Supplemental material of: Detecting and tracking eddies in oceanic flow fields: A vorticity based Euler-Lagrangian method

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S1 Seeded eddy model

The model is inspired by the seeded eddy model of a turbulent flow described in Abraham (1998) and the model of Jung et al. (1993). The model consist of n eddies and a background flow u_0 in positive x-direction.

The stream function for an eddy field with n eddies is given by

$$\psi(\boldsymbol{x},t) = \sum_{i=1}^{n} (-1)^{m_i} \cdot w_i \cdot \exp(-\kappa_i((x-x_i)^2 + \alpha(y-y_i)^2)) + u_0 \cdot x \tag{1}$$

where $\kappa_i^{-1/2}$ is linked to the radius of eddy i with vortex strength w_i. The eddy i emerges at position $(x_i \quad y_i)$. The radii of the eddies are drawn from a distribution of 1000000 eddies possessing a power law $1/r^3$. The distribution is normalized and allows eddies between $R_{min} = 10$ km and $R_{max} = 50$ km. Because we are interested in mesoscale eddies (15-25 km radii) we first draw a sample of 100 radii from the distribution and then pick randomly 10 sample radii that are between 15 km and 25 km. The position of these eddies is drawn from an uniform distribution. Half of the eddies rotates counter-clockwise and the other half clockwise. The factor m_i decides if the eddy rotates clockwise or counter-clockwise. If m_i is even the eddy rotates clockwise if m_i is odd the eddy rotates counter-clockwise.

The model is parametrized such that it yields a dimensionless model. Lengths are measured in units of the maximal eddy radius of the mesoscale eddies and time is in units of the lifetime of an eddy which is the same for all eddies. The parametrization is given in Table S1.

S2 The hydrodynamical model of a vortex street

The hydrodynamical model of the vortex street is an adapted version of the model by Jung et al. (1993). Jung et al. (1993) models the flow in a channel behind a cylinder in the middle of the channel (cf. Jung et al. (1993) and Sandulescu et al. (2006) for details).

Table S1. Dimensional and dimensionless parameters of the seeded eddy model

Parameter	r	T_c	α	κ	u_0	w
dimensional	25 km	30 d	1	$1/r^2$	$0.18\,\mathrm{ms}^{-1}$	$55 \cdot 10^3\,\mathrm{m}^2\mathrm{s}^{-1}$
dimensionless	1	1	1	1	18.66	200

Table S2. Dimensional and dimensionless parameters of the hydrodynamical model of a vortex street

Parameter	r	T_c	α	κ	L	u_0	y_0	w
dimensional	25 km	30 d	1	$1/r^2$	150 km	$0.18\,\mathrm{ms}^{-1}$	12.5 km	$55 \cdot 10^3\,\mathrm{m}^2\mathrm{s}^{-1}$
dimensionless	1	1	1	1	6	18.66	0.5	200

Here this model is modified in such a way that two counter rotating eddies that develop at times t and $t + T_c/2$ at position $(1, y_0)$ and $(1, -y_0)$ respectively, travel a distance L in positive x-direction within their lifetime T_c and fade out. The model is artificial because the impact of the cylinder on the eddy formation and its shading are neglected here making the eddy formation non-physical out of the blue. However, since all quantities to be estimated by the eddy tracking tool are then given analytically, makes this artificial model an ideal test bed for numerics.
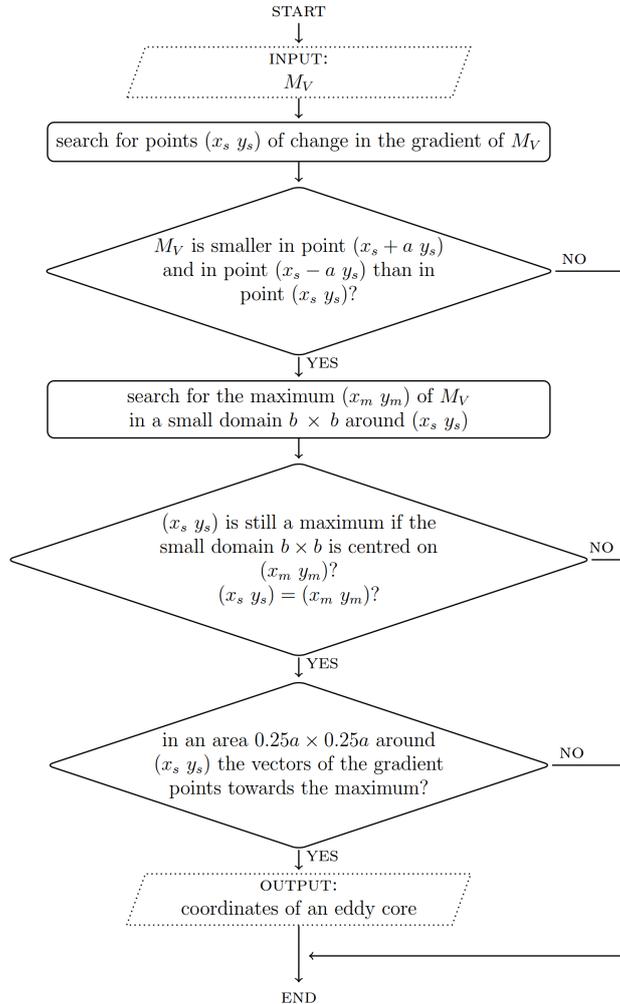
Hence, the model is simplified as follows:

$$\Psi(x,y,t) = -w h_1(t) g_1(x,y,t) + w h_2(t) g_2(x,y,t) + u_0 y. \tag{2}$$

The first two terms describe the life cycle of the eddies of opposite sense of rotation. The vortex strength is given by w. The dynamics of the eddy evolution is modelled as the modulation of the amplitudes by the function $h_1(t) = |\sin(\pi t/T_c)|$ resp. $h_2(t) = h_1(t - T_c/2)$. The function $g_i(x,y,t) = \exp(-\kappa_0((x - x_i(t))^2 + \alpha(y - y_i(t))^2))$ with $i = 1,2$ models the Gaussian shaped effect of the eddies on the stream function. The movement of the eddy centres is expressed by $x_1(t) = 1 + L(t/T_c$ mod $1)$, $x_2(t) = x_1(t - T_c/2)$ and $y_1(t) = y_0 = -y_2(t)$. (In case of type 3 noise, the noise is added to the y_1 and y_2 terms.) The factor $\kappa_0^{-1/2}$ is the radius and determined as a characteristic linear size of the eddies and α is the ratio between the elongations of the eddy in x and y direction. In our case α is set to 1 (circular eddies). The last term describes the background flow with the velocity u_0.

The parametrization of the flow is chosen as in Sandulescu et al. (2006). Lengths are measured in units of the eddy radius r and time in units of the lifetime T_c of an eddy. The dimensional and dimensionless parameters are given in Table S2. The vortex strength is furthermore varied to study its impact on the eddy evolution.
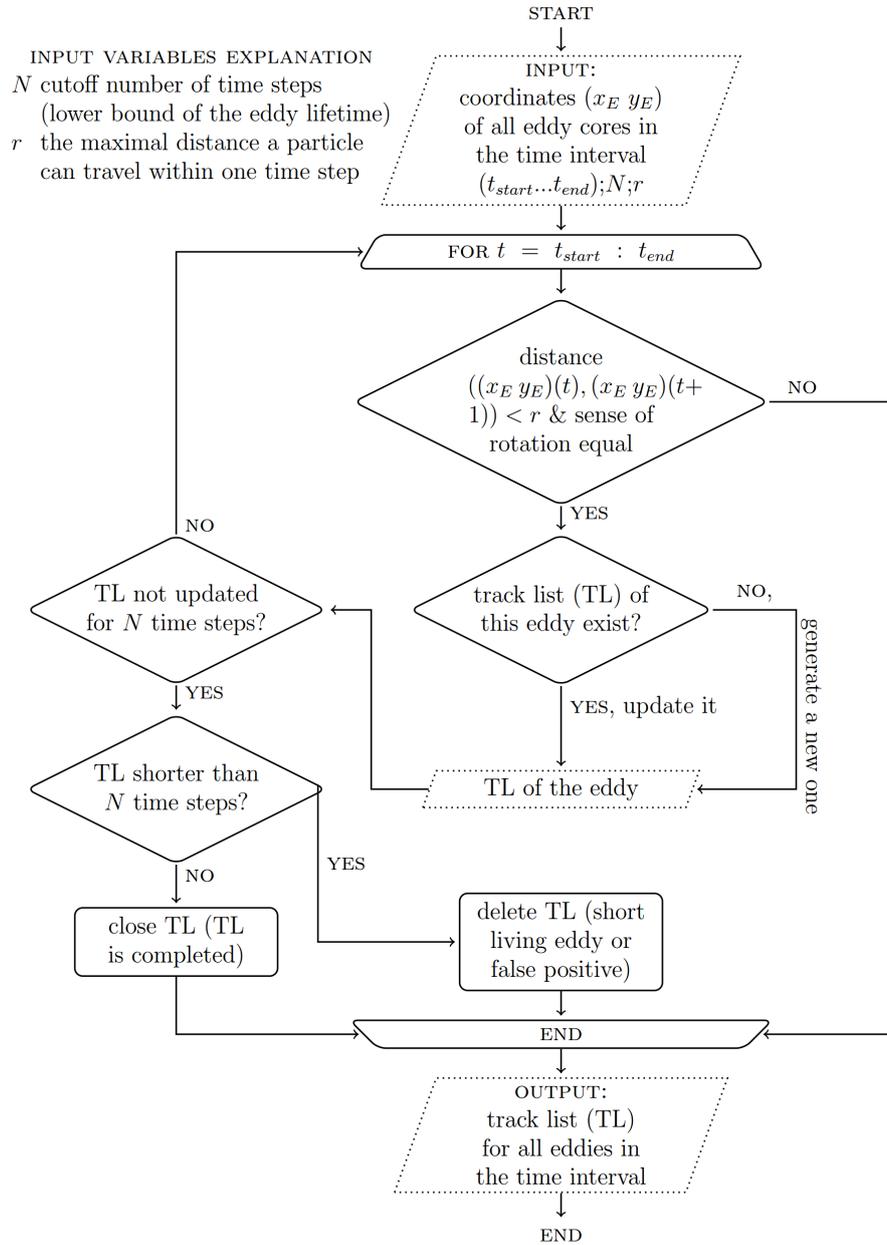
START

INPUT:
M_V

search for points $(x_s\ y_s)$ of change in the gradient of M_V

M_V is smaller in point $(x_s + a\ y_s)$ and in point $(x_s - a\ y_s)$ than in point $(x_s\ y_s)$? — NO

YES

search for the maximum $(x_m\ y_m)$ of M_V in a small domain $b \times b$ around $(x_s\ y_s)$

$(x_s\ y_s)$ is still a maximum if the small domain $b \times b$ is centred on $(x_m\ y_m)$? $(x_s\ y_s) = (x_m\ y_m)$? — NO

YES

in an area $0.25a \times 0.25a$ around $(x_s\ y_s)$ the vectors of the gradient points towards the maximum? — NO

YES

OUTPUT:
coordinates of an eddy core

END

Figure S1. Schematic sketch of the eddy detection algorithm based on Euler-Lagrangian descriptor M_V.

S3   Algorithm of the eddy tracking

The key ideas of the algorithm (eddy detection, eddy tracking and eddy shape) inspired by the eddy tracking package by Nencioli et al. (2010) are schematically presented in this section.

Fig. S1 presents the concept of eddy detection. Similar algorithms can be constructed for the modulus of the vorticity and for the Okubo-Weiss criterion (with respect to the fact that eddy cores are minima in case of Okubo-Weiss).
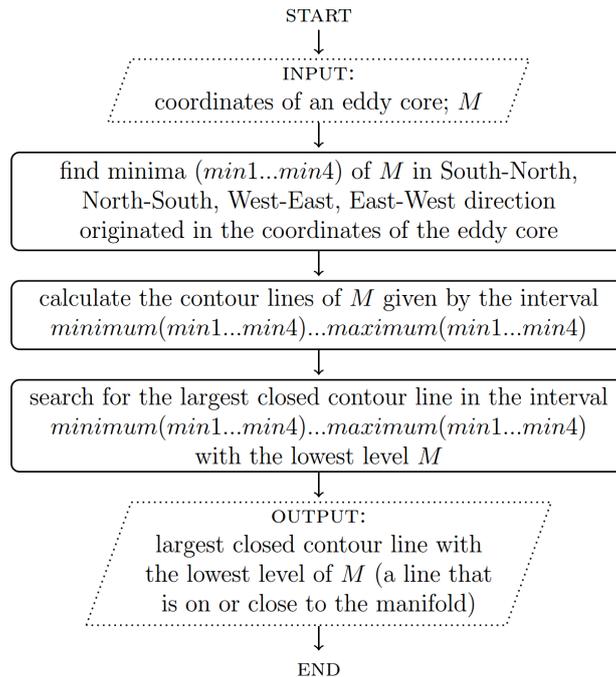
Fig. S2 shows idea of the eddy tracking. Fig. S3 presents the eddy shape detection based on the combination of the Euler-Lagrangian descriptor M_V and the Lagrangian descriptor M.

START

INPUT VARIABLES EXPLANATION

N cutoff number of time steps
(lower bound of the eddy lifetime)
r the maximal distance a particle
can travel within one time step

INPUT:
coordinates $(x_E\ y_E)$
of all eddy cores in
the time interval
$(t_{start}...t_{end}); N; r$

FOR $t = t_{start} : t_{end}$

distance
$((x_E\ y_E)(t), (x_E\ y_E)(t+1)) < r$ & sense of
rotation equal

NO

YES

NO — TL not updated for N time steps?

track list (TL) of this eddy exist?

NO, generate a new one

YES

YES, update it

TL shorter than N time steps?

TL of the eddy

NO

YES

close TL (TL is completed)

delete TL (short living eddy or false positive)

END

OUTPUT:
track list (TL)
for all eddies in
the time interval

END

Figure S2. Schematic sketch of the eddy tracking algorithm based on Euler-Lagrangian descriptor M_V.

References

Abraham, E. R.: The generation of plankton patchiness by turbulent stirring, Nature, 391, 577–580, 1998.

Jung, C., Tél, T., and Ziemniak, E.: Application of scattering chaos to particle transport in a hydrodynamical flow, Chaos, 3, 555–568, 1993.

START

↓

INPUT:
coordinates of an eddy core; M

↓

find minima $(min1...min4)$ of M in South-North,
North-South, West-East, East-West direction
originated in the coordinates of the eddy core

↓

calculate the contour lines of M given by the interval
$minimum(min1...min4)...maximum(min1...min4)$

↓

search for the largest closed contour line in the interval
$minimum(min1...min4)...maximum(min1...min4)$
with the lowest level M

↓

OUTPUT:
largest closed contour line with
the lowest level of M (a line that
is on or close to the manifold)

↓

END

Figure S3. Schematic sketch of the eddy shape algorithm based on the combination of the Euler-Lagrangian descriptor M_V and the Lagrangian descriptor M.

Nencioli, F., Dong, C., Dickey, T., Washburn, L., and McWilliams, J. C.: A Vector Geometry-Based Eddy Detection Algorithm and Its Application to a High-Resolution Numerical Model Product and High-Frequency Radar Surface Velocities in the Southern California Bight, J Atmos Ocean Tech, 27, 564–579, 2010.

Sandulescu, M., Hernández-García, E., López, C., and Feudel, U.: Kinematic studies of transport across an island wake, with application to

5   Canary islands, Tellus A, 58, 605–615, 2006.