



## Scale and space dependencies of soil Nitrogen variability

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15 **Abstract.** In this study we use the multifractal analysis, through generalized dimensions ( $D_q$ ) and the relative entropy ( $E(\delta)$ ) to investigate residual effects on wheat and grain, biomass and nitrogen content, of fertigation treatments applied to a previous crop. The wheat crop covered nine subplots from a previous experiment on melon response to fertigation. Each subplot had previously received a different level of applied nitrogen and plants from the previous melon crop had already taken up part of it. Many

20 factors affect these variables, causing it to vary at different scales creating a non uniform distribution. The  $D_q$  were used to study the relation between scales meanwhile  $E(\delta)$ , and their increments between scales, were used to identify the scale at which the variable had a maximum structure and compare with the scaling behavior of the nitrogen applied. The  $E(\delta)$  is particularly appropriate for this because of does not require any prior assumptions to the structure of the data and it is easy to calculate.

25 The four variables studied presented a weak multifractal character presenting a low variation in  $D_q$  values, around the unit, that wasn't relevant for the study. On the other hand, the  $E(\delta)$  and the increments in



$E(\delta)$  help us to detect changes in the scaling behavior of all the variables studied. To this respect, the results showed that the applied nitrogen through fertirrigation dominated the wheat and grain biomass response as well as nitrogen content of the whole plant; surprisingly grain nitrogen content didn't show the same structure than the applied nitrogen. At the same time, there was a noticeable structure variation  
5 in the biomass and nitrogen content at the smaller scales that correspond to the previous cropping root arrangement due to uptake of the applied nitrogen.

**Key words:** relative entropy, multifractal analysis, sink crop

## 1. Introduction

Soils can be seen as the result of spatial variation operating over several scales. This observation points to  
10 “variability” as a key soil attribute that should be studied (Burrough et al., 1994). Soil variability has often been considered to be composed of “functional” (explained) variations plus random fluctuations or noise (Goovaerts, 1997 and 1998). However, the distinction between these two components is scale dependent because increasing the scale of observation almost always reveals structure in the noise (Logsdon et al.,  
2008). Geostatistical methods and, more recently, multifractal/wavelet techniques have been used to  
15 characterize scaling and heterogeneity of soil properties among others coming from complexity science (de Bartolo et al., 2011).

Multifractal formalism, first proposed by Mandelbrot (1982), is suitable for variables with self-similar distribution on a spatial domain (Kravchenko et al., 2002). Multifractal analysis can provide insight into spatial variability of crop or soil parameters (Kravchenko et al., 2002 and 2003; Vereecken et al., 2007).  
20 This technique has been used to characterize the scaling property of a variable measured along a transect as a mass distribution of a statistical measure on a spatial domain of the studied field (Zeheke and Si, 2004; López de Herrera, 2016). To do this, it divides the transect into a number of self-similar segments. It identifies the differences among the subsets by using a wide range of statistical moments.

Wavelets were developed in the 1980s for signal processing, and later introduced to soil science by Lark  
25 and Webster (1999). The wavelet transform decomposes a series; whether this be a time series (Whitcher, 1998; Percival and Walden, 2000), or as in our case a series of measurements made along a transect; into components (wavelet coefficients) which describe local variation in the series at different scale (or



frequency) intervals, giving up only some resolution in space (Lark et al., 2003). Wavelet coefficients can be used to estimate scale specific components of variation and correlation. This allows us to see which scales contribute most to signal variation, or to see at which scales signals are most correlated (Lark et al, 2004). This can give us an insight into the dominant processes

5 An alternative to both of the above methods has been described recently. Relative entropy and increments in relative entropy has been applied in soil images (Bird et al., 2006) and in soil transect data (Tarquis et al., 2008) to study scale effects localized in scale and provide the information that is complementary to the information about scale dependencies found across a range of scales. We will use them in this work to describe the spatial scaling properties of a set of biomass data measured on a common 80-m transect  
10 across a wheat crop field. This is an indirect way to study the N variability left in the soil by the previous crop.

The site of this work is located near Mancha Occidental aquifer (U.H.04.04, 6.953km<sup>2</sup>) and Campo de Montiel aquifer (U.H. 04.06, 3.192 km<sup>2</sup>) with high contamination problems and soils with a restrictive subsurface “caliche” pan layer (petrocalcichorizon). Prior to the wheat crop, the plots were used for melon  
15 crop experiments to optimize fertirrigation using different level of nitrogen, which is reported in Castellanos et al. (2010). These treatments constituted a known contribution to the variation of soil nitrogen at predominantly larger scales. During melon crop development a proportion of the nitrogen was taken up adding a second factor of variability also known at smaller scales. After the melons were harvested the wheat was sown across the plots and harvested at the end of the crop season. The wheat was  
20 used effectively as a nitrogen sink crop and allowed us to evaluate the soil nitrogen residual.

In this study, we have analyzed transect data for Nitrogen content and weight of the grain and the whole plant of the wheat crop. First, correlations between these four variables and the different Nitrogen application doses in the previous crop, without considering spatial structure, were estimated. Then, multifractal and relative entropy analyses were applied to investigate the structure among the scales. This  
25 is the first time that both type of analysis are applied on the same data set.

## 2. Material and Methods



## 2.1. Field Experiment

Field trials were carried out in *La Entresierra* field station of Ciudad Real in the central of Spain (3° 56' W; 39° 0' N; 640 m of altitude) during May- September season of 2006. The soil of experimental site, classified as Alfisol Xeralf Petrocalcic in the USDA system, (Soil Survey Staff, 1999), presented very low vertical variability up to 60 cm of depth from which one finds a fragmented petrocalcic horizon. The soil was sandy-loam in texture, slightly basic (7,9 pH), medium in organic matter (2,2%), rich in potassium (0,9-1,0 meq L<sup>-1</sup>, ammonium acetate) and with a medium level of phosphorous (16,4 to 19,4 ppm, Olsen) with EC<sub>w</sub>. 0,1-0,2 dS m<sup>-1</sup>.

The area is characterized by a continental Mediterranean climate, with widely fluctuating daily temperatures (for more details see Castellanos et al., 2010).

## 2.2. Crop Melon Experiment

The species most widely cultivated in Spain is the melon type called "Piel de sapo" (*Cucumis melo L.*, var. inodorous, cv. Sancho). Melon seeds were germinated in a greenhouse in April until they had sprouted two or three real leaves. They were subsequently transplanted onto plastic mulch at a density of 4444 plants ha<sup>-1</sup> (1,5 x 1,5 m<sup>2</sup>) at 24 May 2006. A randomized complete block design was used, with three nitrogen treatments and three irrigations. Each treatment was replicated four times in plots measuring 10.5 x 12 m<sup>2</sup>. The plots had seven rows of eight plants each. Three adjacent plots were used to conduct the experiment (Fig. 1).

Each row was drip irrigated from a line with emitters spaced at 0,5 m, that dripped water at a rate of 2 l h<sup>-1</sup>. In order to facilitate the crop establishment, all plots received 30 mm of water. The irrigation schedule was calculated from 15 to 105 DAT with a single daily irrigation of 60% (TR1), 100% (TR2) or 140% (TR3) of melon crop evapotranspiration (ET<sub>c</sub>) depending on the irrigation treatment. Crop evapotranspiration (ET<sub>c</sub>) was calculated daily following the FAO method (Doorenbos and Pruitt 1977) as:

$$ET_c = K_c \times ET_o \quad (1)$$

where K<sub>c</sub> is the crop coefficient, that was obtained in the same area for melon crop in earlier years (Ribas et al. 1995) and ET<sub>o</sub> is the reference evapotranspiration calculated by the FAO Penman-Monteith method



(Allen et al. 2002). The total irrigation applied was 344, 554 and 757 mm for TR1, TR2 and TR3 respectively. The irrigation water quality was measurement weakly through a chemical analysis to estimate the Nitrogen content of the water ( $N_w$ ) (Table 1).

5 The fertilizer treatments consisted in different N doses: 0 ( $A_0$ ), 150 ( $A_1$ ) and 300 ( $A_2$ ) kg ha<sup>-1</sup>. The N was applied in the form of ammonium nitrate during 10 weeks of the crop cycle (from June to August), from a single pool at one end of the field where irrigation water was mixed with the respective doses of N (Table 1).

10 The plots were fertilized with 120 kg of P<sub>2</sub>O<sub>5</sub> ha<sup>-1</sup> (phosphoric acid) for the season, added to the irrigation water and injected daily, from three weeks after transplanting until the last week of August. A standard disease- and insect-control programme was implemented throughout the growing period in accordance with usual management practice to ensure that the response to N fertilizers would not be masked by other factors.

Melons were harvested when there was a significant amount of ripe fruit in the field from 26 July to 7 September with a total of seven harvests.

### 15 2.3. Wheat crop

Winter wheat (cv. Soissons) was grown on the same experimental sites where the melon crop was before (Fig. 2). It was sown 20 December of 2006 in rows spaced 0.15 m apart at a population of 400 seeds m<sup>-2</sup>. Post emergence herbicides were used to control weeds. No fertilizer or organic amendments were use for the cereal crop. Wheat crop was harvested 6 June 2007.

20 At this time a transect was selected in the field that went through several plot treatments as showed in Fig.1. Each 0.5 m a frame of 1x1 m<sup>2</sup> was placed on the soil and the wheat plants captured were harvested and placed in labelled samples. A total of 160 samples were collected traversing a length of 80 m.

### 2.4. Wheat Dry weight

25 In each sample, wheat grain was placed apart from the rest of the plant to obtain separately the dry weight of each sample. The grain dry weight ( $GW$ ) and plant dry weight ( $PW$ ) were determined by oven drying at 80 °C to a constant weight and the data is showed in Fig. 3B and 3D.



## 2.5. Wheat N uptake

Sub-samples of the dry plants and wheat grain were ground to a fine powder to determine the N content using the Kjeldahl method (Association of Official Analytical Chemists, 1990). The N uptake by the plant ( $PN$ ) and by the grain ( $GN$ ) was obtained as a product between N concentration and biomass. The resulted data is showed in Fig. 3A and 3C.

## 2.6. Correlation with Nitrogen applied

A simple analysis, regardless the spatial position, were applied to the data collected. The correlation ( $r$ ) point out the as well as the determination coefficient ( $R^2$ ) between the Nitrogen applied and each variable ( $PW$ ,  $GW$ ,  $PN$ ,  $GN$ ) were estimated and plotted.

## 2.7. Multiscale analysis through Generalized Dimensions

The aim of a multifractal analysis (MFA) is to study how a normalized probability distribution of a variable ( $\mu_i$ ) varies with scale. In this sense, the density levels of these probabilities are evaluated through the behaviour of a range of statistical moments of the partition function ( $\chi(q, \delta)$ ). Let's consider a grid segment of length  $\delta$  covering a part of transect, with total length  $L$ . The measure of the  $i^{\text{th}}$  segment is defined  $M_i(\delta)$ . We now perform a weighted sum over all segments that yield to:

$$\mu_i(q, \delta) = \frac{M_i^q(\delta)}{\sum_{j=1}^{N(\delta)} M_j^q(\delta)} \quad (2)$$

For a multifractal measure,  $\chi(q, \delta)$  will have scaling properties (Evertsz and Mandelbrot, 1992), namely

$$\chi(q, \delta) \sim \delta^{\tau(q)} \quad (3)$$

20 Being  $\chi(q, \delta) = \sum_{j=1}^{N(\delta)} \mu_j^q(\delta)$  (4)

where  $\tau(q)$  is a nonlinear function of  $q$  (Feder, 1989). For each  $q$ ,  $\tau(q)$  may be obtained as the slope of a log-log plot of  $\chi(q, \delta)$  against  $\delta$ . A generalized dimension function  $D_q$  is then derived as (Hentschel and Procaccia, 1983):



$$D_q = \tau(q)/(1-q) \quad (5)$$

for  $q \neq 1$ . The case  $D_1$  is defined as the limit  $D_1 = \lim_{q \rightarrow 1} D_q$ . This leads to the scaling relation of entropy given by:

$$S(\delta) = -\sum_{i=1}^{n(\delta)} \mu_i \ln(\mu_i) \sim D_1 \ln(\delta) \quad (6)$$

- 5 The dimension  $D_1$ , known as entropy dimension, can then be extracted from a plot of entropy against  $\ln(\delta)$ .

### 2.8. Multiscale analysis through Relative Entropy

Given these definitions and the behaviour to expect in case of a multifractal measure, we are going to focus in the scaling properties of entropy as a tool to quantify the heterogeneity of coarse grained measure  $\mu_i(\delta)$ , or signal, derived from the transect data as it has been applied previously to black and white soil thin sections (Bird et. al., 2006).

We consider a transect of length  $L$  for a bin size  $\delta$  the entropy ( $S(\delta)$ ) is defined by equation (6). We use here a relative entropy ( $E(\delta)$ ) in order to establish what difference exists from the entropy of a uniform measure, given by

$$15 \quad E(\delta) = \sum_i \mu_i(\delta) \ln \mu_i(\delta) - \ln \frac{\delta}{L} \quad (7)$$

where the second term is the entropy of the uniform measure. Plotting this against the resolution of observation  $\delta$ , then reveals how heterogeneity in the signal evolves with increasing resolution (Tarquis et al., 2008). We may use this simple procedure to identify multiscale signals arising from the superposition of structure at different scales and assess the degree of this scale dependent structure.

- 20 Here we consider some special cases. When we increase the resolution by a factor of 2 we observe that

$$E(\delta/2) = E(\delta) + \sum_i \mu_i (p_i \ln p_i + q_i \ln q_i) \quad (8)$$

where  $p$  and  $q$  control the distribution of the measure in the finer partition and  $p + q = 1$ . Then



$$\Delta E(\delta) = E(\delta/2) - E(\delta) = \sum_i \mu_i (p_i \ln p_i + q_i \ln q_i) + \ln 2 \quad (9)$$

If  $p$  and  $q$  are independent of  $i$  then

$$\Delta E(\delta) = (p \ln p + q \ln q) + \ln 2 \quad (10)$$

This increases as the difference between  $p$  and  $q$  increases and more structure is observed in the data at  
 5 this scale. If  $p = q = 0.5$ , namely there is no structure revealed on increasing resolution then  
 $\Delta E(\delta) = 0$ .

Further if  $p$  and  $q$  are independent of  $\delta$  then we arrive at a binomial cascade. This is a multifractal  
 measure and relative entropy scales logarithmically as

$$E(\delta) = (D_1 - 1) \ln(\delta/L) \quad (11)$$

10

### 3. Results and Discussion

#### 3.1. Correlations

Studying the relation between  $GW$ ,  $PW$ ,  $GN$  and  $PN$  with the Nitrogen applied (Nap) during the melon  
 crop season we have plotted these variables without considering any spatial factor (Fig. 4). All of them  
 15 show a tendency, as we expected, to increase their values as the Nitrogen applied increases. The  
 correlation coefficient ( $r$ ) for the four variables varying from 0.66 ( $GN$  case) up to 0.76 ( $PN$  case)  
 demonstrating that there are statistically significant correlations with Nap. However, we can observe that  
 at each of the Nap values the variables show variability. This is a consequence of a set of processes  
 occurring from melon fertigation to wheat harvest, such as nitrogen uptake by the melon crop, organic  
 20 soil nitrogen mineralization, nitrogen leaching, horizontal diffusion of soluble nitrogen forms and  
 nitrogen uptake by the wheat crop (Milne et al., 2010).

#### 3.2. Generalized Dimensions

A multifractal analysis was applied to the four variables. In all the cases a  $\tau(q)$  function reflected a  
 hierarchical structure from one scale to the other with a value for  $q=1, \tau(1)=0$  indicating the conservative  
 25 character of the variables (Fig. 5A). Therefore, we estimated the  $D_q$  in an interval of  $q=\pm 4$  (Fig. 5B). The





results show a weak variation in the values around 1 highlighting the difficulty characterizing the multiscale heterogeneity applying this type of analysis. In this case, the scale dependency found across a range of scales is not strong enough to show a robust variation in  $D_q$  versus  $q$  and  $\tau(q)$  presents an almost linear trend. There are several works in soil transect data that present similar results (Caniego et al., 2005; Zeleke and Si, 2006).

### 3.3. Relative Entropy

In order to compare the spatial scaling behavior of these four variables with the Nap behavior,  $E(\delta)$  was calculated shown the results in Fig. 6A and Fig. 7. The trend in each case is not log-linear as we would expect for a pure multifractal measure. In the case of Nap the range of values achieved -0.20 (Fig. 6A), and in the rest they approach -0.06 (*GW* and *PW*) or -0.11 (*GN* and *PN*) (Fig. 7).

We have plotted for each variable (Fig. 7)  $E(\delta)$  based on  $D_1$  estimated in the above section using equation [11]. At certain scales both present the same value but at most of the scales there are variations (Fig. 7).

The increments of the  $E(\delta)$  calculated for Nap and the four variables are shown in Fig. 6B and Fig. 8 respectively. *PN*, *GW* and *PW* present a similar scaling trend, with a maximum structure revealed at scale  $\delta=10$  that correspond to a distance of 5 m. This is the same behavior found in Nap. In the case of *GW* the maximum structure is found at  $\delta=20$  (10 m) indicating that the interaction of other factors have influence in this variation and the Nap is not the main one.

All the variations of  $E(\delta)$  at the smallest scales,  $\delta=5$ , 2 and 1 (2.5, 1 and 0.5 m), show an increase giving the second maximum value for *GN*, *GW* and *PW*. This suggests that at those scales the variation is mainly due to the melon cropping as the uptake of the applied nitrogen by this crop left a lower amount of available Nitrogen for the wheat crop.

### 4. Conclusions

We have carried out a multifractal analysis on a transect data which indicates a weak multiscale structure in the four variables studied, biomass and Nitrogen content of wheat and grain. In this case, the generalized dimensions didn't give us relevant information on multiscale heterogeneity.

A relative entropy analysis was used to identify local maxima within the data structure. Grain and plant weight present a maximum structure at a scale of 5 m that correspond with Nap treatment, as well as grain



Nitrogen content. Contrary, in grain weight case the maximum structure is found at 10 m revealing that Nap is not the main factor explaining its variation.

The proposed approach provides information about scale dependencies related to factors that created spatial variability and it is complementary to the multiscale analysis.

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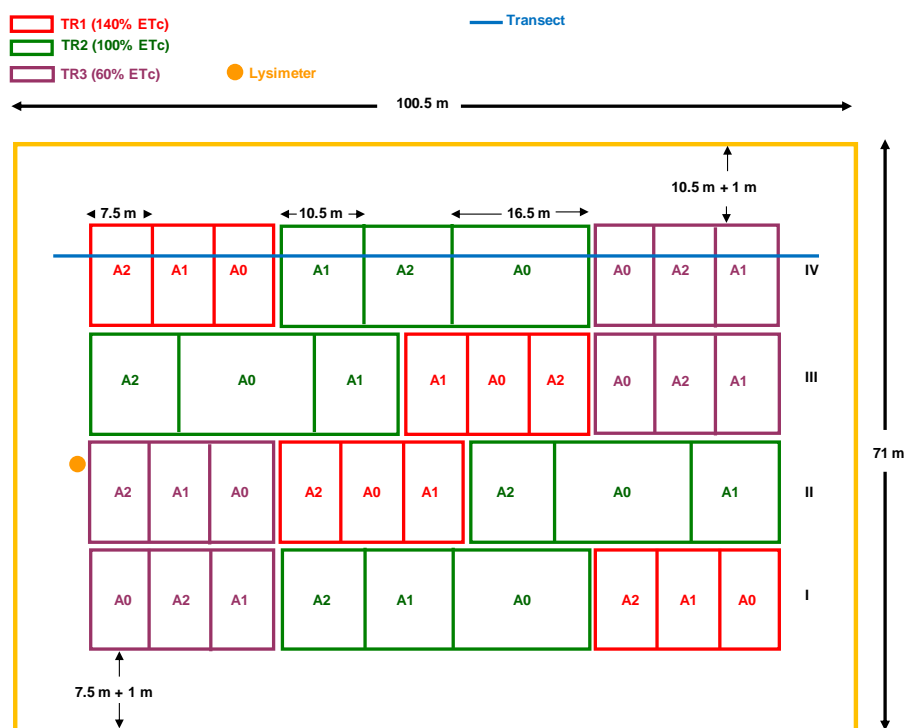
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**Table 1.** The treatments applied to the melon crop, reference evapotranspiration  $ET_0$  (15 to 104 DAT) and estimated crop evapotranspiration  $ET_c$  values (15 to 104 DAT), total irrigation (applied irrigation, taking initial establishment irrigation into account, in the different treatments: 60%  $ET_c$  (W1), 100%  $ET_c$  (W2) and 140%  $ET_c$  (W3) and applied nitrogen information. From Milne et al. (2010) with permission.

Treatment		N applied ( $\text{kg N ha}^{-1}$ )						
Irrigation	Fertilizer	$ET_0$ (mm)	$ET_c$ (mm)	Irrigation (mm)	Total Rain (mm)	Irrigation Water	Fertilizer	Total
	N0						0	55.58
W1	N1	572.12	251.81 (60%)	344.06		55.58	150	205.58
	N2						300	355.58
	N0						0	92.78
W2	N1	572.12	419.68 (100%)	554.27	19.50	92.78	150	242.78
	N2						300	392.78
	N0						0	129.46
W3	N1	572.12	587.55 (140%)	757.20		129.46	150	279.46
	N2						300	429.46

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**Figure 1:** A croquis of the experimental melon crop layout. The nine subplots of the melon crop experiment through which the wheat transect ran are shown. The wheat transect is shown by the dark green line. The fertilizer levels are shown on the figure: N0, N1, N2. represent 0, 150 and 300 kg N ha<sup>-1</sup> respectively. The three different irrigation levels are indicated by the colour of the subplot lines: light blue is W1, the light green W2, and the orange W3 corresponding to 60%, 100%, and 140% of the estimated crop evapotranspiration (Ec) respectively.

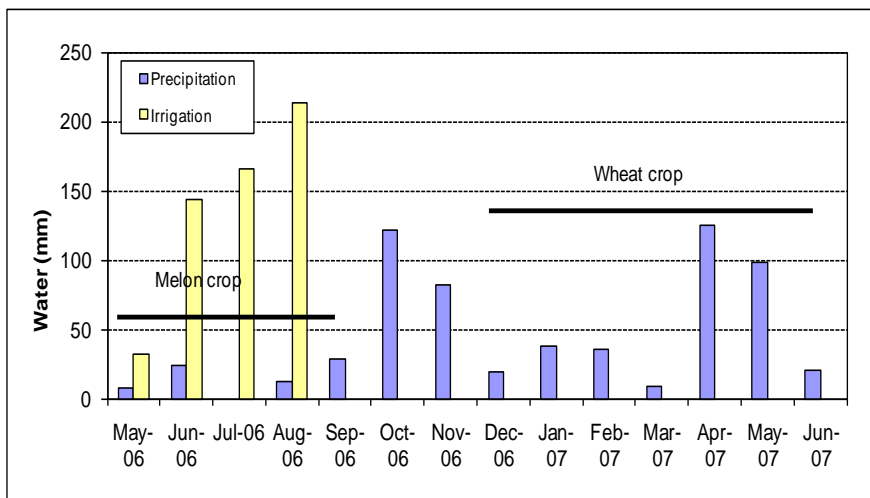
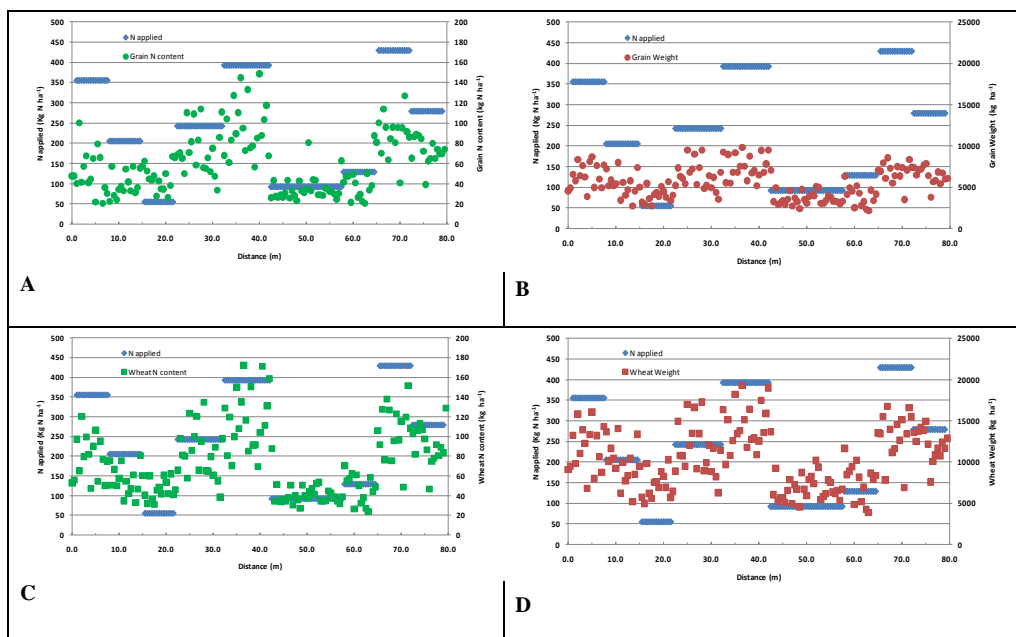


Figure 2: Monthly precipitation and irrigation applied, in mm, for melon and wheat crop.

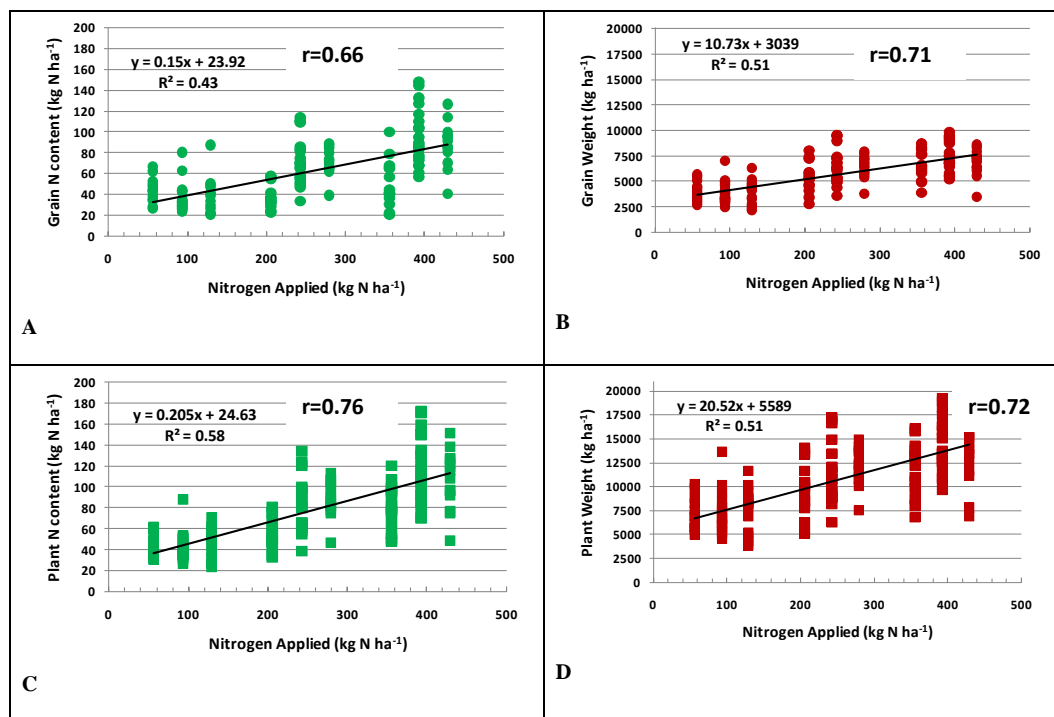
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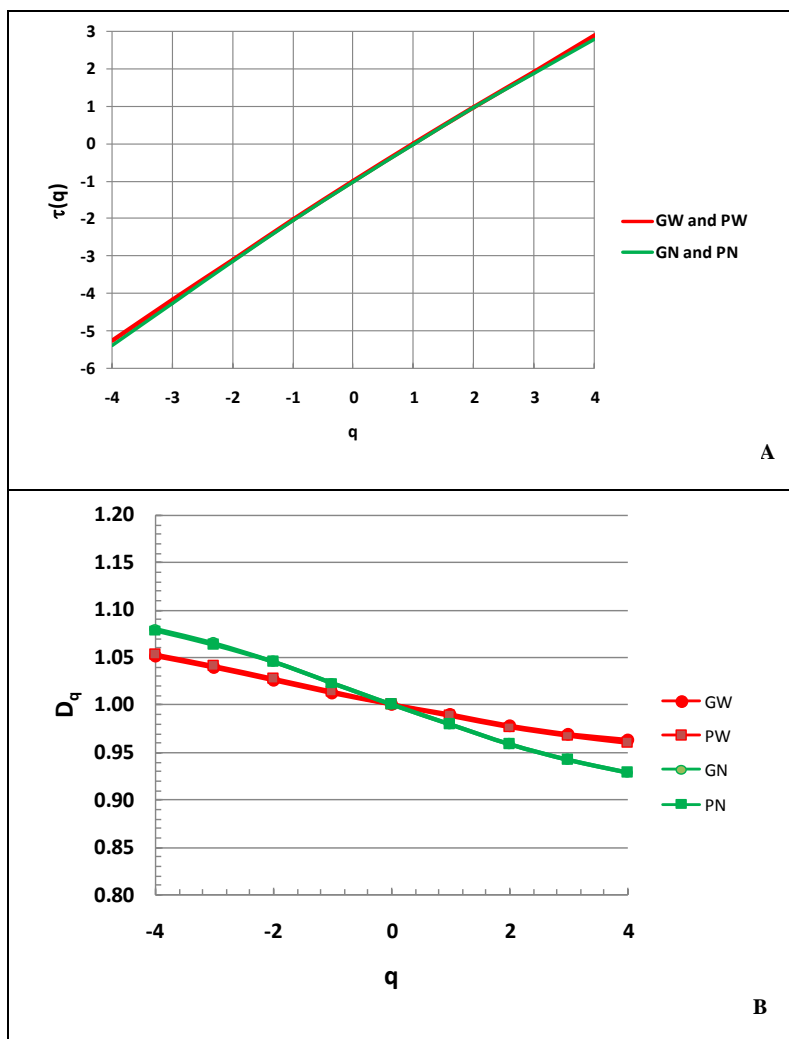
**Figure 3:** Original data of the four variables studied including the Nitrogen doses applied in the melon crop along the transect: A) Grain Nitrogen content (*GN*), B) Grain Weight (*GW*), C) Wheat Nitrogen content (*PN*) and D) Wheat Weight (*PW*).

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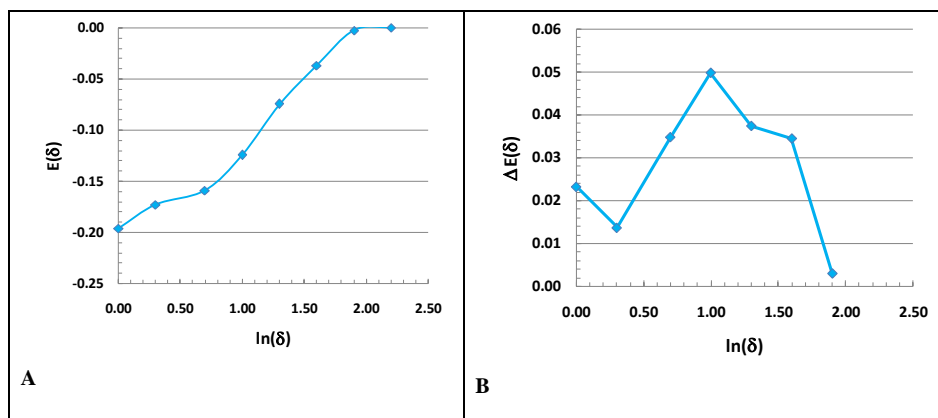




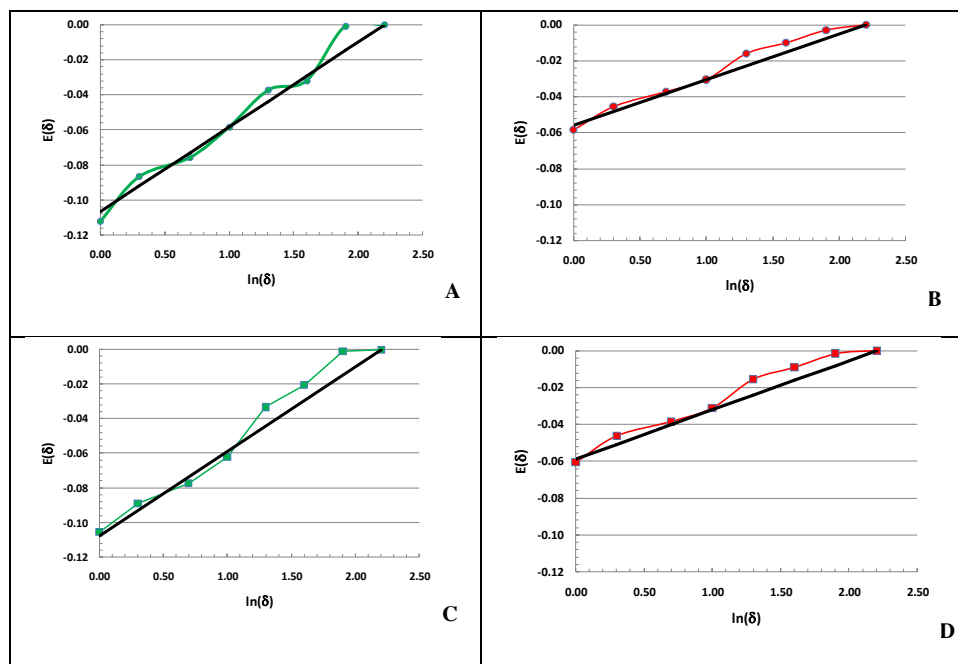
**Figure 4:** Correlations with Nitrogen applied (Nap) of each variable: A) Grain Nitrogen content (GN), B) Grain Weight (GW), C) Wheat Nitrogen content (PN) and D) Wheat Weight (PW).



**Figure 5:** Multifractal analysis of the four variables studied: A) Function  $\tau(q)$  versus  $q$ , B) derived generalized dimensions ( $D_q$ ) from  $\tau(q)$ . The plotted variables are Grain Nitrogen content (*GN*), Grain Weight (*GW*), Wheat Nitrogen content (*PN*) and Wheat Weight (*PW*).

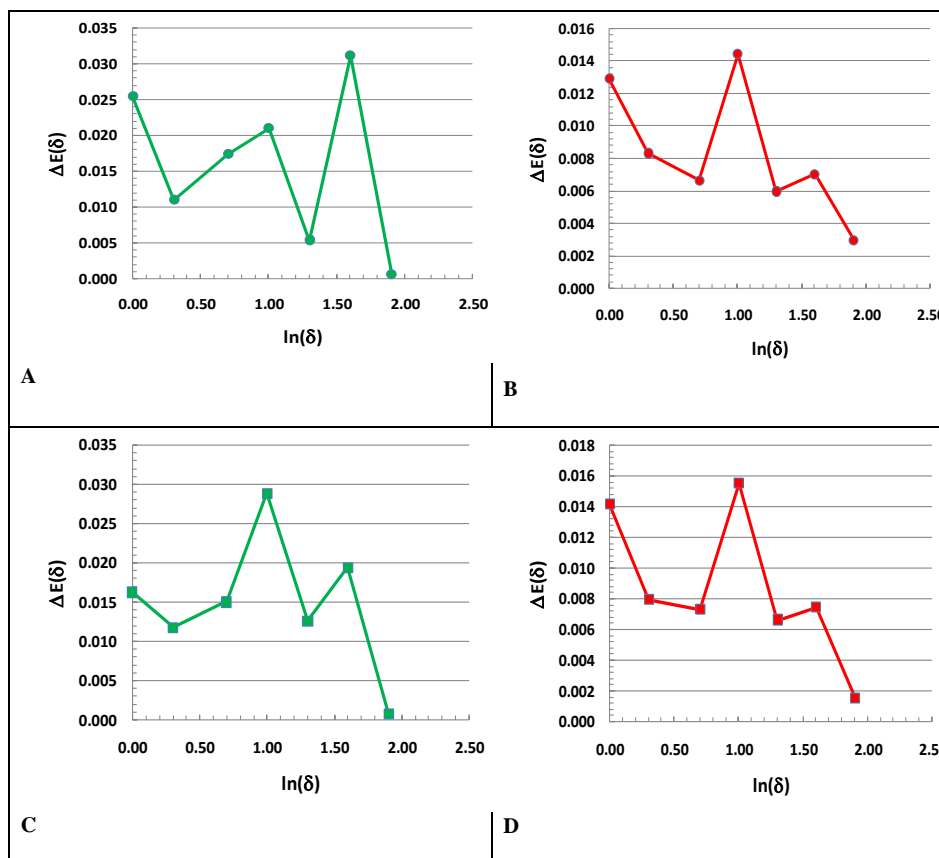


**Figure 6:** Entropy study: A) relative entropy,  $E(\delta)$ , of Nitrogen applied (Nap), B) increment of relative entropy,  $\Delta E(\delta)$ , of Nap.



**Figure 7:** Relative entropy ( $E(\delta)$ ) of: A) Grain Nitrogen content ( $GN$ ), B) Grain Weight ( $GW$ ), C) Wheat Nitrogen content ( $PN$ ) and D) Wheat Weight ( $PW$ ). Black lines represents  $E(\delta)$  based on entropy dimension ( $D_1$ ) of each variable.

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**Figure 8:** Increment of relative entropy ( $\Delta E(\delta)$ ) of: A) Grain Nitrogen content (GN), B) Grain Weight (GW), C) Wheat Nitrogen content (PN) and D) Wheat Weight (PW).