scattering theory that be offered here?

3. p2, line 34 - This is the famous Feynman diagram from particle physics, correct? If so, no change is necessary. I just wanted to confirm.

4. p3, line 8 - What are the characteristics of Axiom A dynamical system that make the theory of Dyson series appropriate? (I assume it is the chaotic nature of Axiom A systems, but maybe a sentence here would be helpful)

5. p3, line 9 - Can you clarify what is meant by a physical measure? My immediate intuition leads me to think this means either a delta-function measure or a finitely integrable measure. I assume the author means the latter, as a delta-function measure does not generally apply to stochastic systems.

6. p8, line 10 - Does f^{τ} have a particular signifcance? Or is it just a place holder function? It appears to be a time-evolution operator to me. If so, then what is the variable 's' on the following line? This may require some clarification.

7. p11, line 3 - Can the authors comment on the solvability condition? i.e. Is the condition that the mean of the system drift is equal to zero?

8. p12, line 20 - For formal equivalence, you are comparing the coefficients of the homogenized model and Markovian weak coupling model, correct? I'm not sure that "formal equivalence" between the full and reduced equations is the most clear way to explain this. Would it be more appropriate to describe the equivalence as "statistical" or "weak" (as opposed to "strong").

9. I would be interested in any comments the authors can make about the asymptotic nature of the weak behaviour of the reduced systems compared to the full systems in terms of the long time behaviour. (i.e. For Figure 3, what happens as theta goes to infinity? Does the superiority of the weak coupling model continue to hold?) The comments made through the additive examples suggest that for epsilon small enough the weak coupling model converges to the homogenization model but the lines in Figure 3 give me the impression that for large epsilon, the weak coupling model may do better initially, but the homogenization may give a better long time approximation in terms of distribution width.

10. I have a concern about the scaling of the fast-slow system (1) compared to system (3). The powers of epsilon are not equivalent between the two systems and I suspect that this has implications for the scalings in systems (2) and (5) which are equivalent in epsilon. Should the reader be concerned about the difference in scalings?

11. For completeness, I recommend that the authors include the ensemble sizes used to generate the curves in the figures.

12. Regarding Tables 1-4, a mean and standard deviation are probably sufficient to demonstrate the superiority of the weak coupling method for the systems considered, but I think it would be interesting to see the distribution of exit times in terms of a PDF. If this is easily done, I would be curious to see it.

Technical –

1. Graphs were challenging to read on paper because the axis labels and lines were small and/or thin. If the authors could make the plots more readable, I'm sure readers would appreciate it.

2. Equations are not consistently labeled. (i.e. 20 / (20) / Eq. 20). I also notice that the authors sometime make reference to the full triad systems without equation refs, where as the reduced systems often have equation refs. For increased readability, I think it would be good to include equation references to the full systems where possible.

3. p6, line 20 - Should $R(\tau)$ be $R(\tau, z)$? There are a few other instances where the memory function, R, is specified with either one or two arguments. It would be good to keep this consistent.

4. p15, line 5 - The variables σ_{1}, σ_{2} have already been used on page 4 as scaling coefficients for the noise terms. I think a different variable would help the clarity here.

5. p5, line 6 - Regarding the previous point, I also think it would be appropriate to change the variable $\sigma(\tau)$ to something else.