The authors have clearly made a conscientious effort to address my original concerns, and they have, for the most part succeeded. While the new version is much improved, I still can't say that I understand it completely. I think that, at this point, it's just a matter of wording, and the authors should be able to correct what seem to me to be obvious problems and produce an entirely acceptable manuscript fairly easily.

First, the authors should be careful to treat the Lebesgue measure rigorously correctly. A few examples:

1. On page 5: "Generally a Lebesgue measure on $\mathbb{R}^n$ assumes that A is any subset of $\mathbb{R}^n$." This is not true. The σ-algebra $\mathcal{F}$ in the definition of the triplet $(\Omega, \mathcal{F}, \mu)$ (line 6 on p5) that defines the measure does not include all subsets of $\mathbb{R}^n$. There are subsets of $\mathbb{R}^n$ to which a Lebesgue measure cannot be consistently assigned. Construction of these so-called "unmeasurable sets" is described in the standard texts, as the authors know.

2. Same paragraph: instead of "Thus if A is any subset of $\mathbb{R}^n$, one can collect ..." I suggest "for any $A \in \mathcal{F}$ one can collect ..."

3. p5, toward the bottom: "The Lebesgue measure of any subset in $\mathbb{R}^n$ also coincides with its volume." Again, there are subsets of $\mathbb{R}^n$ to which a Lebesgue measure cannot be consistently assigned.

4. p8: "...any bounded closed domain A" As before, A must be Lebesgue measurable.

5. p11 lines 7-8 instead of "$A \in \mathbb{R}^n$", you want "$A \in \mathcal{L}^2$"

The definition of scale in section 3 is much better than the original, but it is still not clear. At this point some careful attention to the exposition should be sufficient. Here is the problem:

p8, lines 4-5: "We use Lebesgue measure on $\mathbb{R}^2$, i.e., $\mu_{iv}(A) = m^2(A) = inf\big(\sum_{i=1}^{+\infty} I^2(A_i)\big)$ where ... From a geometric perspective, the measure function refers to the shape of the subset, and the scale further indicates the size." OK, I understand that, say, in figure 1, you mean to say that disks C_1, C_2 and C_3 have the same shape, but they have different scales because they are different sizes. But the Lebesgue measure is the area, and you have defined it as it is defined in the books. By your definition of m^2, referring to figure 1, $m^2(C_2) > m^2(C_1)$, but on line 10 you write "$m^2_{C1} = m^2_{C_2} = m^2_{C_3}$ because they are the same function." I don't understand this at all. You mention a function you call f but it plays no part in the definition of m^2. The statement "$m^2_{C1} = m^2_{C_2} = m^2_{C_3}$" is inconsistent with the stated definition of m^2.

I'm guessing that C_1, C_2 and C_3 are examples of footprints. If so, would you please say this explicitly?

What, exactly, are the functions associated with C_1, C_2 and C_3?