Dear Prof. Talagrand,

Thank you for your hard working and constructive suggestions, which significantly improved our study. We have made the corresponding revisions. The point-by-point responses to the suggestions are in the following text.

Best wishes,

Feng Liu

Authors' Responses to the Comments from Editor

Comment 1.

P. 6, Eq. (4). x(t) is here an Ito process, just as I(t) before. I suggest you use the same notation.

Comment 2.

P. 6, l. 30. ... the well-accepted Bayesian theory of data assimilation.

Comment 3.

P. 13, l. 4. The two equations on this line state the same result for two different arguments sX and sY. Actually, in view of the eqs (15) and (16) that follow, I think the corresponding sentence is useless. The sentence These formulas prove ... can be put after Eqs (15) and (16).

Comment 6.

P. 16, l. 1. Why not put an absolute value for the variance of X (and remove the sentence It should be noted ...) ?

Comment 7.

And, finally, change the first line of acknowledgments to We thank the editor, Dr Talagrand, and ...

Response:

Thanks for your suggestions. We have adopted all the above suggestions. Please find the detail information in the new manuscript.

Comment 4.

P. 13, l. 24. I understand x in H(s, x) is the same thing as X before. Use the same notation.

Response:

Actually x in $H(s, x)$ is different from X. Here x is an argument of H. Although x always indicates the state vector, it is not the same as an instance of state vector X. Furthermore, the first- and second-order derivatives of H are notated as H_x and H_{xx}, respectively, where the notation x cannot be replaced by X.

But we can use another letter to replace x in order not to cause confusion. As comment 1, x was replaced by I. Therefore H can also be written as $H(s, I)$, and its first- and second-order derivatives are H_I and H_{II}.

Changes in the manuscript:

In the new manuscript, all the related notations were changed accordingly. But in Sect. 3.4, SRTE is still defined as $H(s, x(s))$, because we used $I(\tau)$ to present the state vector radiation intensity (I is the widely accepted notation for radiation intensity in a radiative transfer equation). In this section we believed that x and $I(\tau)$ cannot be mixed up.

Comment 5.

It seems that, from the last line of p. 14, σX is assumed to be equal to 1. Why not keep an explicit σX ?

Response:

The reasons that we did not keep an explicit σ_X are, on the one hand, the formulas with explicit σ and φ were given in Eqs. (21) and (23), and on the other hand, in Eqs. (24) and (25), we tried to deduce the simplest version of Eqs. (21) and (23) to present the direct relationship between the scale transformation and $p(Y|X)$, where σ and φ should not be included in these equations. So the state X is assumed to be only with the scale-dependent Gaussian noises, resulting σ_X=1 and φ_X=0.