Authors' Response to Referee

May 27, 2016

Overall

Thanks for the suggestions from the referee. The paper will be better organized after we incorporate the modification based on the comment of the referee. The essential details will be added in the final version.

Responses to Specific Comment

5. Page 2, line 16. Ls is not properly defined. The authors should explain a bit more precisely what it is meant by "critical threshold" ?

Response: In Whartenby et al's paper and this paper, different dimensions of the observation have been tested to achieve the accurate prediction. When the dimension is larger than the threshold Ls, the prediction quality is reasonably acceptable, and the root mean square error (RMSE) can be reduced to a relative low order. However, for any dimensions smaller than Ls, the RMSE increases abruptly, which indicates the approach to the accurate prediction fails.

(Whartenby, W., J. Quinn, and H. D. I. Abarbanel, "The Number of Required Observations in Data Assimilation for a Shallow Water Flow," 20 Monthly Weather Review 141, 2502-2518, (2013).

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6. Page 3, line 5. Notation or text must be improved. Is $\mathbf{y}$ a L-dimension vector?

Response: $\mathbf{y}$ is an L dimensional vector, which is constructed by the measurement on the L grids.

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7. Page 3, line 13. The equation $y_l(t) = x_l(t) + noise$ suggests that $L = D$ which is not the case in your experiments, and the fact that $L << D$ is indeed one of your key point. You should say that you use an operator H that only observes a portion of the state-vector (mainly the heights in the experiments that follow).

Response: As indicated in the paper, the index l is in the range of $\{1, 2, ...L\}$. L is the dimension of the observation state. The operator H is an identical matrix which is omitted.

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10. Page 7, line 15. How is Ls obtained? Does it come from the simple nudging case Eq.(2) ?

Response: Ls is the critical threshold of the simple nudging method. When the dimension of observation L bigger than Ls, the synchronization will be achieved. As a benchmark of the traditional method, it has been compared with the sufficient dimension L in this study.

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12. Page 8, line 20. The sentence about parameter estimation relates to the chosen " perfect model" scenario. It would be better if the author states this clearly.

Response: Perfect model scenario means the model is error free ($R_f = 0$). Only the observation error is considered in the study here.

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13. Page 8, line 22-24. You might want to say something more on this regard. How is it optimized? What does it mean average mutual information? Is it the time decorrelation scale?

Response: Average mutual information in the delay embedding theorem is used for determining the embedding delay parameter. From the average mutual information, the independent coordinate can be constructed* **.

* Abarbanel, Henry D. I., Analysis of Observed Chaotic Data, Springer-New York (1996)
** Rey, Daniel, Michael Eldridge, Mark Kostuk, Henry D. I. Abarbanel, Jan Schumann-Bischoff, and Ulrich Parlitz, "Accurate State and Parameter Estimation in Nonlinear Systems with Sparse Observations", Physics Letters A 378, 869-873.(2014)..

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14. Page 9, line 4-6. Are you computing the error using only the observed components of the state vector also for $t > T$?

Response: In this paragraph, the error in the prediction window ($t > T$) is computed by the observed components (h). In the following analysis, the error is computed by the unobserved components (u, v).
Since all information (observed and unobserved variables) is known in the twin experiment, The errors of the unobserved components are also the criteria of a valid assimilation.

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16. Page 10, line 5-6. Do you have suggestions on how to select the coupling?

Response: The coupling strength is tested by a continuous increment. The results displayed here are compared with the results g = 0.5 and g = 1.5. As discussed above, when the conditional Lyapunov exponent is negative, the synchronization will be achieved.

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17. Page 10, line 11-13. In fact the values chosen for L are very close to each other and results highlight a strong sensitivity of your method to this (error diverge when L = 248). Can you comment more on this? Another aspect regards how those observations are placed. One can always achieve a better control of the error by a proper deployment of the observations (possibly with the use of target observations). What would it happen if observations were denser in the proximity of the most dynamically active areas?

Response: In this study, the variations of variables (after normalization) at different grids are in the same order. There is no "more dynamically active areas" in this simulation. Moreover, the observation locations are randomly chosen on the grids. As the referee's comment, it's the fact that a proper deployment of observations will induce a higher prediction quantity. This may be included in our future study.

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18. (2) the conclusion relating the model resolution and observations network is unclear. Even with a high resolution model one may still necessitate a growing number of observations to keep under control the unstable modes. Please clarify this point.

Response: In the study, the resolutions 16x16, 32x32 and 64x64 were tested, we found the proportion of the necessary observation L to the dimension of the system D will reduce in some systems with higher resolutions. While, the conclusion that the necessary dimension of time delayed nudging method will be much less than the one in the traditional nudging method always holds.

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19 (3) In the experiments so far you have only observed h, why are you then showing how observational noise is simulated for the velocities if the latter are not assimilated? (4) What are the values of C_{height} and C_{data} and how are they chosen? We do not know how these values scales with respect.

Response: (3) Those are not observational noise for velocities. This is the initialization error of the stream function. (4) C_{height} and C_{data} are empirical choices which are in the rational scale of observation error. The assimilation results are not very sensitive to the choices of these parameters.

General Technical Corrections

Other questions focus on the rephrasing, referencing and formatting. We will make the changes and polish the paper in the revision of our paper. Thanks for the thorough reading and detailed suggestions.